The Pricing of Idiosyncratic Cash Flow Risk with Heterogeneous Beliefs

Michael Gallmeyer* Hogyu Jhang† Hwagyun Kim‡

Current Draft: July 2021

ABSTRACT

This paper investigates an exchange economy in continuous time, where investors perceive idiosyncratic cash flow risks through subjective expectation on firms' cash flows to impact equilibrium quantities. We empirically document that cash flow fluctuations are mainly idiosyncratic, and stocks with higher idiosyncratic cash flow risks have the higher belief dispersion and larger book-to-market ratios. This mechanism helps explain a data-consistent equilibrium cross section. The model implies that assets with higher cash flow volatilities and belief differences have higher expected returns, producing the value premium. A growth premium prevails without belief dispersion. Furthermore, the model can generate a downward-sloping equity term structure.

Keywords: Belief difference, Idiosyncratic cash flow risk, Value premium, Equity term structure, General equilibrium

JEL classification: G00, G02, G10, G11, G12

We would like to thank Hongjun Yan, Daniel Andrei, Robert Ready, Ngoc-Khanh Tran, John Wei, Qi Zeng, Wei Yang, Byeong-Je An, Burton Hollifield, Tano Santos, Lu Zhang, Motohiro Yogo, Jeff Pontiff, Dana Kiku, Ilan Cooper, Paul Gao, Bing Han, Jason Chen, Yong Chen, Emmanuel Alanis, Junghoon Lee, Jungwook Kim, and seminar participants at the North American Summer Meeting of the Econometric Society, China International Conference in Finance, European Finance Association Annual Meeting, North American Winter Meeting of the Econometric Society, Finance Down Under, Financial Intermediate Research Society, FMA Asian Meeting, Econometric Society World Congress, KFMA Joint Finance Conference, Texas A&M University, HKUST, KAIST College of Business, University of Exeter ,Georgia Tech, Seoul National University CBA, CNU, SKKU, KDI, and numerous other places for their helpful comments and discussions. Jhang acknowledges the financial support from IREC, The Institute of Finance and Banking, at Seoul National University. We would also like to thank Kimberley Berdy for editorial assistance.

*The McIntire School of Commerce, University of Virginia, mgallmeyer@virginia.edu.

†School of Business, Chungnam National University, jhogyu@gmail.com.

‡Mays Business School, Texas A&M University, hagenkim@tamu.edu.

I. Introduction

In this paper, we study a continuous-time asset pricing model that features heterogeneous beliefs regarding idiosyncratic cash flows of firms. The model produces both time series and cross section of stock returns consistent with data. This task is of critical importance in developing a workhorse model of asset pricing, and the value premium anomaly illustrated in Figure 1, is a key issue in this enterprise for the following reasons.

[Insert Figure 1]

The cash flow risk, defined as the covariance between a firm's cash flow and the aggregate cash flow, plays an instrumental role in explaining the cross section of stock returns, as emphasized in the literature.[1] When prototypical asset pricing models produce a cross-sectional return variation associated with cash flows, a puzzling feature arises in that these models predict a growth premium that is opposite to the empirically observed value premium. Simply put, value stocks have a smaller risk premium in light of discount risk than growth stocks, because value (growth) stocks tend to have the shorter (longer) cash flow duration. Thus, economic models that explain the time series properties of market returns struggle to match empirical cross-sectional return variations, as pointed out by Lettau and Wachter (2007) and Santos and Veronesi (2010). Related, Binsbergen, Brandt, and Koijen (2012) show that widely used, standard asset pricing models such as Bansal and Yaron (2004), Campbell and Cochrane (1999), and Gabaix (2012) generate either upward sloping or flat equity term structures while the data show that the historical equity term structure is downward sloping.

To justify the value premium, Lettau and Wachter (2007, 2011) let discount effect not translate into the cross section. Thus, while investors do not fear stocks with long duration cash flows that are subject to discount effect, they perceive stocks with short duration (value

[1] Abel (1999), Da (2009), and others study theoretical aspects. Bansal, Dittmar, and Lundblad (2005), Santos and Veronesi (2006), Yang (2007), Cohen, Polk, and Vuolteenaho (2009), Campbell, Polk, and Vuolteenaho (2010), and many others study empirical aspects of cash flow risks in the cross section of stock returns. Notably several papers have tried to link cash flow risk and cash flow duration to cross-sectional return variation. For instance, Campbell and Vuolteenaho (2004), Bansal, Dittmar, and Lundblad (2005), Kiku (2007), Hansen, Heaton, and Li (2008), Zhang (2005), Lettau and Wachter (2007), Da (2009), Santos and Veronesi (2010), and Choi, Johnson, Kim, and Nam (2013) develop structural models that directly associate cash flow risk or cash flow duration with book-to-market ratios and expected stock returns to this end.

stocks) as riskier, because those assets interact with front-loaded cash flows; hence, value stocks have higher expected returns. Alternatively, under the circumstance that growth stocks have higher expected returns due to strong discount effect, Santos and Veronesi (2010) counterfactually magnify the exposure to aggregate cash flow risk especially for value stocks: since cash flow risk effect dominates discount effect in the cross section, it can undo the growth premium.

To motivate our study, Table I displays the snapshots of cash flow risk reflected in the decile portfolios sorted by the book-to-market ratio. The first row shows the overall sizes of cash flow risk, proxied by the variance of dividend growth in each portfolio, the second row refers to the covariances between individual dividend growth and the aggregate dividend growth, and the third row reports sample variance of the residuals of individual dividend growth netting out the effect from the aggregate dividend growth. The last row reports analysts' earnings forecast dispersion of the decile portfolio firms using the I/B/E/S data set.

[Insert Table I]

Table I states that the most of cash flow fluctuations come from the idiosyncratic component, and the systematic risk exposures appear to be quite small to produce the data-consistent value premium. Plus, value stocks have much higher idiosyncratic risk of cash flow than growth stocks, and data tell that the dispersion in analyst forecasts on earnings for the value firms are markedly higher. Previous studies find that value firms are subject to higher distress risk and less room to grow. Taken together, it appears logical and reasonable to model belief differences on firms' idiosyncratic cash flow risks.

In this vein, we incorporate investors' different beliefs into individual cash flow dynamics with the following features. First, we model both aggregate and individual cash flow processes consistently. For the aggregate cash flow process, we use an exogenous model that is impacted by aggregate risk only. However, an individual firm's cash flow process is subject to idiosyncratic risk in addition to aggregate risk. The dynamics of the cash flow processes (including idiosyncratic risk exposure) implies that our model is constructed such that in the aggregate, idiosyncratic cash flow risks cancel out via quantities.

Second, we introduce investors' heterogeneous beliefs into cash flow processes. The key assumption is that investors have different opinions on the long-run mean of the cash flow share process with respect to firm- or asset-specific risk.[2] This differs from the most of the heterogeneous beliefs literature where investors update their perceptions of the drift of underlying processes through aggregate risk. Since the market offers a sufficient number of assets to trade on these belief differences, idiosyncratic cash flow risk is priced in equilibrium through belief differences. As a result, individual expected stock returns are *positively* affected by idiosyncratic cash flow risk through belief differences, which is one of our main results.[3] The higher the belief difference, the stronger the effect of idiosyncratic cash flow risk on equilibrium stock returns. Individual equilibrium quantities are affected by the model's state variables that capture cash flows, investor preferences, and belief differences. Cross-sectional return variations result from different exposures to these variables. Thus, the theory connects firm characteristics and related investor behaviors to the cross-section of stock returns. Quantitative study of our model reveals that the cross-sectional return variation is largely attributed to the pricing of idiosyncratic cash flow risk in equilibrium. When belief differences are modeled in this way, we show that sorting assets based on price-to-fundamental ratios endogenously picks up stocks with higher (idiosyncratic) cash flow risk and higher degrees of belief differences in the cross section leading to the value premium. Simulation experiments further support this claim in that our model produces a growth premium if idiosyncratic cash flow risk is not priced when belief difference is turned off, illustrating the importance of the belief difference channel.

The pricing of idiosyncratic cash flow risk through belief differences also helps explain the downward sloping equity term structure, shown by Binsbergen, Brandt, and Koijen (2012). To investigate the shape of equity term structure in our model, we compute the cash flow duration using both a direct accounting method following Dechow, Sloan, and Soliman (2004) and an indirect method using the price elasticity with respect to the cash flow. We find two distinctive features. First, regardless of the measures, value (growth)

[2] This assumption is similar to Basak (2000) where investors have different beliefs about an extraneous process unrelated to economic fundamentals. Similarly, Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2017) study how inflation disagreement can impact real quantities and prices.

[3] Babenko, Boguth, and Tserlukevich (2016) show that in an option-theoretic setting, idiosyncratic cash flow risk negatively affects equilibrium stock returns. High idiosyncratic cash flow shocks positively affect a firm's profit, which in turn increases the firm size. When the firm size increases, the price of risk, measured by the CAPM beta, decreases so that the expected excess return decreases. On the other hand, Cochrane, Longstaff, and Santa-Clara (2008), Martin (2013), Choi, Johnson, Kim, and Nam (2013), and Choi and Kim (2014) show that in an equilibrium setting, larger idiosyncratic cash flows can increase the market price of risk because of under-diversification.

stocks have the shorter (longer) cash flow duration in our model. Second, assets with the shorter (longer) cash flow duration have the higher (lower) expected returns since they have higher (lower) idiosyncratic cash flow risk and larger (smaller) belief dispersion, featuring a downward sloping equity term structure. Therefore, our economic mechanism by which the value premium arises generates a downward sloping equity term structure as well.

The paper is organized as follows. Section 2 introduces our model. Section 3 provides the economics of cash flow characteristics as well as the reason for high (low) belief difference for value (growth) firms. Section 4 derives equilibrium and discusses rich implications of the pricing of idiosyncratic cash flow risk via investors' different beliefs. Section 5 shows and discusses equity term structure. We provide proofs in appendix. Online appendix provides data descriptions, and other supporting materials.

II. The Model

This section spells out the model, with preferences of investors, characteristics of cash flows, and beliefs.

A. *Investor Preference*

The model assumes that investors hold a constant relative risk aversion utility function with an external habit. Risk aversion is identical across investors for simplicity. Investor k's utility function is

$$u(c_k(t)) = \frac{1}{1-\gamma}\left(\frac{c_k(t)}{X(t)}\right)^{1-\gamma}, \quad k = 1, 2. \tag{II.1}$$

where $X(t)$ represents a ratio habit as in Abel (1989). The habit process X follows the form of Constantinides (1990), Detemple and Zapatero (1991), and Santos and Veronesi (2010):

$$X_t \equiv \delta \int_0^t e^{-\delta(t-\tau)} D_\tau d\tau, \tag{II.2}$$

where D_t is the aggregate cash flow. In particular, we use Santos and Veronesi (2010) by defining the mean-reverting process, $H_t \equiv (D_t/X_t)^{(1-\gamma)}$ with dynamics

$$dH_t = h_1(\bar{H} - H_t)dt + h_2 H_t dB_A(t), \tag{II.3}$$

where $h_1 > 0$, $h_2 > 0$.

B. Endowment

The aggregate cash flow is given by

$$\frac{dD_t}{D_t} = \mu_D dt + \sigma_{D,A} dB_A(t), \tag{II.4}$$

where $dB_A(t)$ represents aggregate Brownian risk. For simplicity, we assume that there are two sectors or trees, called ″stocks″ in our setting that are claims to distinct cash flows. For the specification of the tree process, we follow Menzly, Santos, and Veronesi (2004) by taking the cash flow share process as exogenous to describe the relative movement of individual cash flow processes in the economy.

Assumption 1. *The share, s_t, is defined as the individual cash flow ($\equiv D_s(t)$) divided by the aggregate cash flow ($\equiv D(t)$). The process s_t follows*

$$ds_t = \phi_s(\bar{s} - s_t)dt + s_t \boldsymbol{\sigma}(s_t) d\boldsymbol{B}_t', \quad \phi_s > 0, \tag{II.5}$$

where

$$\begin{aligned} &\boldsymbol{\sigma}(s_t) \equiv (\sigma_{s,A}(t), \sigma_{s,I}(t)), \\ &\sigma_{s,j} \equiv v_{s,j} - s_t v_{s,j} - (1 - s_t) v_{(1-s),j}, \quad j = A, I \\ &d\boldsymbol{B}_t \equiv (dB_A(t), dB_I(t)), \\ &\phi_s > 0, \end{aligned} \tag{II.6}$$

where ϕ_s is the rate of mean reversion, $\bar{s}$ is the long run mean of the share of the asset under consideration, $B_A(t)$ and $B_I(t)$ represent the aggregate Brownian risk and the idiosyncratic Brownian risk respectively, and $v_{s,j}$ and $v_{(1-s),j}$ are the diffusion coefficients of individual *assets with share s_t and share $(1 - s_t)$.*

Assumption 1 states that a single asset cannot dominate the entire market in the long run as shown in Menzly, Santos, and Veronesi (2004). This stationarity enables us to analyze the cross section of stock returns in the long run. This specification is a simple version of Menzly, Santos, and Veronesi (2004) and Santos and Veronesi (2010), as we only consider two stocks, driven by two risk exposures, i.e., aggregate risk and idiosyncratic risk to define an individual cash flow process. An online appendix provides the details of the individual cash flow share process. Given s_t, the remaining process $(1 - s_t)$ is naturally defined as it should make up the total market - market's cash flow share is 1 - together with s_t. Thus, the process $(1 - s_t)$ is rather a redundant share process. In other words, when we introduce assets in the market, we only need to consider the market and an asset with the share s_t as risky assets since the asset with $(1 - s_t)$ is naturally defined. We use this observation in the later section II.D and II.E.

C. *Belief Difference*

This subsection introduces heterogeneous beliefs associated with cash flow share processes. The importance of modeling investors' heterogeneous beliefs is emphasized early by Lintner (1965), Miller (1977), and Harrison and Kreps (1978). Later work studied the impact of economic agents' different beliefs about underlying fundamental economic processes on equilibrium quantities. Detemple and Murthy (1994) study the effect of belief differences in a production economy. For exchange economies, key contributions include Zapatero (1998), Basak (2000), Basak (2005), Jouini and Napp (2007), Gallmeyer and Hollifield (2008), David (2008), Dumas, Kurshev, and Uppal (2009), Weinbaum (2009) and Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2017). Similarly, we assume that investors have different beliefs about the fundamental, i.e., the long-run mean of the cash flow share $(\bar{s})$, which can be viewed as a measure of growth potentials. The following assumption formalizes.

Assumption 2. *Investors receive the same information about the underlying cash flow processes including both aggregate and individual cash flow processes, but agree to disagree about the long run mean of individual cash flow share processes, $\bar{s}^{(k)}$. This induces investor-specific* *idiosyncratic risk shocks, $B_I^{(k)}$. We write investors' perceived share process as follows:*

$$\frac{ds_t}{s_t} = \phi_s\left(\frac{\bar{s}^{(k)}}{s_t} - 1\right)dt + \sigma_{s,A}(t)dB_A(t) + \sigma_{s,I}(t)dB_I^{(k)}(t), \tag{II.7}$$

where $k = 1, 2$ refers to the individual investors. Then, the innovation process $B_I(t)$ is given as

$$dB_I^{(k)}(t) \equiv \eta_t^{(k)}dt + dB_I(t), \;\; \text{where } \eta_t^{(k)} \equiv \frac{\phi_s(\bar{s} - \bar{s}^{(k)})}{\sigma_{s,I}(t)s_t}.^4 \tag{II.8}$$

Note that $\eta_t^{(k)}$ measures the difference between the true long run mean of the share and k-th investor's perceived long run mean of the share.

A theoretical foundation for the "agree-to-disagree" assumption appears in Varian (1985), Harris and Raviv (1993), Morris (1994), and Morris (1996). Although Assumption 2 is similar to the assumption of belief difference in aforementioned studies, there is a major difference: investors hold different beliefs on the long-run mean of the cash flow only through idiosyncratic risk, and they agree to the impact of aggregate risk on individual cash flows. Specifically, we assume that investors have disagreement in interpreting firm-specific information rather than aggregate information because of their differences in education, cultural background, expertise, cognitive capabilities, and etc. This assumption can be viewed as a special form of rational inattention theory of Sims (2003) which asserts that investors' information processing capacity is a scarce resource.[5] In particular, a theory of rational inattention suggests that economic agents process the most important information first, and if they still have remaining information processing capacity, only then they deal with other information that has (probably) less importance. If investors process aggregate or systematic information first, then firm-specific information, and the assessment of the latter tends to differ due to the scarcity of capacity to process information, our model shares the spirit of rational inattention.

Although individual cash flow processes are subject to both aggregate and idiosyncratic risks, the aggregate cash flow process has exposure only to aggregate risk. Thus, equation

[4]This result can be easily derived from the optimal filtering theory. For additional details, one can refer to Liptser and Shiryaev (2001).

[5]Hong, Stein, and Yu (2007) and Hong and Stein (2007) provide in-depth discussions about this. For other applications of rational inattention, see Sims (2006) and Xiong and Peng (2006).

(II.4) implies that in the aggregate, idiosyncratic risks are diversified away such that both individual and aggregate cash flows are modeled consistently.

According to the definition of the share process s_t, an individual cash flow process is defined as the product of the share process and the aggregate dividend. By applying Ito's lemma to the product of s_t and D_t, we can write the perceived individual cash flow process as:

$$\frac{dD_s(t)}{D_s(t)} = \mu_{D_s}^{(k)}(t)dt + \sigma_{D_s,A}(t)dB_A(t) + \sigma_{D_s,I}(t)dB_I^{(k)}(t) \quad k = 1, 2, \tag{II.9}$$

where

$$\begin{aligned} \mu_{D_s}^{(k)}(t) &\equiv \mu_D + \phi_s \left(\frac{\bar{s}^{(k)}}{s_t} - 1 \right) + \Xi_s^{CF} - s_t \Xi_s^{CF} - (1 - s_t)\Xi_{(1-s)}^{CF}, \\ \sigma_{D_s,A}(t) &\equiv \sigma_{D,A} + \sigma_{s,A}(t), \\ \sigma_{D_s,I}(t) &\equiv \sigma_{s,I}(t), \end{aligned} \tag{II.10}$$

where $\Xi_j^{CF} \equiv v_{j,A} \cdot \sigma_{D,A}$, with $j = s, (1-s)$. Ξ_s^{CF} is the unconditional covariance between the share process and the aggregate cash flow process. We define Ξ_s^{CF} as a fundamental cash flow risk parameter following Menzly, Santos, and Veronesi (2004). Ξ_s^{CF} plays an important role in our quantitative study, because it enables us to estimate individual cash flow parameters, $v_{s,A}$ and $v_{s,I}$, and clarifies the magnitude of systematic cash flow risk in the cross section.[6]

D. The Market

This economy deals with two risky assets and one riskless asset. Without loss of generality, we consider the market portfolio and an asset with the share process s_t for the risky assets. As mentioned earlier, an asset with the share $(1 - s_t)$ is redundant as it makes up the market with an asset with the share s_t. An asset with the share s_t is referenced as an asset s, while M denotes the market portfolio. The price process of the market portfolio is expressed as

$$\frac{dP_{M,t} + D_t}{P_{M,t}} = \mu_{P_M}(t)dt + \sigma_{P_M,A}(t)dB_A(t). \tag{II.11}$$

[6] Identification of individual cash flow risk parameters such as $v_{s,A}$, $v_{s,I}$, $v_{(1-s),A}$ and $v_{(1-s),I}$ is explained in the Appendix.

Accordingly, the perceived price of asset s, denoted as P_s is given by

$$\frac{dP_{s,t}+D_{s,t}}{P_{s,t}} = \mu^{(k)}_{P_s}(t)dt + \sigma_{P_s,A}(t)dB_A(t) + \sigma_{P_s,I}(t)dB^{(k)}_I(t) \quad \text{for } k=1,2, \tag{II.12}$$

where

$$\mu^{(k)}_{P_s}(t) \equiv \mu_{P_s}(t) - \frac{\sigma_{P_s,I}(t)\phi_s(\bar{s}-\bar{s}^{(k)})}{\sigma_{s,I}(t)s(t)}, \tag{II.13}$$

and, r_t refers to the riskless rate of return.

E. *Equilibrium*

Turning to the consumption-portfolio problem of an individual investor, investor k's wealth $W^{(k)}(t)$ evolves as

$$\begin{aligned} dW^{(k)}_t =& W^{(k)}_t \left[r_t - \tilde{c}_{k,t} + \pi^{(k)}_M(t)(\mu_P(t)-r_t) + \pi^{(k)}_s(t)(\mu^{(k)}_{P_s}(t)-r_t)\right] dt \\ &+ W^{(k)}_t \left[\pi^{(k)}_M(t)\sigma_{P,A}(t) + \pi^{(k)}_s(t)\sigma_{P_s,A}(t)\right] dB_A(t) \\ &+ W^{(k)}_t \left[\pi^{(k)}_s(t)\sigma_{P_s,I}(t)\right] dB^{(k)}_I(t), \end{aligned} \tag{II.14}$$

where $\tilde{c}_{k,t}$ is the consumption fraction of the k-th investor, $c_{k,t}/W^{(k)}_t$. The quantities $\pi^{(k)}_M$ and $\pi^{(k)}_s$ are the k-th investor's risky investment fractions of wealth in the market portfolio and the asset that corresponds to the share process, s_t. The riskless investment is defined as $b_k(t) \equiv 1-\pi^{(k)}_M(t)-\pi^{(k)}_s(t)$. Following Dybvig and Huang (1988), we impose a non-negativity condition on the wealth process to rule out arbitrage strategies.

Investor-specific state price densities are specified as follows:

$$d\xi^{(k)}_t = -\xi^{(k)}_t \left[r_t dt + \theta_A(t)dB_A(t) + \theta^{(k)}_I(t)dB^{(k)}_I(t)\right] \quad \text{for } k=1,2, \tag{II.15}$$

where θ_A is the market price of aggregate risk and $\theta^{(k)}_I$s is the perceived market price of idiosyncratic risk for investor k. By no arbitrage, the market prices of risks are:

$$\begin{aligned} \theta_A(t) &\equiv \frac{\mu_P(t) - r_t}{\sigma_{P,A}}, \\ \theta_I^{(k)}(t) &\equiv \left[-\frac{\sigma_{P_s,A}}{\sigma_{P_s,I}}\theta_A(t) + \frac{1}{\sigma_{P_s,I}}\left(\mu_{P_s} - r\right) - \eta_t^{(k)} \right]. \end{aligned} \tag{II.16}$$

Thus, the following link exists between the two idiosyncratic market prices of risks:

$$\theta_I^{(1)}(t) - \theta_I^{(2)}(t) = \eta_t^{(2)} - \eta_t^{(1)} = \bar{\eta}_t. \tag{II.17}$$

Simply for convenience, we assume the second investor is always more optimistic than the first one such that $\bar{s}^{(2)}$ is bigger than $\bar{s}^{(1)}$, which leads to our next assumption.

Assumption 3. *Belief difference exists in the economy. That is,*

$$\bar{\eta}_t \equiv \eta_t^{(2)} - \eta_t^{(1)} = \frac{\phi_s(\bar{s}^{(1)} - \bar{s}^{(2)})}{\sigma_{s,I}(t)s_t} < 0.$$

Assumption 3 simply states that agent 1 (2) is the pessimist (optimist), and the term, $\bar{\eta}$ represents the belief difference. Note that by definition, $\bar{\eta}$ is always *negative*, if there exists belief difference.

Investors are assumed to be infinitely lived and the market is complete in our economy. Thus, we can formulate an individual optimization problem using martingale methods as follows:

$$\begin{aligned} &\max_{c_k} \; E^{(k)}\left[\int_0^\infty u_k(c_k(t))\mathrm{d}t\right] \\ &\qquad \text{subject to} \\ &\qquad E^{(k)}\left[\int_0^\infty \xi^{(k)}(t)c_k(t)\mathrm{d}t\right] \le W^{(k)}(0) \equiv w_k P(0), \end{aligned} \tag{II.18}$$

where $P(t)$ is the total wealth held by both investors at time t since it is the value of the market portfolio. Also note that $W^{(1)}(t) + W^{(2)}(t)$ is the total wealth in the economy such that it equals $P(t)$. The quantity w_k is the initial fraction of wealth held by investor k of the market portfolio. From the maximization problem in (II.18), the optimality condition for investor k's consumption is given by:

$$c_k(t) = I_k\left(\frac{\xi^{(k)}(t)}{\lambda_k}\right) = \left(\frac{1}{X_t}\right)^{\frac{1-\gamma}{\gamma}} \left[\frac{\xi^{(k)}(t)}{\lambda_k}\right]^{-\frac{1}{\gamma}}, \tag{II.19}$$

where $1/\lambda_k$ is the Lagrange multiplier for investor k's optimal consumption-portfolio choice problem and $I_k(\cdot)$ is the inverse of investor k's utility function. From the static budget constraint of investor k's problem, we have:

$$\lambda_k = \left[\frac{E^{(k)}\left[\int_0^\infty \left\{\xi^{(k)}(t)X_t\right\}^{\frac{\gamma-1}{\gamma}} dt\right]}{w_k P_M(0)}\right]^{-\gamma}. \tag{II.20}$$

Then, equilibrium in this economy is defined as follows:

Definition 1. *Given preferences, endowments, and beliefs structures, an equilibrium in this economy is a collection of allocations* $\left(\overset{*}{c}_k, \overset{*}{\pi}_M^{(k)}, \overset{*}{\pi}_s^{(k)}, \overset{*}{b}_k\right)_{k=1,2}$ *and a supporting price system* $\left(r, \mu_P, \mu_{P_s}^{(k)}, \sigma_P, \sigma_{P_s}\right)$ *such that 1)* $\left(\overset{*}{c}_k, \overset{*}{\pi}_M^{(k)}, \overset{*}{\pi}_s^{(k)}, \overset{*}{b}_k\right)$ *optimally solves investor k's consumption-portfolio choice problem given his/her perceived price processes, 2) security prices are consistent across investors, and 3) all markets clear for* $t \in [0, T]$*:*

$$\sum_{k=1}^{2} \overset{*}{c}_k(t) = D(t), \sum_{k=1}^{2} \overset{*}{\pi}_M^{(k)}(t) = 0, \sum_{k=1}^{2} \overset{*}{\pi}_s^{(k)}(t) = 0. \tag{II.21}$$

III. Cash Flow Risks and Belief Difference

Quantitative analyses of the model require the estimates of the parameters describing cash flow dynamics and the difference in beliefs. This section reports parameter estimates in equations (II.5) and (II.8) to discuss cash flow dynamics. In so doing, we focus on the cash flow share ratio. Details of the data and estimation method are in the Appendix.

The key parameters in cash flow dynamics are estimated by the maximum likelihood method, and other statistics such as average share ratio, ($Avg(\bar{s}/s_t)$), and the coefficient of variation, $CV(s_t)$ (the ratio of the standard deviation of the share to its mean) are directly measured, following their definitions. Tables II and III display the estimates of various cash flow characteristics including risk parameters and belief differences. In Table II, Panel A shows the estimation result without I/B/E/S data, and Panel B shows the result with I/B/E/S data to show the degree of belief difference in the cross section.

[Insert Table II]

Table II indicates that growth firms have a higher long-run mean of the share, $\bar{s}$, than value firms. This is plausible since the long-run mean of the share represents a firm's long-run growth, measuring how well a firm is expected to perform in terms of total payout. Coherent with the labels of value and growth firms, a growth firm may have a higher long-run mean of the cash flow share than a value firm. How about the conditional expected growth in cash flow share or $E_t(ds_t/s_t)$? From the equation (II.5), $E_t(ds_t/s_t) = \phi_s(\bar{s}/s_t - 1)dt$ holds. Thus, both the share ratio ($\bar{s}/s_t$) and the mean reversion (ϕ_s) matter to measure the expected growth of payout. Table II illustrates that the estimate of the share ratio, $\bar{s}/s_t$, is *higher* in value firms than growth firms. To check the robustness of this observation, we measure several different estimates of the share ratio in value deciles: the average of the share ratio over time, the quantile values of the share ratio, and the average of the share ratio in each quantile group. Table III shows that the share ratios of value stocks are still higher than those of growth stocks in all cases.

[Insert Table III]

If $\bar{s}/s_t$ is interpreted only as the cash flow growth (as in the case of time series), then the result appears to be counterintuitive in that the value stocks are associated with a longer distance between $\bar{s}$ and s_t, suggesting a higher share growth.[7] However, if the share ratio also reflects a firm's relative performance measure in terms of the total payout, then a bigger value of share ratio can result from a lower value of the current share (s_t), which is a source of risk. Because it is well known that value firms tend to be distressed and risky (e.g., Fama

[7]This is consistent with a recent empirical finding when the share ratio is used as the expected cash flow growth. Chen (2017) indicates that the value portfolio has higher cash flow growth rate than the growth portfolio in many cases. Lakonishok, Shleifer, and Vishny (2013) also provides a similar implication.

and French (1992), Zhang (2005) and Choi, Johnson, Kim, and Nam (2013)), value firms' current earnings can be much lower than their long-run means than those of growth firms, leading to the higher share ratios. Although our finding that value (growth) stocks have high (low) share ratios runs counter to conventional wisdom, it is robust and yields an important implication for the risk-return trade off in the cross section, going forward.

Second, the mean reversion coefficient, ϕ_s accounts for the different expected growth in the cash flow share as well. Table II shows that the mean-reversion speed, ϕ_s, is particularly low for value stocks in the cross section. This means that value firms' cash flows slowly revert to their mean levels compared to others. Slow mean reversion suggests that value firms' cash flows are subject to large cash flow fluctuations in the long run, leading to high cash flow volatility. $CV(s_t)$ can proxy for the total variability in cash flows. Table II shows that the coefficient of variation is higher on value firms, implying that value firms have larger exposure to cash flow fluctuations. Interestingly, the monotonicity of total cash flow volatility is mostly in line with the monotonicity of idiosyncratic cash flow volatility ($v_{s,I}$).[8] Since our model prices idiosyncratic cash flow risk in equilibrium, total cash flow volatility reflects a source of cash flow risk for pricing assets. To the contrary, conventional asset pricing models allow only systematic cash flow risk to be priced. Thus, when the idiosyncratic cash flow risk is priced in equilibrium, the usual (systematic) cash flow beta consisting of ($v_{s,A}/\sigma_{D,A}$) cannot be the sole driver in producing cross sectional return variation, and the idiosyncratic risk component matters as well.

Now we turn our attention to belief dispersion in the cross section. We construct belief difference measure following Diether, Malloy, and Scherbina (2005) such that investor's difference in beliefs is defined as the ratio of the standard deviation of analysts' earnings forecasts to the mean of analysts' earnings forecasts, $\frac{\text{SD(Earnings Forecasts)}}{\text{Avg(Earnings Forecasts)}}$. This measure can be roughly approximated by $\frac{\text{EF}_{\text{max}}-\text{EF}_{\text{min}}}{\text{Avg(EF)}}$, where EF $\equiv$ Earnings Forecasts where EF_{max} and EF_{min} are investors' largest and smallest expected mean cash flows. To explain the measure, we take an example of firms with growth options in play.

Investors might have more diverse opinions on the cash flow of firms whose growth options are the main assets, because of the uncertain nature of the growth options. In this case, the distance between EF_{max} and EF_{min} can be large, or closely related, a large value of standard

[8]Aggregate cash flow risk parameter, ($v_{s,A}$), does not vary much for the most part in the cross section except two most value-like assets.

deviation of earnings forecasts. However, this distance is likely to be longer (shorter) for bigger (smaller) firms since earnings tend to be larger (smaller) for bigger (smaller) firms. Therefore, the distance measure alone cannot properly represent investors' differences in beliefs due to the firm size effect. To mitigate the size effect, we normalize the distance with the average value of the share, Avg(EF).

The data show that growth firms are usually much bigger than value firms in size (market capitalization). Thus, when applying the normalized distance measure to firms in the cross section, we tend to observe longer distances, i.e., large values of $\mathrm{EF}_{\max} - \mathrm{EF}_{\min}$ for bigger firms or growth firms; hence a bigger SD(earnings forecasts) for growth firms than for value firms. But at the same time, Avg(EF) of growth firms is likely be higher than that of value firms.[9] Indeed, this tendency is captured by the data; growth firms' cash flow share is much larger than that of value firms, i.e., $\bar{s}_{\text{growth}} >> \bar{s}_{\text{value}}$ (see Table II). This is consistent with the conventional wisdom that growth firms have higher growth potentials. In such a case, the measure, $\frac{\mathrm{EF}_{\max}-\mathrm{EF}_{\min}}{\mathrm{Avg(EF)}}$ can be small when the denominator is relatively large compared to the numerator. On the other hand, growth options are usually small for value firms. In such a case, analysts' opinions will mildly fluctuate around those firms' $\bar{s}$ such that the distance $\mathrm{EF}_{\max} - \mathrm{EF}_{\min}$ can be short. However, value firms tend to have a low Avg(EF) due to the lack of growth potential. Then, the measure $\frac{\mathrm{EF}_{\max}-\mathrm{EF}_{\min}}{\mathrm{Avg(EF)}}$ can be large when the numerator is relatively larger compared to the denominator.

Although above discussion can plausibly explain why value stocks have larger belief disagreement than growth stocks in terms of low profitability of value stocks, it is still necessary to investigate data. We looked at value weighted standard deviations and value weighted (absolute) mean(or median) of EPS forecasts of portfolios in value decile.[10] Both measures are increasing from growth stocks to value stocks, which runs counter to conventional intution mentioned above: investors might have more diverse opinions about growth firms since they are exposed to more uncertain future growth opportunities and value firms have lower profitability than growth firms. If the measure of belief difference is the mere reflection of value firm's overall low profitability, then we wouldn't have seen this data. The data implies that value stocks are indeed exposed to more diverse opinions in terms of the standard deviation of EPS forecasts, and they are not necessarily less profitable in terms of per-share earnings

[9] Once growth firms start to pay out, they tend to pay lot more than value firms.
[10] We don't report them here for the brevity

forecasts. Also, the magnitude of mean per-share earnings forecasts is not big enough to offset the size of the standard deviation of per-share earnings forecasts. Thus, our measure of belief difference is increasing from value stocks to growth stocks in value decile.

The reason that we observe this phenomenon can be attributed to the nature of data. Analysts' forecasts are about per share. Thus, value stocks' earnings forecasts per share can be larger than growth stocks' one. And this is usual street wisdom that value firms usually pay more than growth firms if it is measured by per share.[11] We would like to emphasize that our finding of cross sectional pattern of belief difference is not entirely new. It is consistent with Diether, Malloy, and Scherbina (2005). Table IV in Diether, Malloy, and Scherbina (2005) shows that the measure of belief difference is increasing from low book-to-market group to high book-to-market group.

Normalized belief difference measure is in line with our theory in that it is similar to is $(\bar{s}_{optimist} - \bar{s}_{pessimist})/s_t$ when the numerator is proxied by $\mathrm{EF}_{\max} - \mathrm{EF}_{\min}$ and the denominator is proxied by Avg(EF).[12] If we decompose the ratio into $\frac{(\bar{s}_{optimist} - \bar{s}_{pessimist})}{\bar{s}} \frac{\bar{s}}{s_t}$, the empirical proxy captures the first part and it shows that the belief difference is closely related to the cash flow share ratio. It is beyond the scope of the paper to endogenize the link between the degree of belief dispersion and asset characteristics though, our empirical finding suggests that assets with higher idiosyncratic cash flow fluctuations have higher belief dispersion. If the size of belief difference for each asset is dictated by the amount of risk and uncertainty, this empirical link seems plausible in that value stocks have higher cash flow risks, and the wedge between optimists and pessimists are likely to be larger for the value stocks.

[11] Also, the street wisdom says that value firms pay more than growth firms since they do not have future growth opportunities; hence, no need to invest.

[12] The denominator, the average of earnings forecasts, can easily be approximated by the current share. This is the same reason that the current stock return proxies the expected stock return from the previous period since the earnings forecasts are made before the current share is observed.

IV. Equilibrium Results

This section derives and discusses equilibrium results. We find two stochastic discount factors ($\xi^{(1)}(t)$ and $\xi^{(2)}(t)$ that satisfy the following goods market clearing condition.

$$\overset{*}{c}_1(\xi^{(1)}(t)/\lambda_1, t) + \overset{*}{c}_2(\xi^{(2)}(t)/\lambda_2, t) = D(t). \tag{IV.1}$$

For the computational purpose, we define the stochastic weighting process λ_t as follows:

$$\lambda_t \equiv \frac{\lambda_1 \xi_t^{(2)}}{\lambda_2 \xi_t^{(1)}}, \tag{IV.2}$$

where $\lambda_0 = \frac{\lambda_1}{\lambda_2}$, since $\xi^{(k)}(0) = 1$ for $k = 1, 2$. As discussed in Basak (2000), λ_t provides information about the differences in investors' opportunity sets given heterogeneous beliefs.

By applying Itô's lemma to λ_t, we obtain the diffusion process of $\lambda(t)$ as

$$\frac{d\lambda_t}{\lambda_t} = \bar{\eta}_t dB_I^{(2)}(t). \tag{IV.3}$$

Thus, λ_t process is fully described by investors' disagreements, $\bar{\eta}_t \equiv \left[\eta_t^{(2)} - \eta_t^{(1)}\right]$, and the second (optimistic) investor's perceived idiosyncratic Brownian risk $B_I^{(2)}(t)$. For consistency, we continue to use the second investor's Brownian risk afterwards, but one can always rewrite the problem from the pessimist's point of view without loss of generality.

With the definition of the λ_t, we characterize the stochastic discount factors as follows.

Proposition 1.

$$\frac{\xi_t^{(2)}}{\lambda_2} = D_t^{-\gamma} \left(\frac{1}{X_t}\right)^{1-\gamma} \left[1 + \left(\frac{1}{\lambda_t}\right)^{-(1/\gamma)}\right]^{\gamma}, \ \textit{and} \ \frac{\xi_t^{(1)}}{\lambda_1} = \frac{\xi_t^{(2)}}{\lambda_2}\frac{1}{\lambda_t}. \tag{IV.4}$$

Without belief difference, the λ_t process becomes 1.

Proof: See Appendix.

A. Equilibrium Prices and Returns

Because $\xi^{(1)}$ and $\xi^{(2)}$ are linked via λ_t process, the Radon-Nikodym derivative between the two investors' perceived probability measures, it suffices to compute the price of an asset with the share, s_t, using the second investor's state price density. The following proposition shows the equilibrium price-dividend ratio of an asset s.

Proposition 2. *Approximate equilibrium stock price with the share process s_t is given by*

$$\frac{P_s(t)}{D_s(t)} \approx \left[\beta_{s,0} + \beta_{s,1}\left(\frac{\bar{H}}{H_t}\right) + \beta_{s,2}\left(\frac{\bar{s}^{(2)}}{s_t}\right) + \beta_{s,3}\left(\frac{\bar{s}^{(2)}}{s_t}\frac{\bar{H}}{H_t}\right)\right], \tag{IV.5}$$

where coefficients $\beta_{s,k}$'s are the functions of parameters determining cash flow or firm characteristics:

$$\beta_{s,k} \equiv f_k(v_{s,I}, v_{s,A}, \overline{\overline{\eta_t}}, \gamma, h_1, h_2, \phi_s, \bar{s}^{(2)}), \tag{IV.6}$$

for $k = 0, 1, 2, 3$ and $\overline{\overline{\eta_t}}$ is the time series average of belief difference measure for a stock with the share, s_t, and

$$\begin{aligned}
\beta_{s,0} &= -\frac{1}{\overline{\alpha}_1 + \overline{\alpha}_2 v_{s,I} + h_2 v_{s,A} - h_1 - \phi_s}, \\
\beta_{s,1} &= \frac{h_1}{(\overline{\alpha}_1 + \overline{\alpha}_2 v_{s,I} + h_2 v_{s,A} - h_1 - \phi_s) \times (\overline{\alpha}_1 + \overline{\alpha}_2 v_{s,I} - \phi_s)}, \\
\beta_{s,2} &= \frac{\phi_s}{(\overline{\alpha}_1 + \overline{\alpha}_2 v_{s,I} + h_2 v_{s,A} - h_1 - \phi_s)(\overline{\alpha}_1 - h_1)},
\end{aligned}$$

with $\overline{\alpha}_1 = (1-\gamma)/(2\gamma^2\overline{\overline{\eta_t}}^2)$ and $\overline{\alpha}_2 = \overline{\overline{\eta_t}}/\gamma$. $\beta_{s,3}$ is in an appendix due to its length and a small quantitative magnitude The quantities $\bar{s}/s_t$ and $\bar{H}/H_t$ are referred to the share ratio and the habit ratio, respectively.

Proof: See Appendix.

Equation (IV.5) states that two variables mainly determine the equilibrium price-dividend ratio of a stock with the share s_t: the share ratio ($\bar{s}^{(2)}/s_t$) and the habit ratio ($\bar{H}/H_t$) from the standpoint of the second (optimistic) investor. This is a extended version of Gordon growth formula in that the price-dividend ratio depends on discount rate variable, $\bar{H}/H_t$, cash flow variable, $\bar{s}^{(2)}/s_t$, and the interaction variable of the two that governs the so-called convexity. The price-dividend ratio has both time-series and cross-sectional implications in equilibrium. Time series variations in H and s affect equilibrium price-dividend ratio, and nontrivial cross-sectional effects can arise, as the main variables and the beta coefficients differ across assets, depending on cash flow characteristics and especially the degree of belief dispersion.

We first note that all the $\beta_{s,k}$ coefficients are positive under the reasonable conditions such as the existence of belief difference, mean reversion of the share process, and relative risk aversion being greater than 1. Thus, equilibrium price-dividend ratio is positively affected by all the variables. For instance, the habit ratio pro-cyclically affects the price-dividend ratio along the business cycle; equilibrium price-dividend ratio increases in a good economy and vice versa.[13] Plus, the higher the cash flow share ratio, the bigger the equilibrium price-dividend ratio. An increase in the share ratio implies an increase in the expected dividend growth (see equation (II.10)). Higher expected dividend growth translates into a higher value of the equilibrium price-dividend ratio, due to $\beta_{s,2}$ being positive.[14] In addition, qualitatively speaking, an increase in the expected cash flow growth rate has a positive effect on the price-dividend ratio through the interaction with the habit ratio (positive $\beta_{s,3}$). The left panels of Figure 2 show the simulated time-series property of price-dividend ratio with respect to the share ratio of our calibrated model for the value and growth stocks. The last row of the left column in figure 2 draws the same contents on the same graph for comparison. We draw scatter plots for the ease of exposition. Consistent with the previous studies, a positive relation between the price-dividend ratio and the share ratio prevails with an important difference: a significant disconnect between the value and the growth stocks exists, and the bottom left row in figure 2 suggests that the cross-sectional link is negative, if the times-series averages of the variables of interest (straight lines) are computed within each of the value or growth stocks.

[Insert Figure 2]

[13]Put differently, when the economy is in a slump, the habit variable H_t is high, and vice versa.

[14]This mechanism can be roughly seen in equation (IV.5). The increase in the share ratio, $\bar{s}^{(2)}/s_t$, is equivalent to the decrease in the share, s_t, as the long-run mean $\bar{s}^{(2)}$ is fixed. Note that equilibrium price-dividend ratio can be seen as $P_s(t)/D_s(t) = P_s(t)/s_t D_t = (P_s(t)/s(t))(1/D_t)$. In the model, D_t is given exogenously. Thus, the decrease in s_t increases the price-dividend ratio, $P_s(t)/D_s(t)$. A positive time-series relation between the share ratio and price-dividend ratio has also been shown in Menzly, Santos, and Veronesi (2004) and Santos and Veronesi (2010).

The beta coefficients play crucial roles in accounting for the above pattern, as they convey different information across assets depending on firm characteristics as well as investors' belief differences. In particular, coefficients are nonlinear functions of the habit parameters (h_1 and h_2), the long-run mean of the cash flow share, $\bar{s}$, and the average value of belief difference, $\overline{\overline{\eta}_t}$, which we give more details in the below.

[Insert Figure 3]

The upper part of Figure 3 confirms the observation in figure 2 to show that our model indeed simulates a negative cross-sectional relation between the share ratio and the price-dividend ratio. This result states that the time-series property of a positive link between the share ratio and the price-dividend ratio for each value deciles does not monotonically translate into the cross section because of nonlinearity. This nonlinearity results mainly from the idiosyncratic cashflow risk, augmented with heterogeneous beliefs.

Recall that the coefficient on the share ratio ($\beta_{s,2}$) governs cash flow dynamics and investors' belief differences for each asset s as seen in equation (IV.6). From the formula of $\beta_{s,2}$ in the proposition, $\overline{\alpha}_1$ and $\overline{\alpha}_2$ are are functions of belief difference, $\bar{\eta}$; they are both negative but increase in absolute values when belief difference increases. When belief difference increases, the denominator of $\beta_{s,2}$ increases as the product of $\overline{\alpha}_1 + \overline{\alpha}_2 v_{s,I}$ with $\overline{\alpha}_1 - h_1$ is positive.[15] With the same logic, the increase of idiosyncratic cash flow risk parameter, $v_{s,I}$, leads to higher value of denominator: hence, lowers the value of $\beta_{s,2}$. In short, the higher the idiosyncratic cash flow risk and belief differences, the lower the value of $\beta_{s,2}$. In both the data and the simulation, assets with higher share ratios tend to have higher cash flow risk parameters (both $v_{s,A}$ and $v_{s,I}$) and more diverse investors' opinions (See Table II and Internet Appendix). Thus, low values of $\beta_{s,2}$'s correspond to higher values of the share ratios in the cross section and vice versa.[16]

[Insert Figure 4]

Figure 4 shows the cross-sectional relation between share ratios and $\beta_{s,2}$'s. They turn out to be strongly negatively (and nonlinearly) related: high (low) values of share ratios are associated with very low (high) values of $\beta_{s,2}$'s: $\beta_{s,2}|_{\text{low share ratio}} >> \beta_{s,2}|_{\text{high share ratio}}$. Therefore,

[15] h_1 is positive. See Internet Appendix for the parameter calibration of aggregate processes.

[16] Also the mean-reversion coefficient, ϕ_s, is bigger on stocks with low share ratios, which further fortifies the relation.

in the cross section, very low (high) values of $\beta_{s,2}$ strongly suppresses (magnifies) high (low) values of share ratios, leading to a negative cross sectional relation between the share ratio and equilibrium price-dividend ratio. This is a key theoretical result of our model equilibrium and it contrasts with existing studies such as Menzly, Santos, and Veronesi (2004), Santos and Veronesi (2006), and Lettau and Wachter (2007): in those studies, assets with high (low) share ratios have high price-dividend ratio as well as high (low) expected excess returns, which is not consistent with data.

With this in mind, we now compute the (approximate) expected excess return of an asset with share s_t.

Proposition 3. *Equilibrium return process for a stock with the share process, s_t is given by*

$$dR_s \approx \mu_{R_s}^{(2)} dt + \sigma_{R_s,A} dB_A + \sigma_{R_s,I} dB_I^{(2)}, \tag{IV.7}$$

where $\mu_{R_s}^{(2)}$ is the approximate expected excess return, i.e., $E_t^{(2)}[dR_{s,t}]$, and it is given by:

$$E_t^{(2)}[dR_{s,t}] \approx \left[\frac{D_s(t)}{P_s(t)}\right]\left[\mu_{s,t}^{A,I} + \mu_{s,t}^{I}\right], \tag{IV.8}$$

where

$$\begin{aligned} \mu_{s,t}^{A,I} &\equiv \beta_{s,0}\left(\sigma_{D,A} + v_{s,A}\right)\left(\sigma_{D,A} - h_2\right) + \beta_{s,1}\left(\sigma_{D,A} + v_{s,A} - h_2\right)\left(\sigma_{D,A} - h_2\right)\left[\frac{\bar{H}}{H_t}\right] \\ &\quad + \beta_{s,2}\sigma_{D,A}\left(\sigma_{D,A} - h_2\right)\left[\frac{\bar{s}^{(2)}}{s_t}\right] + \beta_{s,3}\left(\sigma_{D,A} - h_2\right)^2\left[\frac{\bar{s}^{(2)}}{s_t}\frac{\bar{H}}{H_t}\right], \\ \mu_{s,t}^{I} &\equiv -\frac{1}{2} v_{s,I}\overline{\overline{\eta}}_t\left(\beta_{s,0} + \beta_{s,1}\frac{\bar{H}}{H_t}\right). \end{aligned} \tag{IV.9}$$

Proof: See Appendix.

Proposition 3 shows a decomposition of the equilibrium expected excess return into two parts (equation IV.9). The first, $\mu_{s,t}^{A,I}$, is the expected excess return that is affected by both aggregate and idiosyncratic cash flow risks, and the second, $\mu_{s,t}^{I}$ is affected only by idiosyncratic cash flow risk via investors' belief differences. We note that the whole term, $\mu_{s,t}^{I}$, is positive mainly because $\overline{\overline{\eta}}_t$ is negative, which we will discuss shortly below in the next subsection. It implies that idiosyncratic cash flow risk increases the expected excess return via investors' differences in beliefs.

The right column of Figure 2 shows that our calibrated model generates a negative time series relation between the share ratio and expected excess returns within each portfolio. As the share ratio and price-dividend ratio are positively related, this implies that *the usual time series return predictability holds as to the share ratio*: low (high) dividend yield induced by high (low) share ratio leads to low (high) expected excess return. However, as the right-bottom of figure 2 and more thoroughly the bottom panel of figure 3 show, this relation is flipped in the cross section such that assets with high average (low) share ratios generate high (low) expected excess returns. That is, within-group and across-group relations are markedly different. Simulated equilibrium cross section indeed matches the data as shown in Figure 5.

[Insert Figure 5]

According to the model, high (low) share ratios are associated with very low (high) $\beta_{s,2}$'s; hence, assets with high (low) share ratios have low (high) price-dividend ratios. Then, equations (IV.8) and (IV.9) imply that assets with high (low) share ratios have high expected excess returns since they have not only high (low) dividend yields in the cross section but also higher belief difference together with idiosyncratic cash flow risk (e.g., $\mu^I_{s,t}$).

B. Cash Flow Risk and the Value Premium

In this subsection, we decompose the expected excess return into two parts - discount rate risk component and cash flow risk component - following the recent trend in the literature.[17] To do that, we first compute the discount rate risk premium and define the cash flow risk premium as the difference between expected excess return and discount rate risk premium. In our model, aggregate discount rate risk is induced by the habit process, H_t.[18] For instance, H_t, from its definition, closely follows the business cycle as H_t^{-1} moves pro-cyclically with the

[17]See Campbell and Vuolteenaho (2004), Campbell, Polk, and Vuolteenaho (2010), Lettau and Wachter (2007), Santos and Veronesi (2010), as well as many others for the study of the importance of discount rate risk component and cash flow risk component respectively in explaining the cross sectional-return variation.

[18]In Campbell and Cochrane (1999) and Santos and Veronesi (2010), the surplus-consumption ratio, $S_t \equiv (C_t - X_t)C_t^{-1}$ induces the aggregate discount rate risk. In our model, the habit process, H_t, plays a similar role.

business cycle. Following Santos and Veronesi (2010), we define discount rate risk premium as the price elasticity of H_t^{-1}.

$$\frac{\partial P_s(t)/P_s(t)}{\partial H_t^{-1}/H_t^{-1}} = \left[\beta_{s,1}\left(\frac{\bar{H}}{H_t}\right) + \beta_{s,3}\left(\frac{\bar{s}^{(2)}}{s_t}\frac{\bar{H}}{H_t}\right)\right]\left(\frac{D_s(t)}{P_s(t)}\right) \tag{IV.10}$$

Thus, in equation (IV.8), the discount rate risk premium is defined as

$$\mu_{s,t}^{DR} \equiv \left(\sigma_{D,A} - h_2\right)^2 \left[\beta_{s,1}\left(\frac{\bar{H}}{H_t}\right) + \beta_{s,3}\left(\frac{\bar{s}^{(2)}}{s_t}\frac{\bar{H}}{H_t}\right)\right]\left(\frac{D_s(t)}{P_s(t)}\right). \tag{IV.11}$$

The resultant cash flow risk premium is then defined as

$$\mu_{s,t}^{CF} \equiv E_t^{(2)}\left[dR_{s,t}\right] - \mu_{s,t}^{DR}. \tag{IV.12}$$

The following proposition summarizes the above.

Proposition 4. *Approximate equilibrium expected excess return of a stock with the share, s_t, is decomposed into discount rate risk premium and cash flow risk premium:*

$$E^{(2)}\left[dR_{s,t}\right] = \mu_{s,t}^{DR} + \mu_{s,t}^{CF}. \tag{IV.13}$$

Simulation of the model with return decomposition as well as other cross sectional moments such as Sharpe ratio are presented below in Table IV and Figure 6.

[Insert Table IV and Figure 6]

Assets are sorted by price-dividend ratios; assets with high (low) price-dividend ratios are categorized as growth (value) stocks. We note that, as the data show in the previous section III, value (growth) like stocks have high (low) share ratios (see upper right corner of Figure 6 and average log price-dividend ratio and average share ratio in the table). Main simulations results are 1) expected excess returns are increasing from growth to value firms, 2) while cash flow risk premium is higher on value firms, discount rate risk premium is higher on growth firms, 3) idiosyncratic cash flow risk premium is distinctively higher on value firms, 4) Sharpe ratio increses from growth to value firms, and 5) total cash flow volatility measured by the coefficients of variation of the cash flow shares, $CV(s_t)$, is increasing from growth to value firms. Thus, our model produces key cross sectional moments, as shown in Table V below.

[Insert Table V]

Return decomposition with equations (IV.11) and (IV.12) is useful in presenting the cross sectional pattern in our model. While cash flow risk premium increases toward value stocks, discount rate risk premium increases toward growth stocks. Growth stocks' expected excess returns distinctively come from discount rate risk premium, which is consistent with assets' nature: growth stocks' cash flows are subject to high discount effect because of their long duration. On the other hand, value stock's expected excess return is mostly governed by cash flow risk premium. Moreover, cash flow risk premium dominates discount rate risk premium in the cross section such that the value premium arises. Then, a natural question arises: what really is responsible for large cash flow risk premium. As was briefly described in 3) above, idiosyncratic cash flow risk premium is one that quantitatively reigns the whole cash flow risk premium. We look at the role of the pure idiosyncratic cash flow risk premium,

$$\mu^I_{s,t} \equiv -\frac{1}{2} v_{s,I} \overline{\overline{\eta}}_t \left(\beta_{s,0} + \beta_{s,1} \frac{\bar{H}}{H_t} \right).$$

It is a function of idiosyncratic cash flow risk parameter, belief difference, and the habit ratio along with two coefficients. As was mentioned before, $\mu^I_{s,t}$ is *positive* as $-v_{s,I}\overline{\overline{\eta}}_t$ is positive; idiosyncratic cash flow risk positively affects expected return of an asset in our model. Table IV above shows cross sectional variation of pure idiosyncratic cash flow risk premium. The premium is negligible for the first 6 decile portfolios that are close to growth stocks. However, it matters for the remaining 4 portfolios and matters most for the value stocks. The reason is that pure idiosyncratic cash flow risk premium is largely determined by the interaction between idiosyncratic cash flow risk parameter, $v_{s,I}$, and the average belief difference, $\overline{\overline{\eta}}_t$. Besides, as Table IV shows, the whole cash flow risk premium is largely determined by pure idiosyncratic cash flow risk premium, which further strengthens the importance of idiosyncratic cash flow risk with belief difference.

Now we realize that the value premium in our model is generated by the cash flow risk premium. Furthermore, we note that the same logic that explains higher expected returns on assets with with higher share ratios in the cross section also explains the value premium. Similar to the data, value stocks have higher share ratios, bigger idiosyncratic cash flow risks, and larger belief differences in our model. Therefore, $\beta_{s,2}|_{\text{low share ratio}} >> \beta_{s,2}|_{\text{high share ratio}}$ directly translates into $\beta_{s,2}|_{\text{growth}} >> \beta_{s,2}|_{\text{value}}$ such that high (low) values of the share ratio is suppressed (magnified) by small (large) values of $\beta_{s,2}$. Thus, a positive cross sectional relationship between the share ratio and the expected excess return directly explains the value premium.

C. Conditional Value Premium

The pricing of idiosyncratic cash flow risk has an additional implication for the value premium. Idiosyncratic cash flow risk is priced only through different beliefs: the higher the belief difference, the larger the effect of idiosyncratic cash flow risk. Therefore the value effect is supposed to be more prevalent for assets with larger degree of belief differences. To check this empirical implication, we double sort assets into 9 groups conditionally: assets are first sorted into three groups based on the magnitude of belief difference, and then in each group, assets are further sorted into three groups based on book-to-market ratio. We compute the value weighted average return in each group.

[Insert Table VI]

Panel B of Table VI clearly shows that the value effect is stronger in the group of assets with higher belief difference. In particular, a significant value effect emerges in the group of large belief difference. On the other hand, in the group of low degree of belief difference, the value effect rarely exists. This result is in line with Yu (2011) in that the value effect is more pronounced for assets with high belief disagreement. Both Yu (2011) and our study use bottom-up measure of belief difference from individual stocks; individual firms' standard deviation (and the mean) of EPS forecasts from I/B/E/S data set. The only difference is that we use the normalized standard deviation. In both studies, stocks are double sorted by book-to-market and respective measures of belief difference.

We compared our measure of belief difference with Yu (2011)'s measure of belief difference, i.e., value weighted standard deviation of EPS forecasts across portfolios in value decile. As was explained before in section III, We find that both measures are in line with each other in terms of direction such that they are increasing from growth stocks to value stocks. This confirms that data sets in both studies largely coincide. Hence, the conditional value effect in each study is the same phenomenon: it is the pricing mechanism that makes our study differ from Yu (2011).[19]

[19]Yu (2011) argues that the value effect is more pronounced because of growth stocks. The way Yu (2011)

D. Belief Difference and Growth Premium Puzzle

In this subsection, we investigate the role of belief difference in-depth, by which we better understand how our model differs from similar asset pricing models. Previous studies such as Santos and Veronesi (2010) and Lettau and Wachter (2007) show that when cash flow risk is not considered in equilibrium, discount rate risk prevails in the cross section such that the growth premium arises. Further, they show that introducing systematic cash flow risk in equilibrium is not enough to generate the value premium.

If we disentangle systematic cash flow risk from our model, we can better understand how idiosyncratic cash flow risk contributes to our model equilibrium, which in turn highlights the role of belief difference. To do that, we need to turn off individual systematic cash flow risk in the model. This can be achieved by setting $\upsilon_{s,A}$ to zero since the systematic cash flow risk - the covariance between the share process and aggregate cash flow process, $Cov\left(ds_t/s_t, dD_t/D_t\right) = \sigma_{s,A}\sigma_{D,A}$ - is approximated by $\Xi_s^{CF} \equiv \upsilon_{s,A}\sigma_{D,A}$. Below is the simulation results of the model when $\upsilon_{s,A} = 0$.

[Insert Figure 7]

Unlike Santos and Veronesi (2010) where the growth premium arises when cash flow risk is turned off, we still have the value premium.[20] What makes this difference? From the previous section, we know that the most of cross sectional stock return variation comes from the variation of cash flow risk premium as shown in Table IV and Figure 6. Hence, for our model to generate a different cross section with the change in systematic cash flow risk, the whole cash flow risk premium must be significantly influenced by the change in $\upsilon_{s,A}$. To check this, we numerically compute the size of both systematic and idiosyncratic cash flow risk in our model. Following the convention, systematic cash flow risk is measured by unconditional covariance between individual cash flow process and aggregate cash flow

investigate the return difference within each group of belief difference is based on Miller (1977). Thus, growth stocks are more overpriced because they are subject to larger belief disagreement. On the other hand, our study explains why assets in high belief difference should exhibit larger value effect by driving equilibrium pricing mechanism of idiosyncratic cash flow risk via belief dispersion.

[20]No cash flow risk in Santos and Veronesi (2010) is equivalent to $\Xi_s^{CF} = 0$ in our model. In Santos and Veronesi (2010) and Menzly, Santos, and Veronesi (2004), individual systematic cash flow risk is measured by θ_{CF}^i that is covariance between an individual cash flow share process and aggregate cash flow process. θ_{CF}^i corresponds to Ξ_s^{CF} in our model.

process, i.e., $Cov\left(\frac{dD_s(t)}{D_s(t)}, \frac{dD_t}{D_t}\right)$. This is approximated by

$$\begin{aligned} Cov\left(\frac{dD_s(t)}{D_s(t)}, \frac{dD_t}{D_t}\right) &= \sigma_{DA}^2 + \sigma_{s,A}\sigma_{D,A} \\ &= \sigma_{DA}^2 + \left[v_{s,A} - s_t v_{s,A} - (1-s_t)v_{(1-s),A}\right]\sigma_{D,A} \\ &\left(= \sigma_{DA}^2 + \Xi_s^{CF} - s_t\Xi_s^{CF} - (1-s_t)\Xi_{(1-s)}^{CF}\right) \\ &\approx \sigma_{DA}^2 + v_{s,A}\sigma_{DA}, \end{aligned} \tag{IV.14}$$

when evaluated at $s_t = \bar{s}$.[21] On the other hand, since idiosyncratic cash flow risk (represented by the parameter, $v_{s,I}$) is priced only through investors' belief differences. Thus, $-v_{s,I}\bar{\eta}_s$ is an appropriate measure of idiosyncratic cash flow risk. Table VII shows the quantitative magnitude of the both cash flow risks in the cross section.[22]

[Insert Table VII]

While aggregate cash flow risk does not fluctuate much in the cross section, idiosyncratic cash flow risk significantly fluctuates. Furthermore, idiosyncratic cash flow risk quantitatively dominates aggregate cash flow risk in the cross section; hence, the fluctuation of total cash flow risk is largely determined by the fluctuation of idiosyncratic cash flow risk. As a result, an experiment with $v_{s,A} = 0$ yields the result that is almost identical to the original simulation. In sum, belief difference plays an important role in our model by pricing idiosyncratic cash flow risk which is crucial for determining the cross section.

We can also assess the importance of belief difference more directly by turning off belief difference in our model (by setting $\bar{\eta}_s$ to zero).

[Insert Figure 8]

Figure 8 shows the cross section generated by the reduced form model with respect to the cash flow share ratio. Unlike the original model, the reduced form model yields so-called the growth premium: stocks with higher share ratios have lower expected excess returns. This should be expected as the reduced form model is a version of Menzly, Santos, and Veronesi

[21]This approximation comes from the identification conditions for the share process. Details can be found in the Appendix.

[22]Since $\bar{\eta}_s$ is negative, we put a negative sign on $v_{s,I}\bar{\eta}_s$ to make it positive.

(2004) or Santos and Veronesi (2010) where growth stocks have higher price-dividend ratios due to higher cash flow growth rates, but also have higher expected returns due to large effect of discount shocks. One thing to note in our reduced form model is that value stocks still have higher share ratios as opposed to Menzly, Santos, and Veronesi (2004) and Santos and Veronesi (2010) since the cash flow parameters used in the experimental simulation at the bottom of figure are the same as those in the original one. However, now idiosyncratic cash flow risk parameter does not affect equilibrium since there is no pricing channel, i.e., belief difference. Thus, discount shock prevails in the cross section; hence, the growth premium arises. In short, without belief difference channel, typical asset pricing models similar to ours cannot generate desired equilibrium cross section.

Our discussions so far highlight the difference between our study and existing ones such as Menzly, Santos, and Veronesi (2004), Santos and Veronesi (2010), and Lettau and Wachter (2007). In those studies, assets with high expected dividend growth (= high share ratio) have high expected returns despite high price-dividend ratio due to strong discount effect, leading to a growth premium. Our model differs from these studies in that it generates a usual-time series return predictability at the fund level; low (high) share ratios leads to high (low) dividend yield, hence high (low) expected returns. But this time series return predictability holds within each group of portfolios and the pattern reverses in the cross section such that assets with high (low) average share ratios have high (low) expected returns. Our analysis suggests that this arises because investors excessively fear high total cash flow risk for assets with high share ratios, which are unlikely to become assets with high growth in cash flows. Market requires a higher premium to bear the risk, reflecting the pessimistic beliefs.

V. The Term Structure of Equity Returns

Beginning with Binsbergen, Brandt, and Koijen (2012), recent studies find that equity term structure is sloping downward: assets with shorter (longer) cash flow durations have higher (lower) average returns. It is also shown that existing asset pricing models such as external habit formation (Campbell and Cochrane (1999)) and long-run risk (Bansal and Yaron (2004)), generate counterfactual upward-sloping equity term structures.[23] These findings impose a critical challenge on existing asset pricing models particularly for the cross-section.[24] A stylized fact – growth (value) stocks have longer (shorter) cash flow durations – implies that downward (upward) sloping equity term structure is equivalent to the value (growth) premium. Thus, equilibrium results in our model suggests a downward-sloping equity term structure. We investigate this using two measures of cash flow durations.

For the first measure, we follow Dechow, Sloan, and Soliman (2004) to define an accounting-based measure of cash flow duration as follows.

$$Dur_{s,t} \equiv \frac{\sum_{t=1}^{T} t \times CF_{s,t}/(1+r)^t}{P_{s,t}} + \left(T + \frac{1+r}{r}\right) \times \frac{\left(P_{s,t} - \sum_{t=1}^{T} CF_{s,t}/(1+r)^t\right)}{P_{s,t}}, \quad \text{(V.1)}$$

where s indicates share s_t, $CF_{s,t}$ is the cash flow of an asset with the share s_t, and r is the common discount rate. Though one can use the simulated aggregate expected (excess) return for r, we use $r_{s,t}$ that is the expected excess return for an asset with the share s_t for more realistic specification.[25] This definition of the cash flow duration comes from the concept of Macaulay duration in the fixed income market.

For the second measure, we define the price elasticity with respect to the cash flow as an indirect measure of cash flow duration:

$$iDur_{s,t} \equiv \frac{\partial P_{s,t}/P_{s,t}}{\partial CF_{s,t}/CF_{s,t}}. \quad \text{(V.2)}$$

We use this measure based on observation that the bond price with longer cash flows have higher price sensitivity with respect to the change in cash flows. We compute two cash flow duration measures for the assets sorted on price-dividend ratio from the model simulation.

[23] On the other hand, a model with a rare disaster such as Gabaix (2012) is shown to generate a flat equity term structure.

[24] Most recent asset pricing models have been successful in generating aggregate equity premium in the time series. However, the challenge by Binsbergen, Brandt, and Koijen (2012) applies to the cross-sectional return variation. Recently, there have been some attempts to resolve this issue. One notable success can be found in Belo, Collin-Dufresne, and Goldstein (2015) that incorporates stationary leverage ratios into dividend dynamics. Also, Hasler and Marfè (2016) generates a downward sloping equity term structure under a rare disaster followed by recovery. Most recent literature includes Gormsen (2019) and Goncalves (2019). For the survey of the literature on equity term structure and related discussions, see Binsbergen and Koijen (2017).

[25] Weber (2018) uses the same procedure.

Although we do not report to conserve space, consistent with Dechow, Sloan, and Soliman (2004) and the stylized fact, both measures of cash flow durations are monotonically increasing from assets with low price-dividend ratios to assets with high price-dividend ratios (or from assets with high share ratios to assets with low share ratios).

Figure 9 shows the relationship between expected excess return and the cash flow duration. Specifically, cash flow durations are recorded on the horizontal axis following the order of price-dividend ratio that is simulated from the model. Indirect durations are proportional to price-dividend ratios, and the horizontal axis represents value decile.

[Insert Figure 9]

The upper panel of Figure 9 shows expected excess return against the direct cash flow duration measure. For a direct measure of cash flow duration, we use $T = 29$ years as we cover 1983 to 2011 in the data.[26] Bottom part of Figure 9 shows expected excess return against indirect cash flow duration. Both figures clearly show downward sloping equity term structures, consistent with data.

Our empirical result shows that assets with low (high) price-dividend ratios have more (less) diverse investors' opinions and larger (smaller) idiosyncratic cash flow risks. In addition, the model simulation matches assets with low (high) price-dividend ratio with short (long) cash flow durations. The economic mechanism that explains the value premium produces equity term structure consistent with data. Finally, we find that our study can satisfy conditions suggested by the latest study about what a desirable asset pricing model should be. Lochstoer and Tetlock (forthcoming) empirically find and argue that an asset pricing model that is consistent with the data should have following properties; 1) cash flow risks have little relation to aggregate market movement, 2) firms' anomaly returns should be closely related to firms' characteristics, and 3) cash flow risk drives firm risk or errors in investors' expectations. In our model, cross-sectional return variation (especially along the value decile) is mainly driven by cash flow risk, not by aggregate discount risk.[27] Plus cash

[26]For discount rate, as mentioned before, we use expected excess return for each asset in the cross section. Though we do not report all the duration results, we observe that varying discount rates (to aggregate expected excess return) and T does not change the reported result.

[27]This is because investors' belief difference prices idiosyncratic cash flow risk which captures the major portion of the whole cash flow risk. In particular, investors' belief difference shows up in the stochastic discount factor as a second pricing component and it has a zero correlation with shocks to aggregate cash

flow risk is governed by idiosyncratic part of cash flows such as firms' characteristics; firms' growth potential (i.e., $\bar{s}$), mean-reversion speed (i.e., ϕ_s), and idiosyncratic cash flow risk (i.e., $v_{s,I}$). Further, heterogeneous beliefs about firms' individual cash flows create and amplify the effect of idiosyncratic cash flow risk. Our model largely satisfies the above suggestions.

VI. Conclusions

We introduce investors' differences in beliefs on individual cash flows in an equilibrium model. Investors interpret the long run mean of individual cash flows differently through idiosyncratic risk. Therefore, idiosyncratic cash flow risk gets priced in equilibrium, contrary to existing asset pricing models. Further, this simple modification can potentially address many issues with the cross-sectional anomalies without affecting aggregate time-series implications of asset pricing models. Indeed, our model generates data-consistent cross-sectional moments, i.e., price-dividend ratios, expected returns, Sharpe ratios, and etc. Moreover, our model also produces comparable size of the value premium observed in the data.

REFERENCES

Abel, Andrew B., 1989, Asset prices under heterogeneous beliefs: Toward a resolution of the equity premium puzzle, *Rodney L. White Center for Financial Research, Working Paper.*

——— , 1999, Risk premia and term premia in general equilibrium, *Journal of Monetary Economics* 43, 3–33.

Babenko, Ilona, Oliver Boguth, and Yuri Tserlukevich, 2016, Idiosyncratic cash flows and systematic risk, *Journal of Finance* 71, 425–456.

Bansal, Ravi, Robert Dittmar, and Christian Lundblad, 2005, Consumption, dividends, and the cross-section of equity returns, *Journal of Finance* 60, 1639–1672.

flows. Thus, it is equivalent to shocks in the stochastic discount factor in a reduced form model of Lettau and Wachter (2007); our study provides the microfoundation to their ad-hoc set up of the stochastic discount factor.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.

Basak, Suleyman, 2000, A model of dynamic equilibrium asset pricing with extraneous risk, *Journal of Economic Dynamics and Control* 24, 63–95.

——— , 2005, Asset pricing with heterogeneous beliefs, *Journal of Banking and Finance* 29, 2849–2881.

——— , and Domenico Cuoco, 1998, An equilibrium model with restricted stock market participation, *Review of Financial Studies* 11, 309–341.

Basak, Suleyman, and Michael Gallmeyer, 2003, Capital market equilibrium with differential taxation, *Review of Financial Studies* 7, 121–159.

Belo, Frederico, Pierre Collin-Dufresne, and Robert S Goldstein, 2015, Dividend Dynamics and the Term Structure of Dividend Strips, *The Journal of Finance* 70, 1115–1160.

Binsbergen, Jules H, and Ralph S J Koijen, 2017, The term structure of returns: Facts and theory, *Journal of Financial Economics* 124, 1–21.

Binsbergen, Jules van, Michael Brandt, and Ralph S J Koijen, 2012, On the Timing and Pricing of Dividends, *American Economic Review* 102, 1596–1618.

Campbell, John, Christopher Polk, and Tuomo Vuolteenaho, 2010, Growth or glamour? fundamentals and systematic risk in stock returns, *Review of Financial Studies* 23, 305–344.

Campbell, John, and Tuomo Vuolteenaho, 2004, Bad beta, good beta, *American Economic Review* 94, 1249–1275.

Campbell, John Y., and John H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–251.

Chen, Huafeng (Jason), 2017, Do cash flows of growth stocks really grow faster?, *Journal of Finance* 72, 2279–2330.

Choi, Seung Mo, Shane Johnson, Hwagyun Kim, and Changwoo Nam, 2013, Dividend policy, investment, and stock returns, *TAMU Mays Business School Working paper.*

Choi, Seung Mo, and Hwagyun Kim, 2014, Momentum effect as part of a market equilibrium, *Journal of Financial and Quantitative Analysis* 49 (1), 107–130.

Cochrane, John H., Francis Longstaff, and Pedro Santa-Clara, 2008, Two trees, *Review of Financial Studies* 21, 347–385.

Cohen, Randolph, Christopher Polk, and Tuomo Vuolteenaho, 2009, The price is (almost) right, *Journal of Finance* 64, 2739–2782.

Constantinides, Goerge M., 1990, Habit formation: A resolution of the equity premium puzzle, *Journal of Political Economy* 98, 519–543.

Cuoco, Domenico, and Hua He, 1994, Dynamic equilibrium in infinite-dimensional economices with incomplete financial markets, *Working paper*.

Da, Zhi, 2009, Cash flow, consumption risk, and the cross-section of stock returns, *Journal of Finance* 64, 923–956.

David, Alexander, 2008, Heterogeneous beliefs, speculation, and the equity premium, *Journal of Finance* 63, 41 – 83.

Dechow, Patricia M., Richard G. Sloan, and Mark T. Soliman, 2004, Implied equity durations: A new measure of equity risk, *Review of Accounting Studies* 9, 197–228.

Detemple, Jerome, and Shashidhar Murthy, 1994, Intertemporal asset pricing with heterogeneous beliefs, *Journal of Economic Theory* 62, 294–320.

Detemple, Jerome, and Angel Serrat, 2003, Dynamic equilibrium with liquidity constraints, *Review of Financial Studies* 16, 597–629.

Detemple, Jerome, and Fernando Zapatero, 1991, Asset prices in an exchange economy with habit formation, *Econometrica* 59, 1633–1657.

Diether, Karl B., Christopher J. Malloy, and Anna Scherbina, 2005, Differences of opinion and the cross section of stock returns, *Journal of Finance* 57, 2113–2141.

Duffie, Darrell, 2001, *Dynamic Asset Pricing Theory* (Princeton University Press) 3 edn.

Dumas, Bernard, Alexander Kurshev, and Raman Uppal, 2009, Equilibrium portfolio strategies in the presence of sentiment risk and excess volatility, *Journal of Finance* 64, 579–629.

Dybvig, Philip H., and Chifu Huang, 1988, Nonnegative wealth, absence of arbitrage, and feasible consumption plans, *Review of Financial Studies* 1, 377–401.

Ehling, Paul, Michael F. Gallmeyer, Christian Heyerdahl-Larsen, and Philipp K. Illeditsch, 2017, Disagreement about inflation and the yield curve, *Journal of Financial Economics, Forthcoming*.

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–465.

Gabaix, Xavier, 2012, Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance, *The Quarterly Journal of Economics* 127, 645–700.

Gallmeyer, Michael, and Burton Hollifield, 2008, An examination of heterogeneous beliefs with a short-sale constraint in a dynamic economy, *Review of Finance* 12, 323–364.

Goncalves, Andrei, 2019, Can Reinvestment Risk Explain the Dividend and Bond Term Structures?, *Working Paper.*

Gormsen, Niels, 2019, Time Variation of the Equity Term Structure, *Working Paper.*

Hansen, Lars Peter, John Heaton, and Nan Li, 2008, Consumption strikes back? measuring long-run risk, *Journal of Political Economy* 116, 260–302.

Harris, Milton, and Arthur Raviv, 1993, Differences of opinion make a horse race, *Review of Financial Studies* 6, 473 – 506.

Harrison, Michael J., and David M. Kreps, 1978, Speculative investor behavior in a stock market with heterogeneous expectations, *Quarterly Journal of Economics* 93, 323 – 336.

Hasler, Michael, and Roberto Marfè, 2016, Disaster recovery and the term structure of dividend strips, *Journal of Financial Economics* 122, 116–134.

Hong, Harrison, and Jeremy C. Stein, 2007, Disagreement and the stock market, *Journal of Economic Perspectives* 21, 109 – 128.

——— , and Jialin Yu, 2007, Simple forecasts and paradigm shifts, *Journal of Finance* 62, 1207–1242.

Huang, Chi Fu, 1987, An inter temporal general equilibrium asset pricing model: the case of diffusion information, *Econometrica* 55, 117–142.

Jouini, Elyés, and Clotilde Napp, 2007, Consensus consumer and intertemporal asset pricing with heterogeneous beliefs, *Review of Economic Studies* 74, 1149–1174.

Kiku, Dana, 2007, Is the value premium a puzzle?, *Working paper, Wharton.*

Lakonishok, Josef, Andrei Shleifer, and Robert W. Vishny, 2013, Contrarian investment, extrapolation, and risk, *Journal of Finance* 49, 1541–1578.

Lettau, Martin, and Jessica Wachter, 2007, Why is long-horizon equity less risky? a duration-based explanation of the value premium, *Journal of Finance* 62, 55 – 92.

——— , 2011, The term structures of equity and interest rates, *Journal of Financial Economics* 101, 90–113.

Lintner, John, 1965, The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics* 47, 13–37.

Liptser, Robert S., and Albert N. Shiryaev, 2001, *Statistics of Random Processes I* (Springer; 2nd rev. and exp. ed. 2001 edition: New York).

Lochstoer, Lars A, and Paul C Tetlock, forthcoming, What Drives Anomaly Returns?, *The Journal of Finance*.

Martin, Ian, 2013, The Lucas orchard, *Econometrica* 81, 55–111.

Menzly, Lior, Tano Santos, and Pietro Veronesi, 2004, Understanding return predictability, *Journal of Political Economy* 112, 1–47.

Miller, Edward, 1977, Risk, uncertainty, and divergence of opinion, *Journal of Finance* 32, 1151–1168.

Morris, Stephen, 1994, Trade with hetereogeneous prior beliefs and asymmetric information, *Econometrica* 60, 1327–1347.

——— , 1996, Speculative investor behavior and learning, *Quarterly Journal of Economics* 111, 1111–1133.

Santos, Tano, and Pietro Veronesi, 2006, Labor income and predictable stock returns, *Review of Financial Studies* 19, 1–44.

——— , 2010, Habit formation, the cross section of stock returns and the cash-flow risk puzzle, *Journal of Financial Economics* 98, 385 – 413.

Sims, Christopher A., 2003, Implications of rational inattention, *Journal of Monetary Economics* 50, 665–690.

——— , 2006, Rational inattention: Beyond the linear-quadratic case, *American Economic Review Papers and Proceedings* 96, 158–163.

Varian, Hal R., 1985, Divergence of opinion in complete markets: A note, *Journal of Finance* 40, 309–317.

Weber, Michael, 2018, Cash Flow Duration and the Term Structure of Equity Returns, *Journal of Financial Economics* pp. 486–503.

Weinbaum, David, 2009, Investor heterogeneity, asset pricing and volatility dynamics, *Journal of Economic Dynamics and Control* 33, 1379–1397.

Xiong, Wei, and Lin Peng, 2006, Investor inattention, overconfidence and category learning, *Journal of Financial Economics* 80, 563–602.

Yang, Wei, 2007, Time-varying exposure to long-run consumption risk, *Working paper, University of Rochester*.

Yu, Jialin, 2011, Disagreement and return predictability of stock portfolios, *Journal of Financial Economics* 99, 162 – 183.

Zapatero, Fernando, 1998, Effects of financial innovations on market volatility when beliefs are heterogeneous, *Journal of Economic Dynamics and Control* 22, 597–626.

Zhang, Lu, 2005, The value premium, *Journal of Finance* 60, 67–104.

Table I An Overview of Cash Flow Risks and Belief Differences – This table displays total cash flow risk, aggregate (systematic) cash flow risk, and idiosyncratic cash flow risk. We investigate cash flows of decile portfolios sorted by book-to-market ratios from 1983 to 2011 with CRSP-COMPUSTAT merged data set. Cash flow is defined by the sum of dividend and share repurchase for each decile portfolio: D_s for $s = 1, \cdots, 10$. Aggregate cash flow is the sum of all cash flows of all 10 portfolios such that $D = \sum_{i=1}^{10} D_s$. A systematic cash flow risk is measured by $Cov\left(\frac{dD_s}{D_s}, \frac{dD}{D}\right)$, and a total cash flow risk is simply measured by the variance of dD_s/D_s. A simple idiosyncratic cash flow risk is measured by the variance of estimated error terms from the regression, $(dD_{s,t}/D_{s,t}) = \beta_0 + \beta_1(dD/D)_t + \epsilon_{s,t}$, where t indicates time. For the measure of investors' belief differences (BD), we compute the coefficient of variation of analysts' earnings forecasts, i.e., standard deviation of earnings forecasts divided by absolute mean of earnings forecasts, from the I/B/E/S data set.

	Cash Flow Risk									
	Growth									Value
Portfolio	1	2	3	4	5	6	7	8	9	10
$Var\left(\frac{dD_s}{D_s}\right)$	0.0472	0.0880	0.0776	0.0468	0.0921	0.0618	0.3529	0.3396	1.6807	1.4288
$Cov\left(\frac{dD_s}{D_s}, \frac{dD}{D}\right)$	0.0152	0.0211	0.0174	0.0113	0.0171	0.0079	0.0121	0.0124	0.0425	0.0347
$\sum_t \hat{\epsilon}_{s,t}^2/(N-2)$	0.0383	0.0708	0.0660	0.0420	0.0810	0.0596	0.3482	0.3346	1.6152	1.3860
BD	0.045	0.053	0.081	0.085	0.102	0.126	0.174	0.160	0.273	0.419

Table II Characteristics of Decile Portfolios – This table shows basic statistics of characteristics of decile portfolios sorted by book-to-market ratios from 1983 to 2011 for CRSP-COMPUSTAT merged data set. For the measure of investors' belief differences, we compute coefficient of variation of analysts' earnings forecasts, i.e., standard deviation of earnings forecasts divided by absolute mean of earnings forecasts, from the I/B/E/S data set. $v_{s,A}$ is pinned down by the unconditional covariance between the share process and the aggregate cash flow process, i.e., $\Xi^{CF} = v_{s,A}\sigma_{D,A}$. $v_{s,I}$ can be computed from the identification (normalization) condition imposed on the share process. Details of the estimation can be found in Appendix A. Mean-reverting coefficients, ϕ_s, are estimated by generalized least squares method. Coefficient of variation of the cash flow share, s_t, is defined as the ratio of the standard deviation of s_t to the mean of the share s_t, and indicated by $CV(s_t)$. $\bar{s}$ is the time-series average of the share s_t.

Cash Flow Characteristics										
	Growth									Value
Portfolio	1	2	3	4	5	6	7	8	9	10
$v_{s,A}$	-0.059	-0.032	-0.052	-0.089	-0.061	-0.107	-0.078	-0.076	0.055	0.044
$v_{s,I}$	0.046	0.033	0.023	0.017	0.005	0.016	0.278	0.263	1.473	1.232
ϕ_s	0.104	0.126	0.312	0.142	0.146	0.159	0.187	0.267	0.185	0.075
$\bar{s}$	0.131	0.108	0.080	0.077	0.078	0.064	0.067	0.059	0.051	0.036
Avg$(\frac{\bar{s}}{s_t})$	1.249	1.214	1.140	1.209	1.224	1.208	1.315	1.290	1.444	2.140
$CV(s_t)$	0.415	0.40	0.31	0.39	0.431	0.40	0.614	0.499	0.713	1.08
BD ($\propto -\bar{\eta}$)	0.045	0.053	0.081	0.085	0.102	0.126	0.174	0.160	0.273	0.419

Table III The Share Ratios of Decile Portfolios – For each of 10 portfolios sorted by book-to-market ratios, we report quantile values of the share ratios of the portfolio, average values of the share ratios in each quantile group of the portfolio, and finally the percentage of the share ratios of the portfolio that are less than 1 from 1983 to 2011 in CRSP-COMPUSTAT(CCM) merged data set.

	CRSP-COMPUSTAT, 1983 to 2011									
	Growth									Value
Portfolio	1	2	3	4	5	6	7	8	9	10
Quantile value	0.821	0.829	0.867	0.867	0.798	0.819	0.900	0.814	0.791	0.817
	1.037	1.016	1.011	1.056	1.079	1.074	1.129	1.075	1.304	1.183
	1.476	1.442	1.228	1.326	1.450	1.373	1.760	1.594	1.794	2.779
	11.107	10.886	8.654	11.020	6.640	6.434	3.912	4.684	5.800	12.115
Average in Quantile	0.660	0.696	0.743	0.694	0.639	0.658	0.614	0.624	0.551	0.566
	0.922	0.909	0.942	0.961	0.960	0.961	1.007	0.952	1.084	0.989
	1.232	1.218	1.109	1.170	1.250	1.192	1.399	1.279	1.54	1.832
	2.184	2.032	1.765	2.012	2.048	2.023	2.239	2.306	2.601	5.172
% of share ratio < 1	45.4	49.1	46.8	42.2	40.8	40.2	36.8	42.0	32.5	37.4

Table IV Model Simulation – This table shows simulation results of the model. 200 firms for 5,000 months were simulated and reported for only the last 2,000 months. Individual expected excess returns are generated according to the equation (IV.8). Individual assets are sorted into decile portfolios based on simulated price-dividend ratios following (IV.5). Relative contributions to the expected excess returns are computed following equations (IV.11) and (IV.12). When simulating individual firms, we take parameter values based on Table II. Average individual aggregate cash flow risk parameters, v_{sA}, are from -0.1177 to 0.0605 and the average individual idiosyncratic cash flow risk parameters, v_{sI}, are from 0.005 to 1.355 across the value-decile. For instance, we create a uniform distribution with $[-10\%, +10\%]$ interval around the given cash flow risk estimate for each portfolio in the first two rows in Table II. Then, we randomly choose 20 values from the uniform distribution. We repeat this 10 times for each aggregate cash flow risk and idiosyncratic cash flow risk. The same procedure is applied to the selection of mean-reversion coefficients, ϕ_s and investors' differences in beliefs either. For aggregate parameters, we use $(\mu_D, \sigma_{D,A}, \gamma, \delta, h_1, h_2) = (0.0172, 0.1524, 3.7, 1.9, 0.0106, 0.095)$. These parameters are chosen to match key aggregate moments such as average return, volatility, aggregate price-dividend ratio, riskless rate, and etc. Calibration procedure for these aggregate parameters is provided in the Internet Appendix. All reported moments are computed based on simulations of individual cash flows, approximate equilibrium price-dividend ratios, and approximate equilibrium expected excess returns based on parameters that are chosen by the procedure described above.

	Growth									Value
Portfolio	1	2	3	4	5	6	7	8	9	10
Expected excess return	0.34%	0.34%	0.35%	0.35%	0.36%	0.38%	0.41%	0.42%	0.50%	1.0%
Average ln(P/D)	8.45	8.22	7.92	7.61	7.39	6.79	6.49	6.39	6.08	5.01
Cash Flow Risk Premium	0.02%	0.02%	0.03%	0.04%	0.05%	0.08%	0.12%	0.13%	0.22%	0.76%
Discount Rate Risk Premium	0.32%	0.32%	0.32%	0.32%	0.31%	0.30%	0.29%	0.29%	0.28%	0.24%
Idiosyncratic Cash Flow Risk Premium	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%	0.02%	0.09%	0.52%
Average Share Ratio	1.003	1.005	0.997	0.984	0.984	0.979	1.126	1.117	3.428	2.929
Sharpe ratio	0.07	0.072	0.077	0.081	0.083	0.089	0.091	0.095	0.108	0.187
$CV(s_t)$	0.0191	0.0124	0.0168	0.0260	0.0168	0.0325	0.0862	0.0822	0.5664	0.5063
Belief Difference $(-\eta)$	0.042	0.052	0.078	0.1	0.115	0.13	0.14	0.14	0.16	0.20

Table V Summary Statistics of the Data – Panel A summarizes basic statistics for the market portfolio from 1983 to 2011 on a monthly basis. Mean excess return of the market, $\overline{R^e_M}$, is the average of excess returns ($\hat{E}(R - r_f)$) on the market portfolio, and r_f is the risk-free rate. Panel B and C summarize key cross sectional moments for decile portfolios sorted on book-to-market ratio from CRSP-COMPUSTAT merged data set (CCM) and CRSP-COMPUSTAT-I/B/E/S merged data set (CCIM) respectively. Returns and volatilities are expressed in percentage.

Panel A: Summary statistics on the market portfolio

$\overline{R^e_M}$	$\overline{\sigma_{R^e_M}}$	Sharpe ratio	$\overline{r_f}$	$\overline{\sigma_{r_f}}$
0.57%	4.57	0.126	0.036%	0.22

Panel B: Statistics on decile portfolio CCM

	Growth									Value
Portfolio	1	2	3	4	5	6	7	8	9	10
$\hat{E}(R - r_f)$	0.89%	0.99%	1.04%	0.99%	0.96%	1.10%	1.07%	1.06%	1.07%	1.41%
Mean B/M	0.15	0.309	0.415	0.514	0.614	0.72	0.84	0.99	1.22	2.43
Mean ln(P/D)	5.07	4.81	4.75	4.61	4.53	4.52	4.43	4.42	4.49	4.52
Sharpe Ratio	0.103	0.13	0.139	0.127	0.121	0.162	0.14	0.146	0.123	0.152

Panel C: Statistics on decile portfolio CCIM

	Growth									Value
Portfolio	1	2	3	4	5	6	7	8	9	10
$\hat{E}(R)$	0.953%	1.00%	1.02%	0.97%	0.91%	1.08%	1.1%	1.01%	1.04%	1.53%
Mean B/M	0.167	0.307	0.416	0.513	0.615	0.721	0.841	0.986	1.218	2.08
Mean ln(P/D)	5.80	5.67	5.50	5.37	5.32	5.29	5.09	5.35	5.19	5.63
Sharpe Ratio	0.1139	0.1271	0.1307	0.1193	0.1075	0.1431	0.1356	0.1228	0.1126	0.1477

Table VI The Value Effect and Belief Difference – Panel A summarizes cross sectional return differences in the value decile in CRSP-COMPUSTAT-I/B/E/S (CCIM) merged data set from 1983 to 2011 on a monthly basis and the corresponding estimates of the average belief differences (BD), and the book-to-market ratios (B/M). Panel B summarizes the value effect in the same data set over the same time period. In panel B, assets are first sorted into three groups based on the magnitude of belief differences from 1983 to 2011. And in each group of belief difference, assets are sorted into three groups based on book-to-market ratio. Then, the value premium in each group is computed as the difference between the average value-weighted return of high B/M group and that of low B/M group.

Panel A: Value deciles portfolios in CCIM

	Growth									Value
Portfolio	1	2	3	4	5	6	7	8	9	10
$\hat{E}(R)$ (%)	0.95	1.00	1.02	0.97	0.91	1.08	1.10	1.01	1.04	1.53
Average B/M	0.17	0.31	0.42	0.51	0.62	0.72	0.84	0.99	1.22	2.08
BD ($\propto -\bar{\eta}$)	0.05	0.05	0.08	0.09	0.10	0.13	0.17	0.16	0.27	0.42

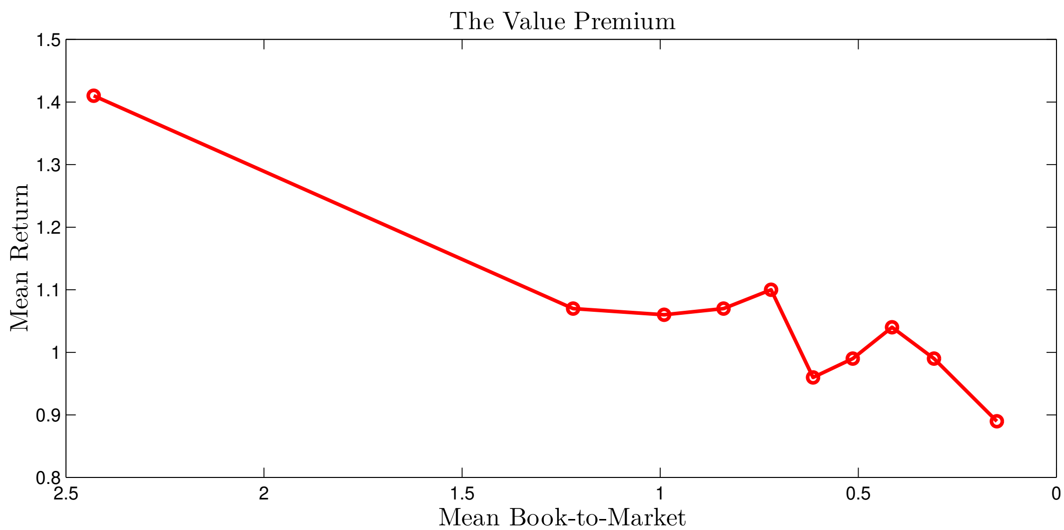
Panel B: Value Premium and Belief Difference in CCIM

	Low BD	Mid BD	High BD
Value Premium	0.225%	0.227%	0.762%**
t-value	1.04	1.258	2.39

Table VII Cash Flow Risks – This table shows models' aggregate cash flow risk and idiosyncratic cash flow risk in the cross section. Aggregate cash flow risk (Agg. CF Risk) in the cross section is measured by unconditional covariance between individual cash flow process and aggregate cash flow process following Menzly, Santos, and Veronesi (2004). Unconditional covariance, that is obtained by evaluating the conditional covariance, is approximated as $Cov_t\left(\frac{dD_s(t)}{D_s(t)}, \frac{dD_t}{D_t}\right) = \sigma_{DA}^2 + \Xi_s^{CF} - s_t\Xi_s^{CF} - (1-s_t)\Xi_{(1-s)}^{CF} \approx \sigma_{D,A}^2 + v_{s,A}\sigma_{D,A}$ at $s_t = \bar{s}$. Thus, aggregate cash flow risk in the cross-section is approximately represented by $\sigma_{D,A}^2 + v_{s,A}\sigma_{D,A}$. Idiosyncratic cash flow risk (Idio. CF Risk) parameter, $v_{s,I}$, is priced only through investors' belief differences such that $-v_{s,I}\bar{\eta}_s$ measures the magnitude of priced idiosyncratic cash flow risk in the model.

Cash-Flow Risks in the Cross-Section

	Growth									Value
Portfolio	1	2	3	4	5	6	7	8	9	10
Agg. CF Risk	0.0142	0.0184	0.0513	0.0097	0.0139	0.0069	0.0114	0.0116	0.0316	0.0299
Idio. CF Risk	0.0022	0.0018	0.0015	0.0012	0.0000	0.0018	0.0344	0.0350	0.2087	0.2384



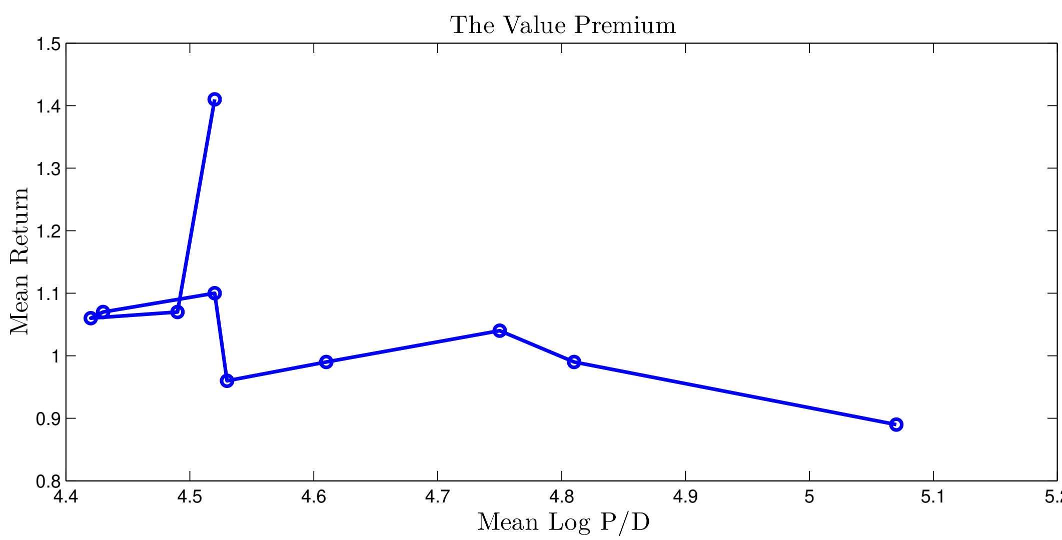


Figure 1. The Value Premium in the Data – January, 1983 to December, 2011. The data is from CRSP-COMPUSTAT merged data set. Stocks are sorted on both book-to-market ratios and price-dividend ratios whose breakpoints are given in Kenneth French's Data Library.

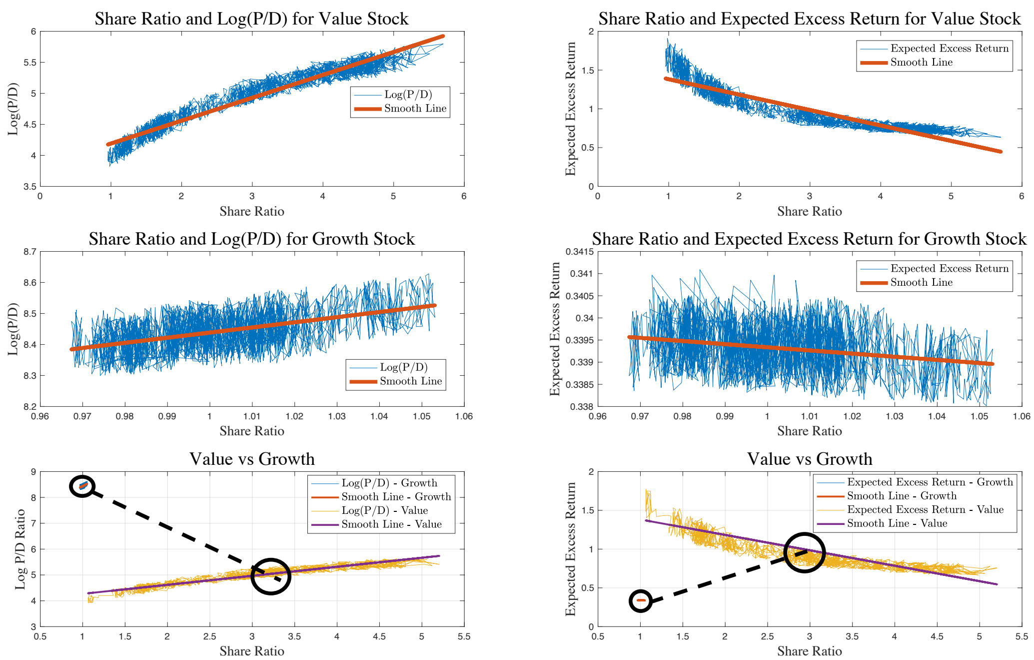
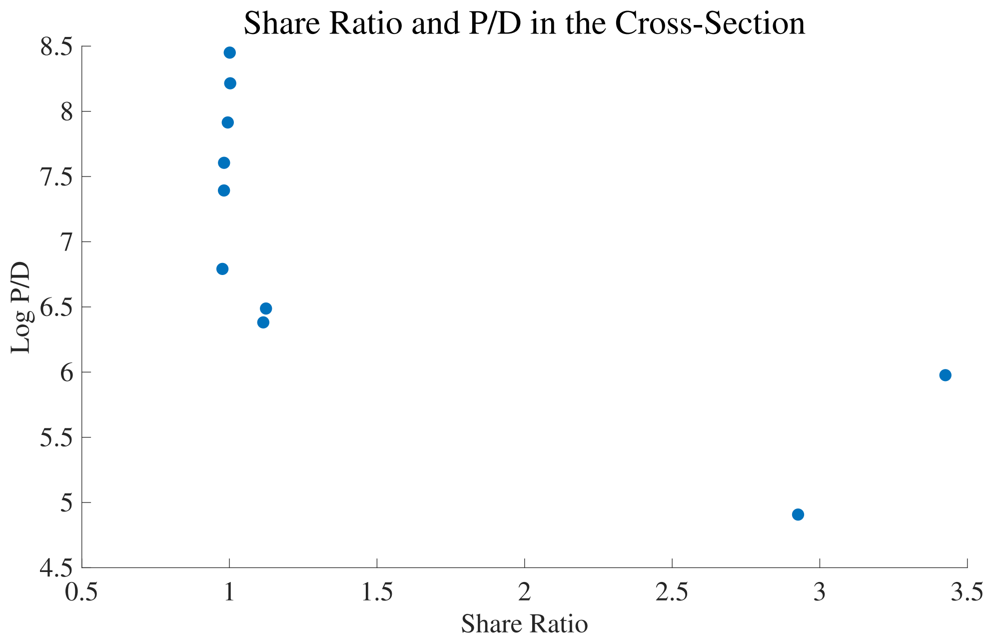


Figure 2. Simulated Time-Series Relations among Price-Dividend Ratio, Share Ratios, and Expected Returns for the Value and Growth Stocks – This Figure shows the time-series relationship between the share ratio, price-dividend ratio, and returns from the simulation of the model. For brevity, we show cases only with value and growth stocks in the value decile. The left column shows the simulated time-series relationship between price-dividend ratios and the share ratios for value portfolio and growth portfolio. The last row draw both graphs on the same axis. The right column shows the simulated time-series relationship between the share ratios and returns for value and growth stocks. The last row displays both value and growth portfolios on the same graph for comparison. Circles indicate time-series averages of the value and growth portfolios, respectively, and the dashed lines connect the circles to illustrate cross-sectional relations. The smaller (bigger) circle refers to the growth (value) portfolio.



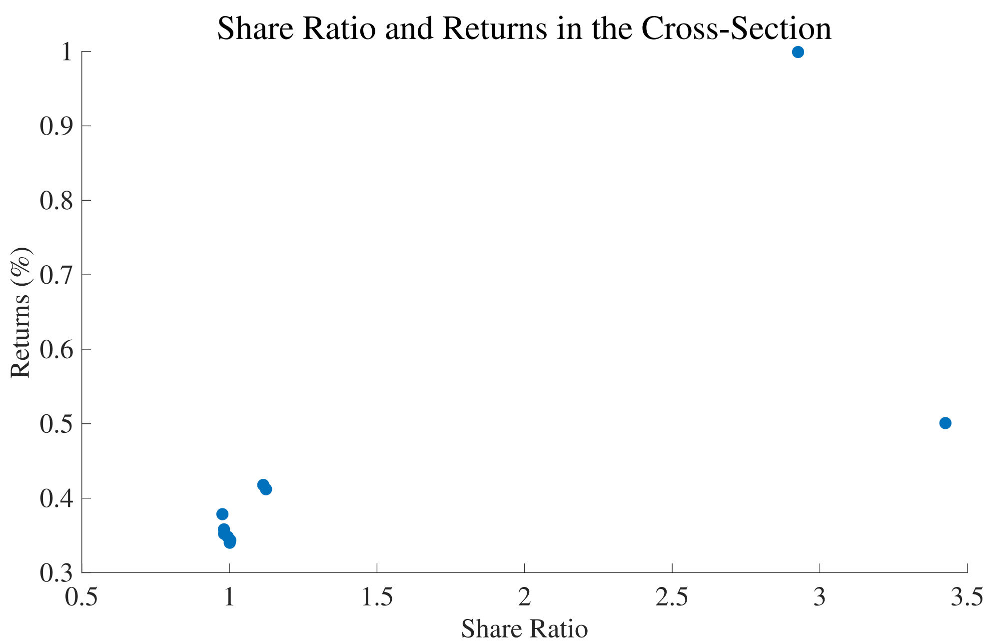


Figure 3. Simulated Cross Sectional Relation between Price-Dividend Ratios (Expected Excess Returns) and the Share Ratios for Value and Growth Stocks – Simulated scatter plots between the share ratios and price-dividend ratios as well as average returns in the value decile are drawn. Upper Figure shows the cross sectional relationship between the share ratio and price-dividend ratio in the value decile. Bottom Figure shows the cross sectional relationship between the share ratio and average returns in the value decile.

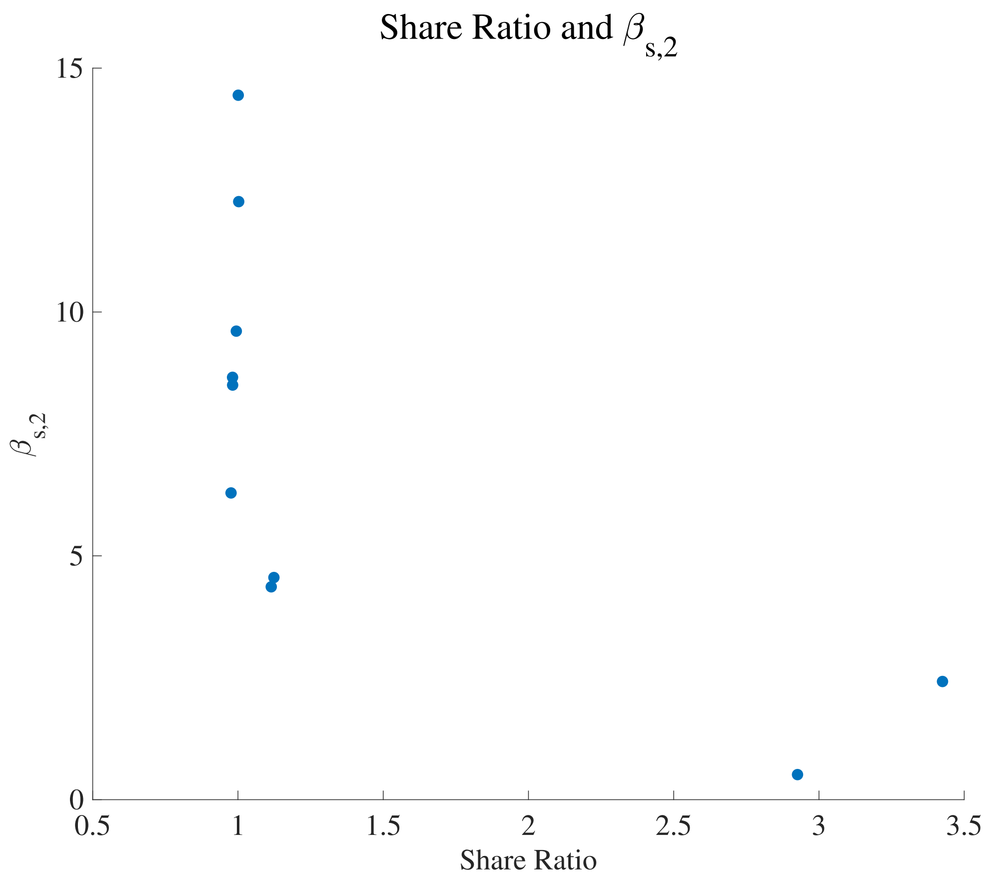
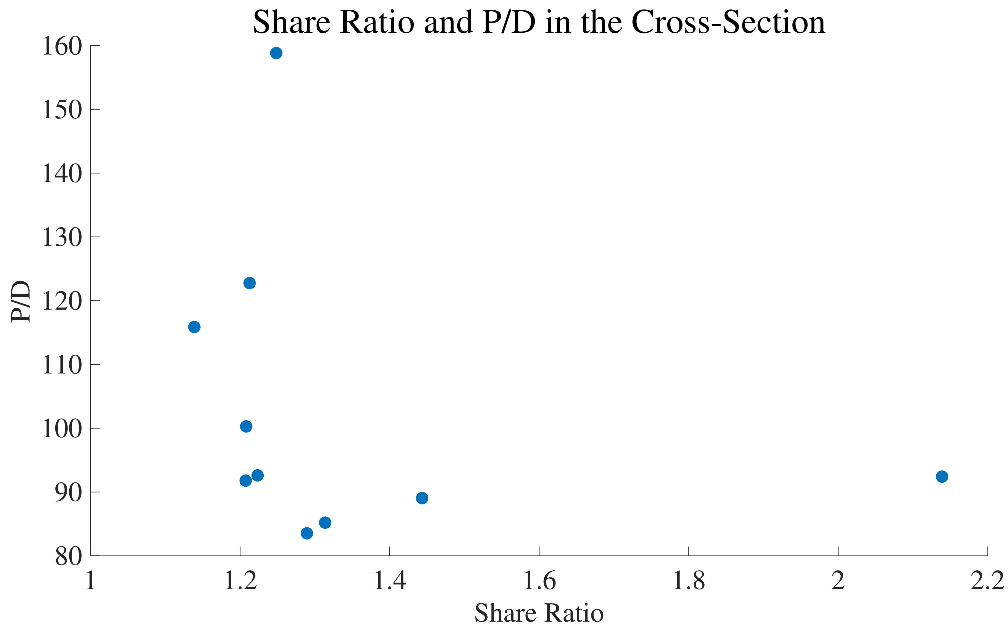


Figure 4. Simulated Cross Sectional Relation between the Share ratio and $\beta_{s,2}$ − Scatter plot between average values of simulated $\beta_{s,2}$ and average share ratios for assets sorted by price-dividend ratios. There is a clear negative relationship between the two. In particular, very high values of $\beta_{s,2}$ are associated with very low share ratios and very low $\beta_{s,2}$'s are associated with very high share ratios.



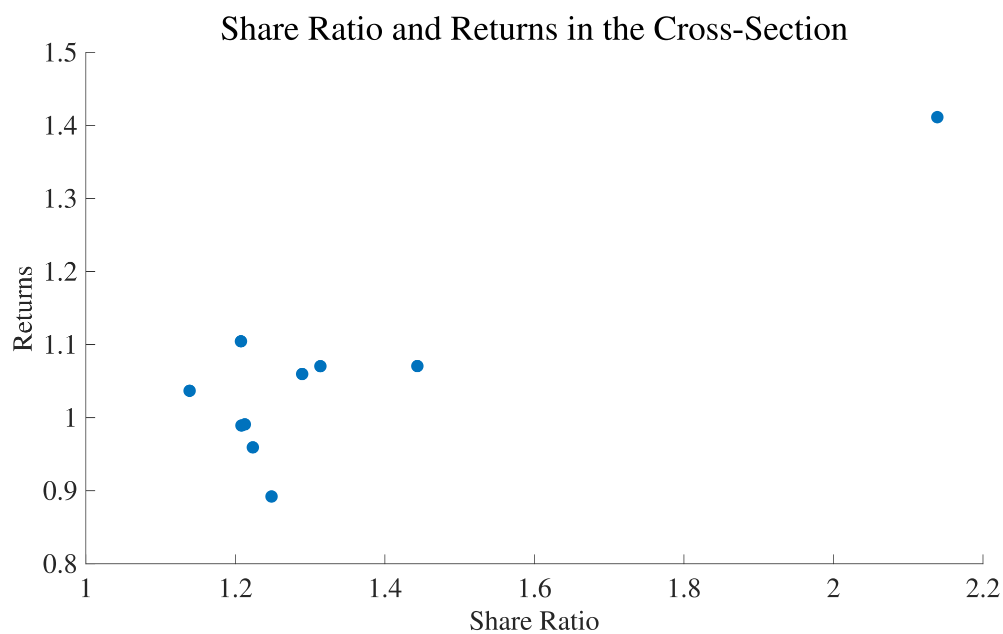


Figure 5. Empirical Cross-Sectional Relation between Price-Dividend Ratios (Average Returns) and the Share Ratios for Value and Growth Stocks (Data) – Emprical scatter plots between the share ratios and price-dividend ratios as well as average returns in the value decile are drawn. Upper Figure shows the cross sectional relationship between the share ratios and price-dividend ratios in the value decile. Bottom Figure shows the cross sectional relationship between the share ratios and average returns in the value decile.

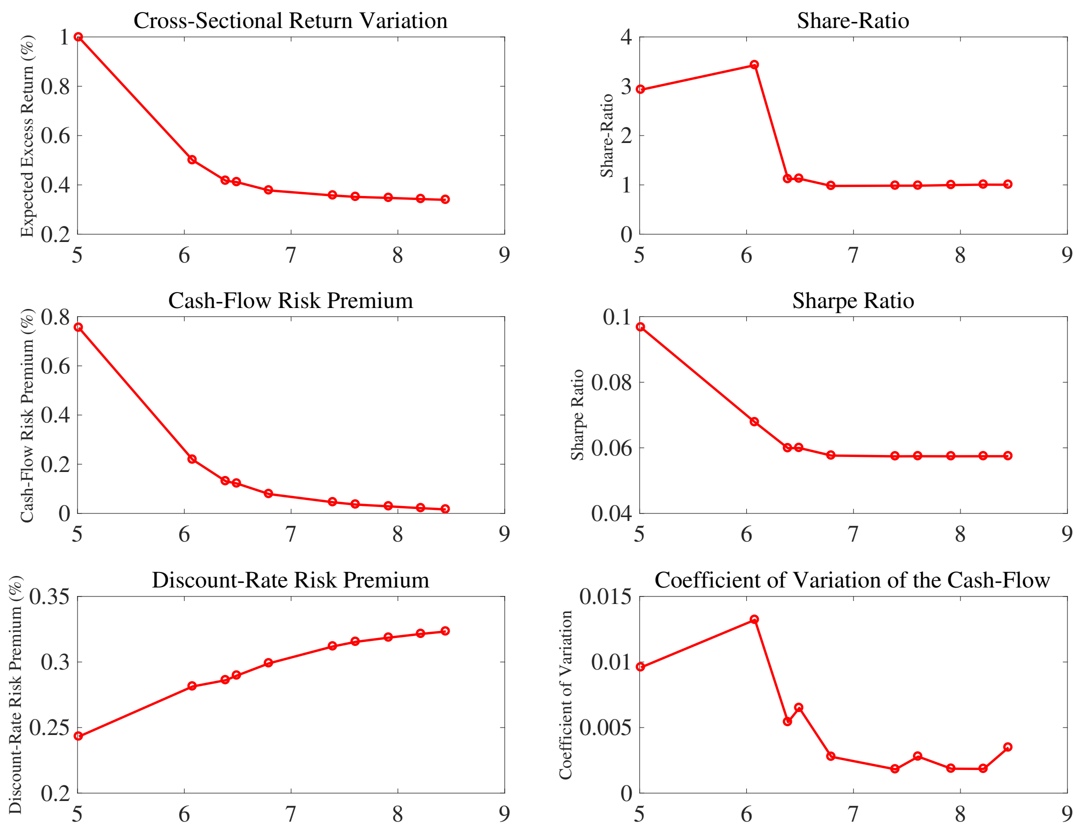


Figure 6. Simulation Results in the Cross Section – The model is simulated with $(\gamma, \delta) = (3.7, 1.9)$ and $(h_1, h_2) = (0.0107, 0.089)$. All the equilibrium quantities for each portfolio in the cross section are drawn against corresponding price-dividend ratio. In particular, all the x-axes are log price-dividend ratios in the cross section. As we use parameters that are estimated from the actual cash flow data, underlying fundamentals such as the share ratio and the coefficient of variation of the cash flow share are consistent with the data. In the simulation, 1. the value premium arises, 2. cash flow risk premium shows the value premium, 3. discount rate risk premium shows the growth premium, 4. cash flow risk premium quantitatively dominates discount rate risk premium in the cross section, and 5. Sharpe ratios are also consistent with the data.

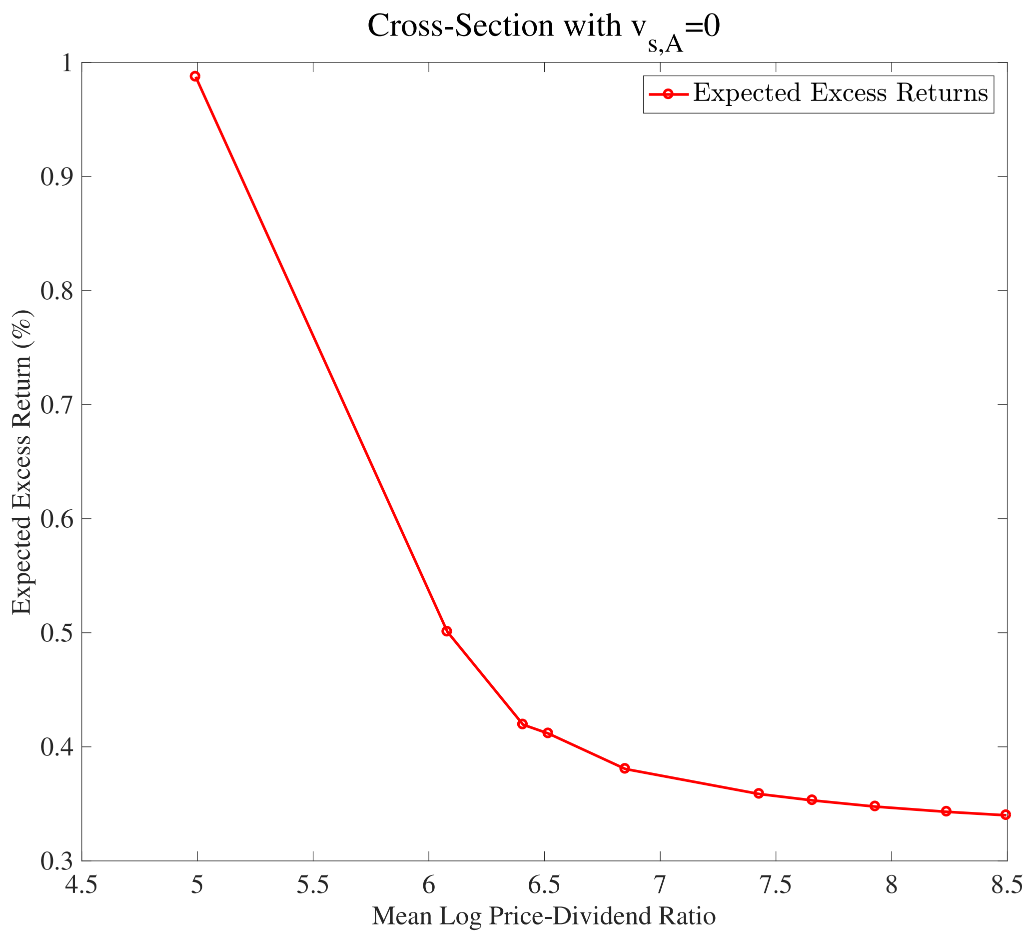


Figure 7. Simulated Cross Sectional Return Variation – The model is simulated with the same parameters that are used in the original model, but with aggregate cash flow risk being turned off: $v_{s,A}$, is set to zero. We also carried some experiments by setting $v_{s,A}$ near zero. Those experiments do not make any differences; hence we omit the results.

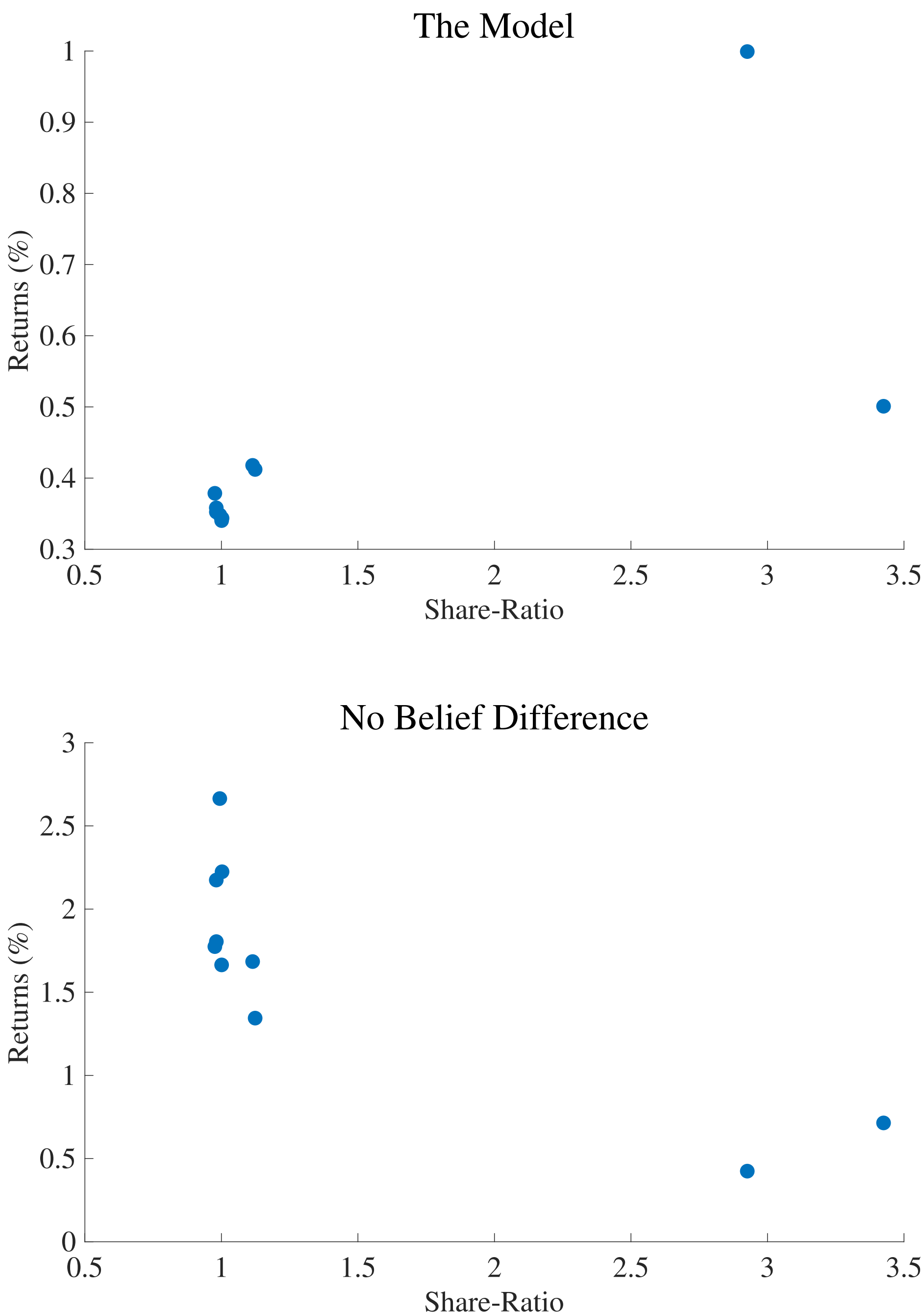
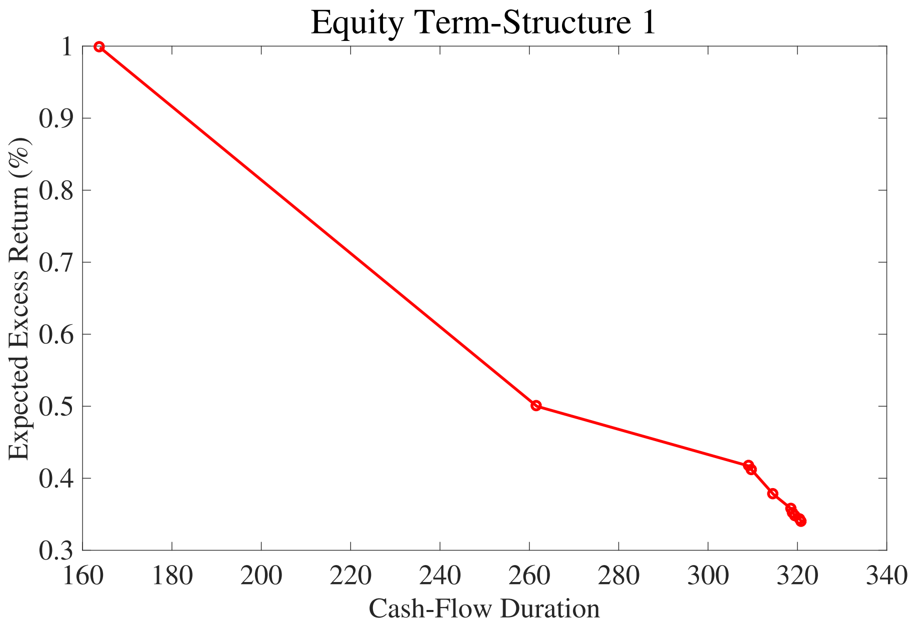


Figure 8. Comparison between the Model with and without Belief Difference – Both models are simulated with the same parameters. Thus, $(\gamma, \delta) = (3.7, 1.9)$ and $(h_1, h_2) = (0.0106, 0.095)$ are used. For the second model, we set the belief difference to zero. The cash flow share ratios are the same for both models, but resulting equilibrium quantities are different.



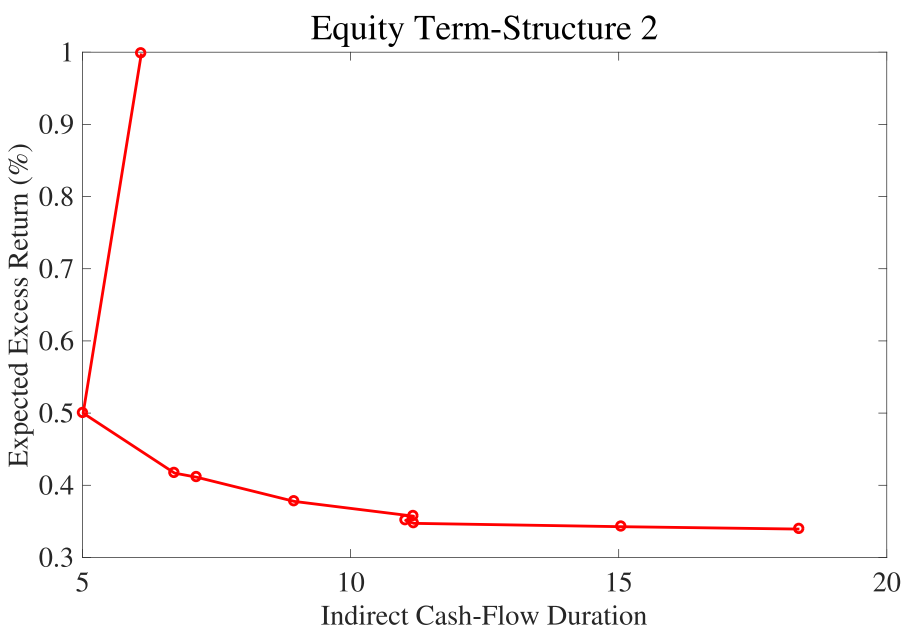


Figure 9. Equity Term Structure – The model is simulated using the same parameters that are used in the original model. Thus, $(\gamma, \delta) = (3.7, 1.9)$ and $(h_1, h_2) = (0.0106, 0.095)$ are used. Upper Figure shows the equity term structure based on accounting measure of the cash flow duration such as Dechow, Sloan, and Soliman (2004). Since we use $T = 29$ for the duration year, the computed cash flow duration is monthly measure. Bottom Figure shows the equity term structure based on the indirect cash flow duration, i.e., price elasticity with respect to cash flow multiplied by 100 for the purpose of visual clarity. Both Figures clearly show downward sloping equity term structures along the cash flow durations.

Appendix A. Basics of the Share Process

We model individual cash flow share process following Menzly, Santos, and Veronesi (2004), but in a simpler way. Specifically, we impose a simple structure on diffusion coefficients such that we treat only one idiosyncratic risk while keeping the flexibility of modeling cash flow processes. The mean-reverting share process is given by (II.5) such that

$$ds_t = \phi_s(\bar{s} - s_t)dt + s_t\boldsymbol{\sigma}(s_t)d\boldsymbol{B}_t', \tag{A1}$$

where

$$\begin{aligned}
&\boldsymbol{\sigma}(s_t) \equiv (\sigma_{s,A}(t), \sigma_{s,I}(t)),\\
&\sigma_{s,j} \equiv v_{s,j} - s_t v_{s,j} - (1-s_t)v_{(1-s),j}, \quad j = A, I\\
&d\boldsymbol{B}_t \equiv (dB_A(t), dB_I(t)),\\
&\phi_s > 0.
\end{aligned} \tag{A2}$$

Menzly, Santos, and Veronesi (2004) show that technical regularity conditions such as $s_t \geq 0$ and $\sum s_t = 1$ are satisfied in their specification. The same regularity conditions hold in our specification as the following conditions are met:

$$\begin{aligned}
&\bar{s} < 1 \quad \text{and} \quad \overline{(1-s)} < 1,\\
&\phi_s > 0,\ \overline{(1-s)} \cdot \phi_{(1-s)} \quad \text{and} \quad \phi_{(1-s)} > 0,\ \bar{s} \cdot \phi_s.
\end{aligned} \tag{A3}$$

Note that $\sigma_{m,n}$ for $m = s, (1-s)$ and $n = A, I$ are parametrically indeterminate. Hence, adding a constant vector to v_s or $v_{(1-s)}$ to the share process does not change the specification of the share process. This observation enables us to normalize v_s and $v_{(1-s)}$ such that we obtain an identification condition as follows.

$$\bar{s}\mathbf{v}_s + (1-\bar{s})\mathbf{v}_{(1-s)} = 0, \tag{A4}$$

where $\mathbf{v}_i$ is the row vector of $v_{i,A}$ and $v_{i,I}$ for $i = s, (1-s)$.[28]

Given the share process, an individual cash flow $D_s(t)$ is defined as the fraction of aggregate cash flow, i.e., $D_s(t) \equiv s_t D_t$. Applying Itô's lemma to $s_t D_t$ yields the diffusion process of an individual cash flow $D_s(t)$:

$$\frac{dD_s(t)}{D_s(t)} = \mu_{D_s}(t)dt + \sigma_{D_s,A}(t)dB_A(t) + \sigma_{D_s,I}(t)dB_I(t), \tag{A5}$$

where

$$\begin{aligned}
&\mu_{D_s}(t) \equiv \mu_D + \phi_s\left(\frac{\bar{s}}{s_t} - 1\right) + \Xi_s^{CF} - s_t\Xi_s^{CF} - (1-s_t)\Xi_{(1-s)}^{CF},\\
&\sigma_{D_s,A}(t) \equiv \sigma_{D,A} + \sigma_{s,A}(t),\\
&\sigma_{D_s,I}(t) \equiv \sigma_{s,I}(t),
\end{aligned} \tag{A6}$$

and $\Xi_s^{CF} \equiv \sigma_{D,A}v_{s,A}$ and $\Xi_{(1-s)}^{CF} \equiv \sigma_{D,A}v_{(1-s),A}$. The covariance between the share process and aggregate dividend (consumption) growth is given by

$$Cov_t\left(\frac{ds_t}{s_t}, \frac{dD_t}{D_t}\right) = \Xi_s^{CF} - \left[\Xi_s^{CF}s_t + \Xi_{(1-s)}^{CF}(1-s_t)\right]. \tag{A7}$$

By computing the unconditional covariance from the data, we obtain

$$E\left[Cov_t\left(\frac{ds_t}{s_t}, \frac{dD_t}{D_t}\right)\right] = v_{s,A} \cdot \sigma_{D,A} \equiv \Xi_s^{CF}, \tag{A8}$$

thanks to the identification condition of the share process, (A4). Thus we can pin down $v_{s,A}$ by computing unconditional covariance between the share process and aggregate cash flow process in the data.

[28] More details can be found in Menzly, Santos, and Veronesi (2004).

Note that the conditional variance of individual cash flow process is given by

$$Var_t\left(\frac{dD_s(t)}{D_s(t)}\right) = \left[\sigma_{D,A} + \sigma_{s,A}(t)\right]^2 + \left[\sigma_{s,I}(t)\right]^2. \tag{A9}$$

Total variability of individual cash flow can be estimated by unconditional variance. We estimate it by evaluating conditional variance (A9) at $s_t = \bar{s}$ together with the identification condition (A4). Thus, we obtain

$$\left(\sigma_{D,A} + v_{s,A}\right)^2 + \left(v_{s,I}\right)^2. \tag{A10}$$

By calculating unconditional variance of individual cash flow process in the data, one can recover $v_{s,I}$ since $v_{s,A}$ is already pinned down previously.

Finally by using (A4), we have

$$v_{(1-s),j} = -\frac{\bar{s}v_{s,j}}{1-\bar{s}}, \quad j = A, I, \tag{A11}$$

for both aggregate and idiosyncratic terms, which completes the estimation of fundamental cash flow risk parameters of individual cash flow process.

Appendix B. Proofs

Derivation of λ_t process. By applying Itô's lemma to λ_t process, we obtain

$$\frac{d\lambda(t)}{\lambda(t)} = \left[-r_t + \mu_{\xi^{(1)-1}(t)} - \theta_A^2(t) - \theta_I^{(1)}(t)\theta_I^{(2)}(t)\right]dt + \theta_I^{(1)}(t)dB_I^{(1)}(t) - \theta_I^{(2)}(t)dB_I^{(2)}(t), \tag{B1}$$

where

$$\mu_{\xi^{(1)-1}}(t) \equiv r_t + \theta_A^2(t) + {\theta_I^{(1)}}^2(t). \tag{B2}$$

By rearranging diffusion terms using the relation between two idiosyncratic Brownian motions, we have $\bar{\eta}_t dB_I^{(2)}(t) - \bar{\eta}_t\theta_I^{(1)}(t)dt$. The drift term in (B1) and $-\bar{\eta}_t\theta_I^{(1)}(t)dt$ are summed up to zero since $\bar{\eta}_t = \eta_t^{(2)} - \eta_t^{(1)} = \theta_I^{(1)}(t) - \theta_I^{(2)}(t)$. Therefore,

$$\frac{d\lambda_t}{\lambda_t} = \bar{\eta}_t dB_I^{(2)}(t). \tag{B3}$$

□

Derivation of market prices of risks. Since the financial market is dynamically complete, we can use price processes of the market portfolio and the asset with the share process, s_t, for determining the market prices of risks as follows.

$$\begin{aligned}
\begin{pmatrix}\theta_A \\ \theta_I^{(k)}\end{pmatrix} &= \begin{pmatrix}\sigma_{P_M,A} & 0 \\ \sigma_{P_s,A} & \sigma_{P_s,I}\end{pmatrix}^{-1}\begin{pmatrix}\mu_{P_M} - r \\ \mu_{P_s}^{(k)} - r\end{pmatrix}\begin{pmatrix}dB_A \\ dB_I^{(k)}\end{pmatrix} \\
&= \frac{1}{\sigma_{P_M,A}\sigma_{P_s,I}}\begin{pmatrix}\sigma_{P_s,I}\left(\mu_P - r\right) \\ -\sigma_{P_s,A}\left(\mu_P - r\right) + \sigma_{P_M,A}\left(\mu_{P_s}^{(k)} - r\right)\end{pmatrix} \\
&= \begin{pmatrix}\frac{\mu_{P_M}-r}{\sigma_{P,A}} \\ -\frac{\sigma_{P_s,A}}{\sigma_{P_s,I}}\frac{1}{\sigma_{P,A}}\left(\mu_{P_M} - r\right) + \frac{1}{\sigma_{P_s,I}}\left(\mu_{P_s}^{(k)} - r\right)\end{pmatrix} \\
&= \begin{pmatrix}\frac{\mu_{P_M}-r}{\sigma_{P_M,A}} \\ -\frac{\sigma_{P_s,A}}{\sigma_{P_s,I}}\theta_A + \frac{1}{\sigma_{P_s,I}}\left(\mu_{P_s} - r\right) - \eta^{(k)}\end{pmatrix}.
\end{aligned} \tag{B4}$$

□

Proof of Proposition 1. To solve for equilibrium, we construct a representative investor's utility function following Huang (1987) and Cuoco and He (1994). This method has been applied in many equilibrium studies such as Basak and Cuoco (1998), Basak

(2000), Detemple and Serrat (2003), Basak and Gallmeyer (2003), and Gallmeyer and Hollifield (2008). λ_t process is a stochastic weight in the representative investor's periodic utility function for computing equilibrium as follows.

$$U(C,\lambda) = \max_{c_1+c_2\le D} \frac{\lambda_t}{\lambda_1}\frac{(c_1/X)^{1-\gamma}}{1-\gamma} + \frac{1}{\lambda_2}\frac{(c_2/X)^{1-\gamma}}{1-\gamma}, \tag{B5}$$

where C is the aggregate consumption; therefore, $C \equiv D$.

Using the stochastic weight process, we can write the consumption goods clearing condition as

$$\overset{*}{c}_1(\xi^{(2)}(t)/[\lambda_2\lambda(t)],t) + \overset{*}{c}_2(\xi^{(2)}(t)/\lambda_2,t) = D(t). \tag{B6}$$

By solving equation (B6), we obtain the required result.

□

Proof of Proposition 2. Note that equilibrium stock price can be represented by either one of state price densities across investors thanks to the Radon-Nikodym derivative, i.e., λ_t process. Equilibrium price of an asset with the share process s_t is represented as follows.

$$\begin{aligned}
P_s(t) &= E_t^{(2)}\left[\int_t^\infty \frac{\xi_\tau^{(2)}}{\xi_t^{(2)}} s_\tau D_t d\tau\right] \\
&= \frac{1}{\left(1+\lambda_t^{1/\gamma}\right)^\gamma \left(\frac{1}{X_t}\right)^{1-\gamma} D_t^{-\gamma}} E_t^{(2)}\left[\int_t^\infty \left(1+\lambda_\tau^{1/\gamma}\right)^\gamma \left(\frac{1}{X_\tau}\right)^{1-\gamma} D_\tau^{-\gamma} s_\tau D_\tau d\tau\right] \\
&= \frac{s_t D_t}{s_t\left(1+\lambda_t^{1/\gamma}\right)^\gamma \left(\frac{D_t}{X_t}\right)^{1-\gamma}} E_t^{(2)}\left[\int_t^\infty s_\tau \left(1+\lambda_\tau^{1/\gamma}\right)^\gamma \left(\frac{D_\tau}{X_\tau}\right)^{1-\gamma} d\tau\right] \\
&= \frac{s_t D_t}{q_t} E_t^{(2)}\left[\int_t^\infty q_\tau d\tau\right] \\
&= \frac{D_s(t)}{q_t} E_t^{(2)}\left[\int_t^\infty q_\tau d\tau\right],
\end{aligned} \tag{B7}$$

where

$$\begin{aligned}
q_t &\equiv s_t z_t H_t, \\
z_t &\equiv \left(1+\lambda_t^{1/\gamma}\right)^\gamma, \\
H_t &\equiv \left(\frac{D_t}{X_t}\right)^{1-\gamma}.
\end{aligned} \tag{B8}$$

In Campbell and Cochrane (1999) and Santos and Veronesi (2010), the consumption surplus ratio, S_t^γ, plays as shocks to aggregate discount rate as a part of stochastic discount factor. Similarly, in our model, H_t plays such a role as it governs aggregate risk. It represents aggregate shock to stochastic discount factor as well as an indicator of economic conditions. When the habit ratio, $\bar{H}/H_t$, is high, the economy goes well and vice versa. By applying Ito's lemma to the process $H_t \equiv (D_t/X_t)^{1-\gamma}$, we get:

$$d\left(\frac{D_t}{X_t}\right)^{1-\gamma} = (1-\gamma)\left(\frac{D_t}{X_t}\right)^{1-\gamma}\left\{\left[\mu_D - \lambda\left(\frac{D_t}{X_t}-1\right) - \frac{1}{2}\gamma\sigma_{DA}^2\right]dt + \sigma_{DA}dB_A\right\}. \tag{B9}$$

Following Campbell and Cochrane (1999), Menzly, Santos, and Veronesi (2004), and Santos and Veronesi (2010), we assume a simpler process of H_t as following:[29]

$$dH_t = h_1(\bar{H} - H_t)dt + h_2 H_t dB_A(t). \tag{B10}$$

Note that the diffusion process of $\lambda_t^{1/\gamma}$ is given by

$$\frac{d\lambda_t^{1/\gamma}}{\lambda_t^{1/\gamma}} = \alpha_1(t)dt + \alpha_2(t)dB_I^{(2)}, \tag{B11}$$

[29]The assumption on the process H_t is very similar to the one in Santos and Veronesi (2010).

where

$$\alpha_1(t) \equiv \frac{1}{2}\frac{1}{\gamma}\left(\frac{1}{\gamma}-1\right)\bar{\eta}_t^2, \quad \alpha_2(t) \equiv \frac{1}{\gamma}\bar{\eta}_t. \tag{B12}$$

Using this, we obtain the diffusion process of $z_t \equiv \left(1+\lambda_t^{1/\gamma}\right)^{\gamma}$ as follows.

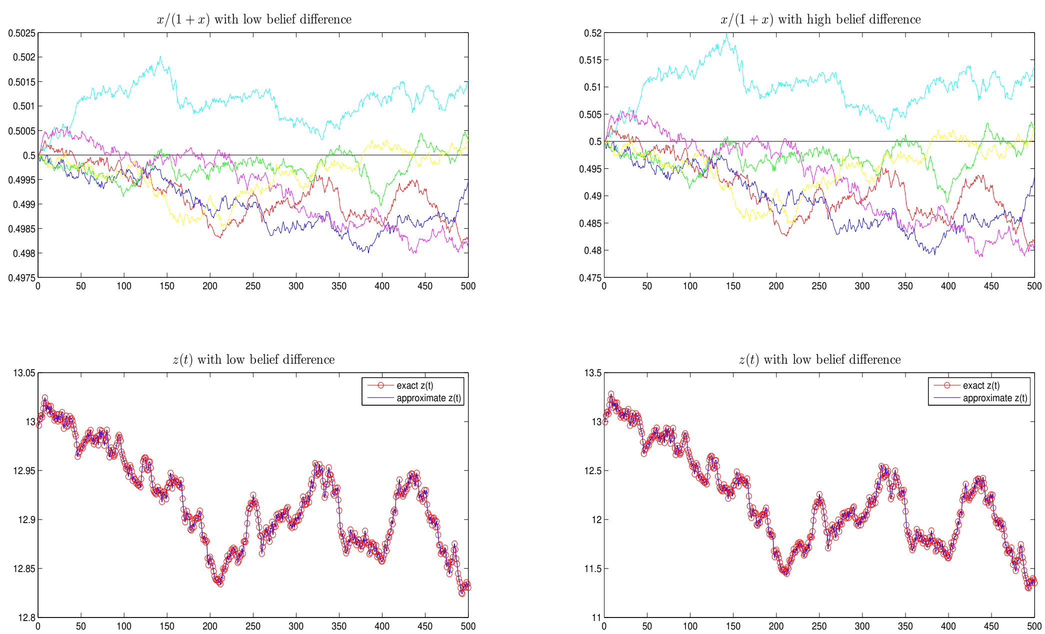
$$\frac{dz_t}{z_t} = \left[\frac{1}{2}\gamma(\gamma-1)\left(\frac{x_t}{1+x_t}\right)^2\alpha_1(t) + \gamma\left(\frac{x_t}{1+x_t}\right)\alpha_2^2(t)\right]dt + \gamma\left(\frac{x_t}{1+x_t}\right)\alpha_2(t)dB_I^{(2)}, \tag{B13}$$

where $x_t \equiv \lambda_t^{1/\gamma}$. For mathematical tractability, we approximate this process by simplifying $x_t/(1+x_t)$. Belief differences, η_t, determines the process of Radon-Nikodym derivative λ_t. In our quantitative study, we use 0.02 to 0.2 of η_t for firms in value decile, whose values are adopted from the data. Simulation shows that $x_t/(1+x_t)$ is very similar to 0.5. By plugging $\alpha_1(t)$ and $\alpha_2(t)$ into the equation above and using the approximation that $x_t/(1+x_t) \approx 1/2$ (see Figure below), we have an approximate process of z_t as follows.

$$\frac{dz_t}{z_t} \approx \tilde{\alpha}_1(t)dt + \tilde{\alpha}_2(t)dB_I^{(2)}, \tag{B14}$$

where

$$\begin{aligned} \tilde{\alpha}_1(t) &\equiv -\frac{1}{8}\left(1-\frac{1}{\gamma}\right)\bar{\eta}_t^2, \\ \tilde{\alpha}_2(t) &\equiv \frac{1}{2}\bar{\eta}_t. \end{aligned} \tag{B15}$$



Accordingly, we use this approximate z_t process for the rest of the proof. In order to get the diffusion process of q_t, we first compute the diffusion process of $z_t H_t$ as

$$\frac{d(z_t H_t)}{z_t H_t} = \mu_{zH}dt + h_2 dB_A + \tilde{\alpha}_2 dB_I^{(2)}, \tag{B16}$$

where $\mu_{zH} \equiv \tilde{\alpha}_1 + h_1\left[\bar{H}/H_t - 1\right]$. Based on this process, we have the diffusion process of q_t as follows.

$$\frac{dq_t}{q_t} = \mu_q(t)dt + \left(\sigma_{s,A}(t) + h_2\right)dB_A + \left(\tilde{\alpha}_2(t) + \sigma_{s,I}(t)\right)dB_I^{(2)}, \tag{B17}$$

where

$$\mu_q(t) \equiv \tilde{\alpha}_1(t) + h_1\left(\frac{\bar{H}}{H_t} - 1\right) + \phi_s\left(\frac{\bar{s}^{(2)}}{s_t} - 1\right) + h_2\sigma_{s,A}(t) + \tilde{\alpha}_2(t)\sigma_{s,I}(t).$$

The drift of dq_t, i.e., $q_t\mu_q(t)$, is expressed by q_t, s_tz_t and z_tH_t as follows.

$$q_t\mu_q(t) \equiv \left(\tilde{\alpha}_1(t) + \tilde{\alpha}_2(t)\sigma_{s,I}(t) + h_2\sigma_{s,A}(t) - h_1 - \phi_s\right)[q_t] + h_1\bar{H}\,[s_tz_t] + \phi_s\bar{s}^{(2)}\,[z_tH_t]\,. \tag{B18}$$

Note that:

$$\begin{aligned} d(s_tz_t) &= \left\{\left(\tilde{\alpha}_1(t) - \phi_s + \tilde{\alpha}_2(t)\sigma_{s,I}(t)\right)[s_tz_t] + \phi_s\bar{s}^{(2)}\,[z_t]\right\}dt + [\cdots]dB_A + [\cdots]dB_I^{(2)}, \\ d(z_tH_t) &= \left\{(\tilde{\alpha}_1(t) - h_1)\,[z_tH_t] + h_1\bar{H}\,[z_t]\right\}dt + [\cdots]dB_A + [\cdots]dB_I^{(2)}. \end{aligned} \tag{B19}$$

Thus, we have a key observation that z_t, q_t, s_tz_t, and z_tH_t exists in the drift of dq_t. Thus, we take a vector process $y_t \equiv [z_t, q_t, s_tz_t, z_tH_t]'$ for computing equilibrium price-dividend ratio. y_t follows a diffusion process as follows.

$$dy_t = Y_1y_tdt + \Sigma(y_t)d\boldsymbol{B'}^{(2)}, \tag{B20}$$

where $\Sigma(y_t)$ is the appropriate matrix of diffusion coefficients, $Y_1 \equiv [y_{ij}]_{4\times4}$ is the matrix of drift coefficients:

$$\begin{pmatrix} \tilde{\alpha}_1(t) & 0 & 0 & 0 \\ 0 & \tilde{\alpha}_1(t) + \tilde{\alpha}_2(t)\sigma_{s,I}(t) + h_2\sigma_{sA}(t) - h_1 - \phi_s & h_1\bar{H} & \phi_s\bar{s}^{(2)} \\ \phi_s\bar{s}^{(2)} & 0 & \tilde{\alpha}_1(t) + \tilde{\alpha}_2(t)\sigma_{s,I}(t) - \phi_s & 0 \\ h_1\bar{H} & 0 & 0 & \tilde{\alpha}_1(t) - h_1 \end{pmatrix}. \tag{B21}$$

To avoid notational abuse, we denote Y_1 as:

$$\begin{pmatrix} y_{11} & 0 & 0 & 0 \\ 0 & y_{22} & y_{23} & y_{24} \\ y_{24} & 0 & y_{33} & 0 \\ y_{23} & 0 & 0 & y_{44} \end{pmatrix}, \tag{B22}$$

where y_{11}, y_{22}, y_{33}, and y_{44} are time-varying functions of $\bar{\eta}_t$, s_t, $\sigma_{s,A}(t)$ and $\sigma_{s,I}(t)$. For the feasibility of the computation on the right hand side of the equation $(B7)$, we approximate all time-varying terms of y_{ij}'s with constants. Approximation with constant is crucial for obtaining approximate analytical solutions for the given Euler equation.

$\bar{\eta}_t$ is the time-varying component in variables that are related to belief difference such as $\tilde{\alpha}_{1,t}$ and $\tilde{\alpha}_{2,t}$. $\bar{\eta}_t$ can be approximated by the time-series average of $\bar{\eta}_t$, i.e., $\overline{\bar{\eta}_t}$. Thus, $\tilde{\alpha}_{1,t}$ and $\tilde{\alpha}_{2,t}$ are approximated with $\overline{\bar{\eta}_t}$, and are expressed by $\overline{\alpha}_1$ and $\overline{\alpha}_2$.

In order to approximate other y_{ij}'s (especially y_{22} and y_{33}), we follow Menzly, Santos, and Veronesi (2004) such that we utilize the normalization condition of the share process that can be found in Appendix A. Normalization conditions for parameters in the share process, i.e., the equation (A4), are given as follows.

$$\begin{aligned} \bar{s}v_{s,A} + (1-\bar{s})v_{(1-s),A} &= 0, \\ \bar{s}v_{s,I} + (1-\bar{s})v_{(1-s),I} &= 0. \end{aligned} \tag{B23}$$

Associated with the condition (B23), $\sigma_{s,A}$ and $\sigma_{s,I}$ are approximated by $v_{s,A}$ and $v_{s,I}$ when they are evaluated at the long-run mean of the share, $s_t = \bar{s}$. Thus we have

$$\begin{aligned} y_{22} &\approx \overline{\alpha}_1 + h_2v_{s,A} + \overline{\alpha}_2v_{s,I} - h_1 - \phi_s, \\ y_{33} &\approx \overline{\alpha}_1 + v_{s,I} + \overline{\alpha}_2 - \phi_s. \end{aligned} \tag{B24}$$

With this way, all the elements in the matrix Y_1 are approximated by constant values. Once approximation is done, then approximated expected value of $E_t^{(2)}[q_\tau]$ can be computed as follows.

$$\begin{aligned} E_t^{(2)}[q_\tau] \approx E_t^{(2)}[\tilde{q}_\tau] = E_t^{(2)}[\tilde{q}_{t+\iota}] &= e_2E_t^{(2)}[\tilde{y}_{t+\iota}] \\ &= e_2\Psi(\iota)\tilde{y}_t, \end{aligned} \tag{B25}$$

where $e_2 \equiv (0, 1, 0, 0)$,

$$\Psi(\iota) = U \exp(\Lambda \cdot \iota) U^{-1}, \tag{B26}$$

Λ is the diagonal matrix with its elements being eigenvalues of $\tilde{Y}_1$ and U is the corresponding eigenvector matrix of $\tilde{Y}_1 \equiv [\tilde{y}_{ij}]$, and finally $\tilde{}$ indicates that the variable is approximated. For mathematical tractability, we assume that all eigenvalues are negative.[30] Thus U is given by:

$$U = \begin{pmatrix} u_{11} & 0 & 0 & 0 \\ u_{21} & 1 & u_{23} & u_{24} \\ u_{31} & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}, \tag{B27}$$

where

$$\begin{aligned} u_{11} &= \frac{\tilde{y}_{11} - \tilde{y}_{44}}{\tilde{y}_{23}}, \\ u_{21} &= \frac{\tilde{y}_{24}(2\tilde{y}_{11} - \tilde{y}_{33} - \tilde{y}_{44})}{(\tilde{y}_{11} - \tilde{y}_{22})(\tilde{y}_{11} - \tilde{y}_{33})}, \\ u_{31} &= \frac{\tilde{y}_{31}(\tilde{y}_{11} - \tilde{y}_{44})}{\tilde{y}_{23}(\tilde{y}_{11} - \tilde{y}_{33})}, \\ u_{23} &= \frac{\tilde{y}_{23}}{\tilde{y}_{33} - \tilde{y}_{22}}, \\ u_{24} &= \frac{\tilde{y}_{24}}{\tilde{y}_{44} - \tilde{y}_{22}}. \end{aligned} \tag{B28}$$

The inverse matrix $U^{-1} \equiv V = [v_{ij}]$ is given by:

$$\begin{pmatrix} v_{11} & 0 & 0 & 0 \\ v_{21} & 1 & v_{23} & v_{24} \\ v_{31} & 0 & 1 & 0 \\ v_{41} & 0 & 0 & 1 \end{pmatrix}, \tag{B29}$$

where

$$\begin{aligned} v_{11} &= 1/u_{11}, \\ v_{21} &= \frac{-u_{21} + u_{24} + u_{23}u_{31}}{u_{11}}, \\ v_{23} &= -u_{23}, \\ v_{24} &= -u_{24}, \\ v_{31} &= \frac{-u_{31}}{u_{11}}, \\ v_{34} &= -u_{34}, \\ v_{41} &= -1/u_{11}. \end{aligned} \tag{B30}$$

Using these quantities, we can compute $\Psi(\iota)$. Hence we get $E_t^{(2)}[\tilde{q}_\tau] = E_t^{(2)}[\tilde{q}_{t+\iota}]$ as follows.

$$E_t[\tilde{q}_{t+\iota}] = e_2 \Psi(\iota) \tilde{y}_t = \Psi_1(\iota) z_t + \Psi_2(\iota) \tilde{q}_t + \Psi_3(\iota) s_t z_t + \Psi_4(\iota) z_t H_t, \tag{B31}$$

where

$$\begin{aligned} \Psi_1(\iota) &= v_{11} u_{21} e^{\tilde{y}_{11}\iota} + v_{21} e^{\tilde{y}_{22}\iota} + v_{31} u_{23} e^{\tilde{y}_{33}\iota} + v_{41} u_{24} e^{\tilde{y}_{44}\iota}, \\ \Psi_2(\iota) &= e^{\tilde{y}_{22}\iota}, \\ \Psi_3(\iota) &= v_{23} e^{\tilde{y}_{22}\iota} + u_{23} e^{\tilde{y}_{33}\iota}, \\ \Psi_4(\iota) &= v_{24} e^{\tilde{y}_{22}\iota} + u_{24} e^{\tilde{y}_{44}\iota}. \end{aligned} \tag{B32}$$

[30] We follow Menzly, Santos, and Veronesi (2004) for this. Our simulation study shows that all diagonal elements are indeed negative.

Therefore

$$
\begin{aligned}
E_t^{(2)}\left[\int_0^\infty q_{t+\iota}d\iota\right] &\approx \int_0^\infty E_t^{(2)}\left[\tilde{q}_{t+\iota}\right]d\iota \\
&= \int_0^\infty e_2\Psi(\iota)\tilde{y}_t d\iota \\
&= \sum_{k=1}^{4}\left[\int_0^\infty \Psi_k(\iota)d\iota\right]\tilde{y}_k(t),
\end{aligned}
\tag{B33}
$$

where $\tilde{y}_k(t)$ is the k-th row vector $\tilde{y}_t$. $\int_0^\infty \Psi_k(\iota)$'s are given by:

$$
\begin{aligned}
\int_0^\infty \Psi_1(\iota)d\iota &= \left[-\frac{v_{11}u_{21}}{\tilde{y}_{11}} - \frac{v_{21}}{\tilde{y}_{22}} - \frac{v_{31}u_{23}}{\tilde{y}_{33}} - \frac{v_{41}u_{24}}{\tilde{y}_{44}}\right], \\
\int_0^\infty \Psi_2(\iota)d\iota &= -\frac{1}{\tilde{y}_{22}}, \\
\int_0^\infty \Psi_3(\iota)d\iota &= -\frac{v_{23}}{\tilde{y}_{22}} - \frac{u_{23}}{\tilde{y}_{33}}, \\
\int_0^\infty \Psi_4(\iota)d\iota &= -\frac{v_{24}}{\tilde{y}_{22}} - \frac{u_{24}}{\tilde{y}_{44}}.
\end{aligned}
\tag{B34}
$$

The integrations above were conducted under the continuing assumption that all the eigenvalues are negative. Thus, approximated equilibrium stock price with the share process s_t is given by:

$$
\begin{aligned}
P_s(t) &\approx \frac{D_s(t)}{\tilde{q}_t}\sum_{k=1}^{4}\left[\int_0^\infty \Psi_k(\iota)d\iota\right]\tilde{y}_k(t) \\
&= D_s(t)\sum_{k=1}^{4}\left[\int_0^\infty \Psi_k(\iota)d\tau\right]\left[\frac{\tilde{y}_k(t)}{\tilde{q}_t}\right] \\
&= D_s(t)\left\{\left[\int_0^\infty \Psi_1(\iota)d\iota\right]\frac{1}{s_t H_t} + \left[\int_0^\infty \Psi_2(\iota)d\iota\right] + \left[\int_0^\infty \Psi_3(\tau)d\iota\right]\frac{1}{H_t} + \left[\int_0^\infty \Psi_4(\iota)d\iota\right]\frac{1}{s_t}\right\} \\
&= D_s(t)\left\{\left[\int_0^\infty \Psi_2(\iota)d\iota\right] + \left[\int_0^\infty \frac{\Psi_1(\iota)}{\bar{s}^{(2)}}d\iota\right]\frac{\bar{s}^{(2)}}{s_t}H_t^{-1} + \left[\int_0^\infty \Psi_3(\iota)d\iota\right]H_t^{-1} + \left[\int_0^\infty \frac{\Psi_4(\iota)}{\bar{s}^{(2)}}d\iota\right]\frac{\bar{s}^{(2)}}{s_t}\right\} \\
&= D_s(t)\left[\beta_{s,0} + \beta_{s,1}\left(\frac{\bar{H}}{H_t}\right) + \beta_{s,2}\left(\frac{\bar{s}^{(2)}}{s_t}\right) + \beta_{s,3}\left(\frac{\bar{s}^{(2)}}{s_t}\frac{\bar{H}}{H_t}\right)\right],
\end{aligned}
\tag{B35}
$$

where $\beta_{s,j}$'s are

$$
\beta_{s,0} \equiv \int_0^\infty \Psi_2(\iota)d\iota,\ \beta_{s,1} \equiv \int_0^\infty \frac{\Psi_3(\iota)}{\bar{H}}d\iota,\ \beta_{s,2} \equiv \int_0^\infty \frac{\Psi_4(\iota)}{\bar{s}^{(2)}}d\iota,\ \beta_{s,3} \equiv \int_0^\infty \frac{\Psi_1(\iota)}{\bar{s}^{(2)}\bar{H}}d\iota. \tag{B36}
$$

Approximate equilibrium price-dividend ratio of the shared stock is, hence, given by

$$
\frac{P_s(t)}{D_s(t)} \approx \left[\beta_{s,0} + \beta_{s,1}\left(\frac{\bar{H}}{H_t}\right) + \beta_{s,2}\left(\frac{\bar{s}^{(2)}}{s_t}\right) + \beta_{s,3}\left(\frac{\bar{s}^{(2)}}{s_t}\frac{\bar{H}}{H_t}\right)\right]. \tag{B37}
$$

□

Proof of Proposition 3. We find diffusion coefficients of $\frac{dP_{s_t}}{P_{s_t}}$ for investor 2. Applying Itô's lemma to $P_s(t)$ that was derived in the previous Proposition, we get diffusion coefficients of $\frac{dP_{s_t}}{P_{s_t}}$ as follows.

$$
\begin{aligned}
dB_A:&\quad \left(\frac{D_s}{P_s}\right)\beta_{s,0}\left(\sigma_{D,A} + v_{s,A}\right) + \left(\frac{D_s}{P_s}\right)\beta_{s,1}\left(\sigma_{D,A} + v_{s,A} - h_2\right)\frac{\bar{H}}{H_t} \\
&\quad + \left(\frac{D_s}{P_s}\right)\beta_{s,2}\sigma_{D,A}\left(\frac{\bar{s}^{(2)}}{s_t}\right) + \left(\frac{D_s}{P_s}\right)\beta_{s,3}\left(\sigma_{D,A} - h_2\right)\left(\frac{\bar{s}^{(2)}}{s_t}\frac{\bar{H}}{H_t}\right), \\
dB_I^{(2)}:&\quad \left(\frac{D_s}{P_s}\right)v_{s,I}\left(\beta_{s,0} + \beta_{s,1}\frac{\bar{H}}{H_t}\right).
\end{aligned}
$$

Diffusion coefficients of a shared asset's excess return - defined as R_s and $dR_s \equiv \frac{dP_{s_t}+D_s(t)}{P_{s_t}} - r_t dt$ - is the same as diffusion coefficients of $\frac{dP_{s_t}}{P_{s_t}}$. Note that equilibrium expected excess return, $E_t[dR_s]$, is given by the negative of the inner product of diffusion coefficient vector of dR_s and diffusion coefficient vector of the state price density $\xi_t^{(2)}$ since equilibrium return is defined by the covariance between the two quantities.[31] Applying Itô's lemma to $\xi_t^{(2)} = (1+\lambda_t^{1/\gamma})^\gamma (1/X_t)^{1-\gamma} D_t^{-\gamma} = z_t H_t D_t^{-1}$ using approximate diffusion process of z_t, yields

$$d\xi_t^{(2)}/\xi_t^{(2)} \approx \mu_{\xi^{(2)}} dt + (h_2 - \sigma_{D,A}) dB_A + \tilde{\alpha}_2(t) dB_I^{(2)}, \tag{B38}$$

where $\mu_{\xi^{(2)}} = \overline{\alpha}_1 + h_1(\bar{H}/H_t - 1) + \sigma_{D,A}^2 - \mu_D - h_2\sigma_{D,A}$.[32] Since expected excess return is determined by the negative of the sum of multiplications of diffusion coefficients given in the equation (B38),

$$E_t^{(2)}[dR_{s,t}] \approx \left[\frac{D_s(t)}{P_s(t)}\right]\left[\mu_{s,t}^{A,I} + \mu_{s,t}^{I}\right], \tag{B39}$$

where

$$\begin{aligned} \mu_{s,t}^{A,I} &\equiv \beta_{s,0}\left(\sigma_{D,A} + v_{s,A}\right)\left(\sigma_{D,A} - h_2\right) + \beta_{s,1}\left(\sigma_{D,A} + v_{s,A} - h_2\right)\left(\sigma_{D,A} - h_2\right)\frac{1}{H_t} \\ &\quad + \beta_{s,2}\sigma_{D,A}\left(\sigma_{D,A} - h_2\right)\frac{\bar{s}^{(2)}}{s_t} + \beta_{s,3}\left(\sigma_{D,A} - h_2\right)^2 \frac{\bar{s}^{(2)}}{s_t}\frac{\bar{H}}{H_t}, \\ \mu_{s,t}^{I} &\equiv -v_{s,I}\overline{\alpha}_2\left(\beta_{s,0} + \beta_{s,1}\frac{\bar{H}}{H_t}\right). \end{aligned} \tag{B40}$$

As was shown above, the diffusion process of the return of an shared asset, $R_s(t)$, is given by

$$dR_s \approx \mu_{R_s}^{(2)} dt + \sigma_{R_s,A} dB_A + \sigma_{R_s,I} dB_I^{(2)}, \tag{B41}$$

where $\mu_{R_s}^{(2)}$ is the expected excess return given above and both $\sigma_{R_s,A}$ and $\sigma_{R_s,I}$ are diffusion coefficients of dP_s/P_s given above. □

[31] For details, see Duffie (2001).

[32] Approximate diffusion process of z_t can be obtained by applying the same method that we use for approximating Y_1, i.e., $\tilde{y}$ and $\tilde{q}$.