

Could Indexation Be a Good Way to Cut Taxes for Stock Investors?

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Keywords: Portfolio choice, Inflation risk, Capital gains tax, Capital gains indexation.

JEL Classification: G11, H24, K34.

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Abstract

We conduct a theoretical analysis of the capital gains indexation proposal, which proposes to cut capital gains taxes by adjusting tax basis for inflation. Our model suggests that with limited use of losses, indexation could make it optimal for the investor to hold onto gains substantially longer even when tax rates are symmetric for long-term and short-term gains. Comparing with taxing nominal gains at the long-term tax rate, taxing indexed gains at the ordinary income tax rate could better improve the investor's welfare, increase the amount of capital gains tax bills, and reduce the costliness of tax naiveté *simultaneously*.

1 Introduction

In many financial markets, capital gains are subject to taxes, and these taxes can pose significant burdens on stock investors (e.g., Poterba (1987)). In the United States, the current tax code primarily applies two methods to cut capital gains taxes for stock investors: (i) setting lower tax rates for long-term capital gains, and (ii) deferring income taxes and waiving capital gains taxes in investors' pension plan accounts (e.g., 401(k) accounts). The literature has extensively studied the economic implications of these methods.¹

Capital gains indexation is another well known method for cutting taxes, with a 40 year history in the United States.² Given the persistent growth of the consumption price level (as shown in Figure 1), it proposes the authorities to cut taxes by adjusting capital gains for inflation. Comparing with other tax deduction methods, the indexation method has a unique feature that it could balance investors' effective tax burdens across different inflation states of the economy. Specifically, indexing capital gains would imply relatively lighter (heavier) tax burdens under states with higher (lower) inflation, where the investors' welfare is likely to be lower (higher).³

¹ For example, Dammon and Spatt (1996) and Dai, Liu, Yang, and Zhong (2015) study the optimal tax-timing strategies with asymmetric long-term/short-term tax rates; Dammon, Spatt, and Zhang (2004), Garlappi and Huang (2006), Huang (2008), and Fischer and Galmeyer (2017) study the optimal asset location and allocation rules with a tax-deferred account.

² In the United States, indexation proposals were floated during the Tax Reforms of 1978 and 1986, but were not included ultimately. More recently in 2018, the U.S. Department of the Treasury considered a regulatory change that would effectively index capital gain (see, e.g., Gravelle (2018)). However, this proposal was not moved forward due to legal reasons.

³ It is worth mentioning that the indexation method is partially implemented as an option for capital gains taxation in Australia. When calculating the capital gains on assets acquired before 21 September 1999, the investors are eligible to use the indexation method. Similar indexation reliefs are also implemented in regions including Ireland and India.

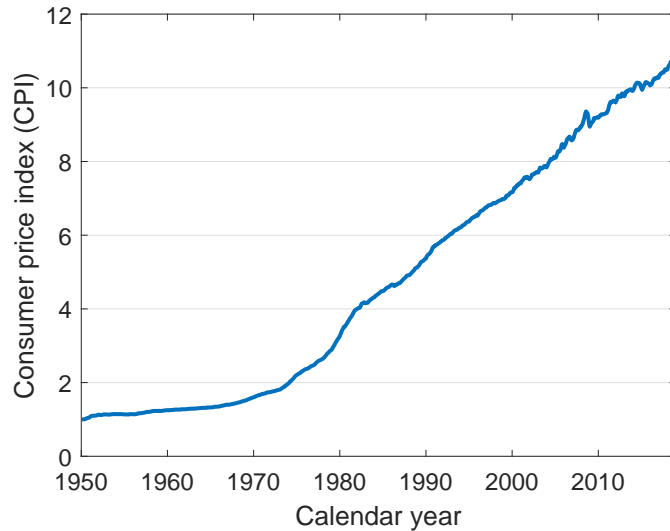


Figure 1: Consumer price index in the United States

This figure shows the consumer price index (CPI) level in the United States over 1950–2018. The data are obtained from the Center for Research in Security Prices. The CPI level at January 1950 is normalized to 1.

Fullerton (1987) discusses possible incentives behind the capital gains indexation proposal, focusing on the corporate sector. Our study complements Fullerton (1987) by conducting a theoretical analysis of this proposal from stock investors’ perspectives. Using a dynamic tax-timing model, we offer the following novel predictions, which could have useful implications for policymakers:

- Indexing capital gains could make it optimal for investors to hold onto gains substantially longer even when the tax rates are symmetric for short-term and long-term gains. This prediction indicates that indexation may not unintentionally introduce speculation and short-termism to the stock market, as is concerned in Gravelle (2018).
- Comparing with the current practice of taxing (long-term) nominal gains at lower rates,

taxing indexed gains at investors' ordinary income tax rates could better achieve the following three goals simultaneously: (i) improve investors' welfare, (ii) increase capital gains tax revenue, and (iii) reduce costliness of investors' tax naiveté.⁴

Our model builds on the growing literature on optimal portfolio and consumption choice with capital gains tax. In particular, we consider the problem of a representative stock investor who dynamically allocates wealth between a risk-free bond and a risky stock and who is subject to capital gains tax. We focus on the case with the limited use of losses, in which capital losses can only be carried forward indefinitely to offset future capital gains.⁵ Unlike the existing models, our model (i) features a stochastic consumption price level to capture the inflation risk, and (ii) allows the tax basis to float with the consumption price level, which enables us to effectively model a market with indexed (or un-indexed) capital gains. This flexibility makes it convenient to compare different tax deduction methods, such as indexation and a tax rate cut, under one unified framework.

We match model parameters to U.S. market data in the postwar era. Then, we provide a rich analysis of model predictions. For purpose of comparison, we consider three economies characterized by different parameterizations of the model. The first economy is a *hypothetical* Benchmark Economy in which capital gains are not indexed and are taxed at an ordinary

⁴ In fact, the original Treasury Study proposed to tax capital gains at ordinary rates and index them for inflation during consideration of the Tax Reform Act of 1986. Our analysis may provide theoretical justifications for this proposal.

⁵ The current U.S. tax code stipulates that an investor can claim a deduction of income taxes on capital losses up to 3,000 dollars per year. Ehling, Gallmeyer, Srivastava, Tompaidis, and Yang (2018) show that the economic impact of this income tax deduction is small for most investors. In an earlier version of this paper, we have included an analysis of the case with full use of losses. Those results are available upon request.

Table 1: Three economies for comparison

This table reports the indexation and taxation features in the three economies considered.

Economy	Benchmark Economy	Indexation Economy	Low Rate Economy
Definition of capital gains	Nominal gains	Indexed gains	Nominal gains
Capital gains tax rate	30%	30%	15%

income tax rate of 30%;⁶ the second is an Indexation Economy in which capital gains are indexed and are taxed at the same ordinary income tax rate; and the third is a Low Rate Economy in which capital gains are not indexed and are taxed at a capital gains tax rate of 15%. By construction, the Low Rate Economy resembles the current practice of capital gains taxation in the United States.⁷ Table 1 summarizes our comparison scheme.

It is well known that in the presence of realization-based capital gains tax, the investor has an incentive to defer capital gains realizations so as to save the time value of taxes (Dammon, Dunn, and Spatt (1989); Chay, Choi, and Pontiff (2006)). We begin with studying the impact of indexation on this tax-deferral incentive. We find that indexation could induce the investor to hold onto (indexed) gains substantially longer. This finding suggests that indexing capital gains and lowering tax rates can have qualitatively different impact on tax-deferral incentives, because lowering tax rates will make the investor realize gains sooner.

We explain the above result as follows. First, with the limited use of losses, deferring gains realizations can provide a particular benefit of making some future losses effectively

⁶ We emphasize that this economy is only used for benchmarking purpose.

⁷ The tax rates considered are representative for U.S. middle income class investors. In our analyses, we consider indexation as a substitute of lower tax rates. We do not compare indexation with the tax-deferral provisions applied to pension accounts, because it is theoretically shown that the optimal stock investment in pension accounts should be small (Dammon, Spatt, and Zhang (2001)).

rebatable (Dai, Liu, Yang, and Zhong (2015)). If capital gains are indexed, experiencing future losses becomes more probable as the cost basis tends to inflate over time; further deferring gains realizations allows the investor to better exploit this benefit. Second, as it takes time for the cost basis to grow, the investor needs to hold the stock for a longer period of time to enjoy greater benefit from tax deduction.

Next, we compare the economic value of deferring taxes (VTD) in three economies. This value measures the economic importance of optimal tax-timing: the smaller the VTD, the less the investor loses from adopting suboptimal tax-timing strategies. Interestingly, we find that although the investor's tax-deferral incentive is the strongest in the Indexation Economy, VTD in this economy is in fact the lowest. For instance, in the base case, VTD equals 6.56% in the Indexation Economy, and it increases to 9.18% in the Low Rate Economy.⁸ The intuition behind this finding is that in the Indexation Economy, heavier tax liabilities are concentrated in low inflation states where the investor's marginal utilities are low; deferring these taxes thus provides less economic value. This result implies that indexation could reduce the costliness of investors' tax naiveté more than the long-term tax rate does. This insight is practically important, as many individual investors are found to follow tax-inefficient trading strategies (Odean (1998), Locke and Mann (2005), and Dhar and Zhu (2006)).

Then, we conduct a welfare analysis of the tax deduction methods implemented in the Indexation Economy or the Low Rate Economy respectively. For a particular method, we con-

⁸ In the Benchmark Economy, VTD is apparently the highest because the tax rate is higher and capital gains are not indexed.

sider two welfare-related measures: the investor's certainty equivalent wealth gain (CEWG) due to tax deduction and the expected capital gains tax bills (CGTB) incurred by the investor. Arguably, CEWG measures the investor's personal welfare improvement, and CGTB can be related to public welfare.

First, we compare the CEWGs when the investor moves from the Benchmark Economy to the Indexation Economy or the Low Rate Economy. The results suggest that indexation could improve the investor's welfare more than the long-term tax rate does. For example, in the base case, the investor could enjoy a CEWG of 1.28% in the Indexation Economy, and a much smaller CEWG of 0.42% in the Low Rate Economy.⁹ Similar patterns are found for a large range of parameter values.

Then, we ask whether the better welfare improvement in the Indexation Economy is achieved at the cost of less capital gains taxes collected from the investor, and find it not the case. In the base case, the present value of the investor's incurred tax bills equals 71% of her initial wealth in the Low Rate Economy, and this figure increases to 134% in the Indexation Economy.¹⁰

Why could the investor derive a higher utility level in the Indexation Economy where she incurs more capital gains taxes? This is because indexation could provide a unique tax benefit to the investor: it could help balance her effective tax burden across different inflation

⁹ These CEWGs are measured as proportions of the investor's initial wealth level.

¹⁰ A rough calculation can help understand why the investor would incur more capital gains taxes in the Indexation Economy. In the base case, reducing the capital gains tax rate from 30% to 15% will roughly reduce the expected tax bills by half; by comparison, indexing capital gains will lead to a 41% decrease in annual taxable returns only ($\mu_P/\mu_S = 0.034/0.083 = 41\%$). We also point out that the reported quantities (e.g., 71% and 134%) are calculated over a relatively long horizon of 100 years, and we have found qualitatively same pattern for other reasonable horizons.

states of the economy. Under high inflation states, the investor has high marginal utilities due to low real consumption levels; thus, the lightened tax burden (due to indexation) can be particularly valuable to her. Under low inflation states, although the investor incurs more taxes, these taxes are less costly due to low marginal utilities. In other words, indexation could provide a partial hedge against the cost of taxation that is interfered with inflation, and this effect is completely absent in the Low Rate Economy.

Furthermore, we show that the above results continue to hold when: (i) we consider a tax naive investor who never defers capital gains taxes, (ii) we match model parameters to data in recent years with lower inflation, and (iii) the price index used for indexation only imperfectly captures the price level of the investor's primary consumption goods.

Taken together, our findings suggest that comparing with the practice of taxing long-term nominal gains at long-term tax rates, taxing indexed gains at ordinary income tax rates could increase the investor's utility and capital gains tax bills simultaneously. They also suggest that improving investors' welfare and increasing tax revenue are not necessarily conflicting goals.

The rest of this paper unfolds as follows. In the next section, we review the studies related to our research. In Section 3, we propose our modeling framework. In Section 4, we present some analytical results. In Section 5, we discuss the results of a numerical analysis of our model. In Section 6, we conclude. All the proofs, technical issues, and some additional results are relegated to the appendix.

2 Literature Review

Capital taxes are found to be an important driver behind asset prices and agents' economic behaviors (See, for instance, Lang and Shackelford (2000), Sinai and Gyourko (2004), Joulfaian (2005), Shan (2011), and Kopczuk and Munroe (2015)). Our study is closely related to the literature on portfolio choice with capital gains taxes. Early studies in this area such as Constantinides (1983, 1984) and DeMiguel and Uppal (2005) focus on tax-timing models with an exact cost basis, which are notoriously difficult to solve because of strong path dependence. With an exact cost basis, it is usually only feasible to solve for a small number of intermediate periods, which impedes the possibility of uncovering richer implications. To overcome this difficulty, Dammon, Spatt, and Zhang (2001) approximate the exact cost basis system with an average cost basis system, which is much more tractable and has been demonstrated to be a good approximation of the exact cost basis system.¹¹ In this study, we also assume an average cost basis system to maintain tractability.

In Dammon, Spatt, and Zhang (2001), it is assumed that the investor can take full advantage of losses credit. However, the current U.S. tax code stipulates that an investor can only claim a tax deduction on a capital loss of up to 3,000 dollars per annum. Ehling, Gallmeyer, Srivastava, Tompaidis, and Yang (2018) show that the economic impact of this income tax deduction is small for most investors. Hence, we follow Marekwica (2012) and Fischer and Gallmeyer (2017) and focus on the case with limited use of losses.

The aforementioned studies are generally conducted under discrete-time frameworks with

¹¹ Such an average cost basis system is implemented in Canada.

largely simplified price generating processes (e.g., binomial trees). Ben Tahar, Soner, and Touzi (2007, 2010) propose a continuous-time version of the model of Dammon, Spatt, and Zhang (2001). Dai, Liu, Yang, and Zhong (2015) extend Ben Tahar, Soner, and Touzi (2010) to the case with asymmetric tax/rebate rates for long-term/short-term investments, which accounts for an important feature of the current U.S. tax code. We also adopt the continuous-time framework, which allows us to conveniently perform theoretical and numerical analyses.

More recent studies of optimal tax-timing focus on the interaction between capital gains tax and asset return process. For example, Cai, Chen, and Dai (2018) study optimal tax-timing in a regime-switching market, where the investment opportunity set may switch between a bull state and a bear state. They develop an asymptotic analysis technique for the tax-timing problem, and demonstrate the existence of a significant cross regime effect in the optimal tax-managing strategy. Lei, Li, and Xu (2020) examine optimal tax trading in the presence of asset return predictability. They argue that as realized returns and expected returns tend to be negatively correlated, a tension between market-timing and tax-timing exists, and that it is economically important to alleviate this tension in an optimal way. They also show that return predictability can significantly affect the welfare implications of capital gains tax and value of deferring capital gains realizations.

Other studies of optimal investment with capital gains tax include Cadenillas and Pliska (1999), who examine the optimal strategy for maximizing long run after-tax growth; Gallmeyer, Kaniel, and Tompaidis (2006), who consider optimal tax management with multiple stocks; Amromin (2003), Shoven and Sialm (2004), Dammon, Spatt, and Zhang (2004), Garlappi and Huang (2006), and Huang (2008), who study asset allocation strategies between a tax-

able account and a tax-deferred account; and Fischer and Gallmeyer (2016), who examine the empirical performance of heuristic tax trading strategies.

Our study is also broadly related to the literature on optimal portfolio choice under inflation risk. Campbell and Viceira (2001), Brennan and Xia (2002), Kothari and Shanken (2004), and Li (2019) examine optimal hedging of inflation risk in portfolio selection. Aoki, Michaelides, and Nikolov (2019) examine how a demand for money affects life-cycle portfolio choice in the presence of inflation. Comparing with these studies, our study focuses on the interaction between inflation risk and capital gains taxation.

3 A Model with Indexed Capital Gains

In this section, we describe our theoretical framework.

Financial Assets. We consider a financial market in an infinite horizon, with time being continuously indexed by $t \geq 0$. Two assets are traded in the market. The first asset is a risk-free bond that pays interest at a constant pre-tax rate of r . The second asset is a risky stock that pays a stream of dividends at a pre-tax rate of δ .¹² The ex-dividend price of the stock, denoted as S_t , is assumed to follow a geometric Brownian motion process:

$$dS_t = \mu_S S_t dt + \sigma_S S_t dB_t, \tag{1}$$

¹² In our main analysis, we assume the stock pays dividends in the form of cash. We have obtained quantitatively similar results when the stock pays dividends in the form of additional stock shares. These results are available from the authors upon request.

where μ_S and σ_S are the mean and volatility of ex-dividend stock returns respectively.

Modeling Indexed Capital Gains. To capture the economic intuition behind indexing capital gains, we assume that the average consumption price level in the economy, denoted as p_t , follows another geometric Brownian motion:

$$dp_t = \mu_P p_t dt + \sigma_P p_t dZ_t. \tag{2}$$

In equations (1) and (2), B_t and Z_t are two correlated Brownian motions with $E[dZ_t dB_t] = \rho dt$ for $\rho \in [-1, 1]$.¹³

Consider a representative stock investor who allocates her wealth between the stock and the bond. When trading the stock, the investor is subject to realization-based capital gains tax.¹⁴ Like many prior studies, we assume a single capital gains tax rate to simplify our analysis. We further assume the investor can only make a limited use of losses in the sense that any capital losses can only be carried forward indefinitely to offset future capital gains.

Let x_t and y_t be the amount of wealth invested in the bond and the stock respectively,

¹³ In a recent proposal for indexing capital gains (cf. Gravelle (2018)), several possible price indexes, including the Consumer Price Index for All Urban Consumers (CPI-U), the chained CPI-U, and the gross domestic product (GDP) deflator, are suggested. Here, we focus on one specific price index to demonstrate our main results.

¹⁴ Capital gains tax is the only market friction in our model. Recent years have witnessed dramatic improvement in market liquidity, and transaction costs have significantly declined as a result (cf. Ben-Rephael, Kadan, and Wohl (2015)). Accordingly, we do not incorporate stock transaction costs in the baseline model. We emphasize that our main results are robust when reasonably small transaction costs are incorporated.

and k_t be the total cost of purchasing the current stock position. Then, we have

$$dx_t = (r(1 - \tau_i)x_{t-} + \delta(1 - \tau_d)y_{t-} - C_t)dt - dI_t + f(0, y_{t-}, k_{t-})dD_t, \quad (3)$$

$$dy_t = \mu_S y_{t-} dt + \sigma_S y_{t-} dB_t + dI_t - y_{t-} dD_t, \quad (4)$$

where τ_i and τ_d are the tax rates on interest and on dividends respectively; $dI_t \geq 0$ is the dollar value of newly purchased stock shares; $0 \leq dD_t \leq 1$ is the proportion of the current stock position that is sold;¹⁵ C_t is the nominal consumption rate; and

$$f(x_t, y_t, k_t) \equiv x_t + y_t - \tau_g(y_t - k_t)^+ \quad (5)$$

is the investor's net wealth level, with τ_g being the capital gains tax rate.

In the standard tax-timing models proposed in the literature, the cost basis k_t remains unchanged when the investor does not trade the stock. This is consistent with the capital gains tax rule currently implemented in the United States. In our study, to examine the potential implications of indexation for stock investors, we deviate from standard models and allow the cost basis k_t to float with the consumption price level p_t when the investor does not trade. For this purpose, we assume its dynamics are given by

$$dk_t = \mu_K k_{t-} dt + \sigma_K k_{t-} dZ_t + dI_t - k_{t-} dD_t + (k_{t-} - y_{t-})^+ dD_t. \quad (6)$$

¹⁵ The constraint $dD_t \leq 1$ implies that the investor is restricted from short selling the stock. In our model, we assume capital gains taxes are paid immediately when the investor realizes gains. In reality, these taxes are paid at the end of each calendar year. Dai, Lei, and Liu (2020) show that the welfare effect of this year-end tax provision is tiny.

This specification implies (i) if we set $\mu_K = \sigma_K = 0$, then we have

$$dk_t = dI_t - k_{t-}dD_t + (k_{t-} - y_{t-})^+dD_t, \quad (7)$$

and the cost basis is frozen to the purchase price in the absence of stock trading (i.e., when $dI_t = dD_t = 0$); and (ii) if we set $\mu_K = \mu_P$ and $\sigma_K = \sigma_P$, then during no-trading periods, we have

$$dk_t = \mu_P k_{t-}dt + \sigma_P k_{t-}dZ_t, \quad (8)$$

indicating that the cost basis tracks the consumption price level. Therefore, our model can nest either the case with or the case without capital gains indexation as special cases.

The Investor's Problem. The investor's solvency region is defined as

$$\Omega = \{(x, y, k, p) : y \geq 0, k \geq 0, p > 0, f(x, y, k) \geq 0\}. \quad (9)$$

Starting from an initial state $(x_0, y_0, k_0, p_0) = (x, y, k, p) \in \Omega$, a triplet of consumption-investment strategy $\{(C_t, I_t, D_t) : t \geq 0\}$ is admissible if (i) the resultant processes $(x_t, y_t, k_t, p_t) \in \Omega$ for all $t > 0$, and (ii) there is no arbitrage opportunity lurking in the model.

Let $\mathcal{A}(x, y, k, p)$ denote the set of all admissible strategies. The investor's objective is to maximize the aggregate discounted utility derived from real consumption by choosing the

optimal (admissible) trading and consumption strategies:

$$\max_{(C_t, I_t, D_t) \in \mathcal{A}(x, y, k, p)} E \left[\int_0^\infty e^{-\beta t} u(C_t/p_t) dt \right], \quad (10)$$

subject to equations (2), (3), (4), and (6), where $\beta > 0$ is the investor's time discounting factor and $u(\cdot)$ is her utility function.¹⁶ Throughout this paper, we assume the investor exhibits a constant relative risk aversion (CRRA) preference:

$$u(w) = \frac{w^{1-\gamma}}{1-\gamma}, \quad (11)$$

where $\gamma > 0$ and $\gamma \neq 1$ is the investor's relative risk aversion coefficient. Moreover, we impose the following restriction on the model parameters to ensure the existence of a well-defined solution in the absence of capital gains tax (see Proposition 1 in Section 4):

$$\tilde{\beta} > (1-\gamma) \left[r(1-\tau_i) + \frac{(\tilde{\mu}_S + \delta(1-\tau_d) - r(1-\tau_i))^2}{2\gamma\sigma_S^2} \right], \quad (12)$$

where

$$\tilde{\beta} = \beta - \mu_P(\gamma-1) - \frac{1}{2}\sigma_P^2(\gamma-1)(\gamma-2), \quad (13)$$

and

$$\tilde{\mu}_S = \mu_S + \rho\sigma_S\sigma_P(\gamma-1). \quad (14)$$

¹⁶ Considering an infinite horizon case allows us to obtain steady-state trading and consumption strategies, which are easier to elaborate on and analyze. Our analyses also apply to the cases with finite horizons.

4 Analytical Results

We first provide some theoretical analyses of the model. The proofs are all in Appendix A.

In the following proposition, we present the solution when capital gains tax is absent.

Proposition 1. *(Solution without capital gains tax.) In the absence of capital gains tax (i.e., $\tau_g = 0$), the value function is given by*

$$V^0(x, y, k, p) = \frac{\bar{K}^{-\gamma}}{1 - \gamma} \left(\frac{x + y}{p} \right)^{1-\gamma}, \quad (15)$$

where

$$\bar{K} = \frac{\tilde{\beta}}{\gamma} - \frac{1 - \gamma}{\gamma} \left[r(1 - \tau_i) + \frac{(\tilde{\mu}_S + \delta(1 - \tau_d) - r(1 - \tau_i))^2}{2\gamma\sigma_S^2} \right], \quad (16)$$

and $\tilde{\beta}$ and $\tilde{\mu}_S$ are defined in (13)-(14). The associated optimal trading and consumption strategies are given by

$$\frac{y}{x + y} = \pi^* \equiv \frac{\tilde{\mu}_S + \delta(1 - \tau_d) - r(1 - \tau_i)}{\gamma\sigma_S^2}, \quad \frac{C}{x + y} = \bar{K}. \quad (17)$$

This proposition suggests that in the absence of capital gains taxes, it is optimal to keep constant stock–wealth and consumption–wealth ratios like in Merton (1969, 1971). We term the quantity π^* the “Merton line” in the presence of inflation risk.

Next, we turn to the case with capital gains tax. In this case, it is well known that the investor can benefit from optimally deferring capital gains realizations. In the following proposition, we first provide information about the value function when the investor never defers capital gains realizations in a tax system with possibly indexed capital gains.

Proposition 2. (Solution without capital gains deferral.) Among the strategies that never defer capital gains realizations, the maximum indirect utility level the investor can achieve, denoted as $V^N(x, y, k, p)$, is characterized as follows:

(i) if $y \leq k$, let $\Psi^N(\zeta)$ be a univariate function defined for $\zeta \in [0, 1]$, which satisfies equation (A-9) and boundary condition (A-10) in Appendix A. Then, we have

$$V^N(x, y, k, p) = \frac{1}{1 - \gamma} \left(\frac{x + y}{p} \right)^{1 - \gamma} e^{(1 - \gamma)\Psi^N\left(\frac{x + y}{x + k}\right)}. \quad (18)$$

(ii) if $y > k$, define a bivariate function $\psi^N(z, b)$ on the domain $\{(z, b) \in \mathbb{R}^2 : 1 - \tau_g z(1 - b) \geq 0, 0 \leq b < 1\}$ as follows:

$$\psi^N(z, b) = \Psi^N(1) + \log(1 - \tau_g z(1 - b)). \quad (19)$$

Then, we have

$$V^N(x, y, k, p) = \frac{1}{1 - \gamma} \left(\frac{x + y}{p} \right)^{1 - \gamma} e^{(1 - \gamma)\psi^N\left(\frac{y}{x + y}, \frac{k}{y}\right)}. \quad (20)$$

The associated investment and consumption strategies are presented in Appendix A.

The optimal trading strategy without deferring any capital gains realizations can be qualitatively described as follows: if a capital gain occurs, realize it immediately and then repurchase some stock shares to re-establish an optimal stock allocation; if a capital loss occurs, then trading the stock continuously to maintain a desirable stock allocation, which is a function of the size of the loss. We will provide a concrete example in Section 5.4.3.

Now, we characterize the value function when the investor can optimally defer capital

gains realizations. Denote the value function by $V(x, y, k, p)$ in this case. Then, standard argument (e.g., Ben Tahar, Soner, and Touzi (2007)) suggests that $V(x, y, k, p)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\max \left\{ \sup_{C \geq 0} \mathcal{L}^C V - \beta V, \mathcal{S}V, \mathcal{B}V \right\} = 0 \quad (21)$$

on the solvency region Ω , where the operators in (21) are given by

$$\begin{aligned} \mathcal{L}^C V &= (r(1 - \tau_i)x + \delta(1 - \tau_d)y - C)V_x + \mu_S y V_y + \mu_P p V_p + \mu_K k V_k \\ &+ \frac{1}{2} \sigma_S^2 y^2 V_{yy} + \frac{1}{2} \sigma_P^2 p^2 V_{pp} + \frac{1}{2} \sigma_K^2 k^2 V_{kk} + \rho \sigma_S \sigma_P y p V_{yp} \\ &+ \rho \sigma_S \sigma_K y k V_{yk} + \sigma_P \sigma_K p k V_{pk} + \frac{(C/p)^{1-\gamma}}{1-\gamma}, \end{aligned} \quad (22)$$

$$\mathcal{S}V = f(0, y, k)V_x - yV_y - [k - (k - y)^+]V_k, \quad (23)$$

and

$$\mathcal{B}V = -V_x + V_y + V_k, \quad (24)$$

respectively.

Given the solution to equation (21), the domain Ω splits into three sub-domains: (1) a Sell region

$$SR \equiv \{(x, y, k, p) \in \Omega : \mathcal{S}V = 0\}; \quad (25)$$

(2) a Buy region

$$BR \equiv \{(x, y, k, p) \in \Omega : \mathcal{B}V = 0\}; \quad (26)$$

and (3) a No-trade region

$$NTR \equiv \{(x, y, k, p) \in \Omega : \mathcal{S}V < 0, \mathcal{B}V < 0\}. \quad (27)$$

Moreover, we infer from (22) that the optimal nominal consumption rate is $C^* = p(pV_x)^{-1/\gamma}$.

Equation (21) involves four state variables, making it difficult to analyze. Thanks to the homogeneity of the CRRA preference and linear dynamics of the state variables (i.e., equations (2), (3), (4), and (6)), the value function can be expressed in the following form

$$V(x, y, k, p) = \frac{1}{1 - \gamma} \left(\frac{x + y}{p} \right)^{1-\gamma} e^{(1-\gamma)\Psi(z, b)}, \quad (28)$$

where $z = \frac{y}{x+y}$ denotes the proportion of gross wealth invested in the stock, $b = \frac{k}{y}$ denotes the basis-price ratio of the current stock position, and $\Psi(z, b)$ is a bivariate function defined on the domain $\{(z, b) \in \mathbb{R}^2 : 1 - \tau_g z(1 - b)^+ \geq 0, b \geq 0\}$. Appendix B presents the equation that governs $\Psi(z, b)$ and a verification theorem.

5 Numerical Analysis of Model Implications

In this section, we explore richer implications of our model through a numerical analysis.

5.1 Parameter Choices

We match asset return and inflation parameters to U.S. market data during the postwar era. Specifically, we obtain monthly data on interest rates, stock returns, and inflation rates over

January 1950–December 2018 from The Center for Research in Security Prices. We use the average T-bill return as an estimate of the pre-tax interest rate, which gives $r = 0.041$. We use the returns on the value-weighted portfolio of the S&P 500 to estimate the dividend yield, expected return, and volatility of the risky stock, and we obtain $\delta = 0.034$, $\mu_S = 0.083$, and $\sigma_S = 0.143$ respectively. We use changes in the CPI level to estimate the average inflation rate and inflation volatility, which yields $\mu_P = 0.034$ and $\sigma_P = 0.013$. The correlation between the changes in the CPI level and in stock market returns is $\rho = -0.063$.¹⁷

We aim to compare two tax deduction methods: indexing capital gains or lowering capital gains tax rate. To facilitate such a comparison, we consider three economies obtained by different parameterizations of the model. The first economy is a *hypothetical* Benchmark Economy in which capital gains are not indexed (i.e., $\mu_K = \sigma_K = 0$) and are taxed at an ordinary income tax rate of 30%; the second is an Indexation Economy in which capital gains are indexed (i.e., $\mu_K = \mu_P, \sigma_K = \sigma_P$) and are taxed at the same ordinary income tax rate; and the third is a Low Rate Economy in which capital gains are not indexed and are taxed at a capital gains tax rate of 15%.¹⁸ By construction, the Low Rate Economy resembles the current practice of capital gains taxation. In each economy, bond interest and stock dividends are taxed at the ordinary income tax rate.

We set the investor’s relative risk aversion coefficient to $\gamma = 6$ and time discounting factor

¹⁷ Later in Section 5.4.3, we will show that our results remain robust when we use more recent data to estimate model parameters. If we use other price index data, such as CPI-U, to estimate the inflation-related parameters, we will obtain quantitatively similar results.

¹⁸ These tax rates are representative for middle income class investors in the United States. For high income class investors whose tax rates are typically higher, our results are even stronger. It should also be noted that under current U.S. tax code, only capital gains with holding periods over one year are eligible for being taxed at the long-term rates. Therefore, the investor’s tax burden in the Low Rate Economy seems slightly lighter than that in reality.

to $\beta = 0.05$. These parameter values satisfy condition (12) and ensure a reasonable wealth allocation to the stock. Table 2 summarizes our default parameter values.

5.2 Optimal Tax-Timing Strategies

In this subsection, we analyze the investor’s optimal tax-timing strategy.

Figure 2 shows the characteristics of the optimal strategies in all three economies. Panel A plots the optimal trading boundaries.¹⁹ When capital gains tax is absent, Proposition 1 suggests that it is optimal to keep a constant proportion of wealth invested in the stock; this optimal allocation is marked by “Merton line” in Panel A. In the presence of capital gains tax, however, continuously trading to maintain this allocation is no longer optimal in general. Take the Benchmark Economy case as an example, with capital gains on paper (i.e., with a basis–price ratio $b < 1$), the investor should sell some stock shares to rebalance her portfolio composition only when her stock allocation (i.e., $y/(x + y)$) rises above the Sell boundary, which is the black solid line above the Merton line. In this case, the investor sells the minimum amount so that the after-sale stock allocation lies exactly on the Sell boundary (signified by the arrow from point A to point B). Similarly, the investor should buy additional stock shares only when her stock allocation drops below the Buy boundary, which is the black solid line below the Merton line. The region delimited by these two boundaries is the No-trade region, inside which the investor is better-off not making any transaction. In this region, realizing gains to rebalance portfolio is not optimal, as the loss on the time value of taxes exceeds the gain from achieving a better risk exposure. When loss

¹⁹ With an infinite horizon, the optimal trading boundaries do not vary over time.

Table 2: Default parameter values

This table summarizes our baseline parameter values. The pre-tax interest rate is estimated from the historical T-bill yields; the expected return, dividend yield, and return volatility of the stock are estimated from the historical returns of a value-weighted portfolio on S&P 500 stocks; the average inflation rate, inflation volatility, and the correlation between the stock returns and inflation rate are estimated from the changes in the CPI level in the United States.

Parameter	Symbol	Value
Subjective preference		
Discounting factor	β	0.05
Relative risk aversion	γ	6
Asset returns		
Pre-tax interest rate	r	0.041
Expected return of the stock	μ_S	0.083
Dividend yield of the stock	δ	0.034
Volatility of stock returns	σ_S	0.143
Inflation		
Expected inflation rate	μ_P	0.034
Volatility of the inflation rate	σ_P	0.013
Correlation between inflation and stock returns	ρ	-0.063
Taxation in the Benchmark Economy		
Expected growth rate of the cost basis	μ_K	0
Volatility of the cost basis	σ_K	0
Tax rates	$\tau_i = \tau_d = \tau_g$	0.3
Taxation in the Indexation Economy		
Expected growth rate of the cost basis	μ_K	0.034
Volatility of the cost basis	σ_K	0.013
Tax rates	$\tau_i = \tau_d = \tau_g$	0.3
Taxation in the Low Rate Economy		
Expected growth rate of the cost basis	μ_K	0
Volatility of the cost basis	σ_K	0
Ordinary income tax rate	$\tau_i = \tau_d$	0.3
Capital gains tax rate	τ_g	0.15

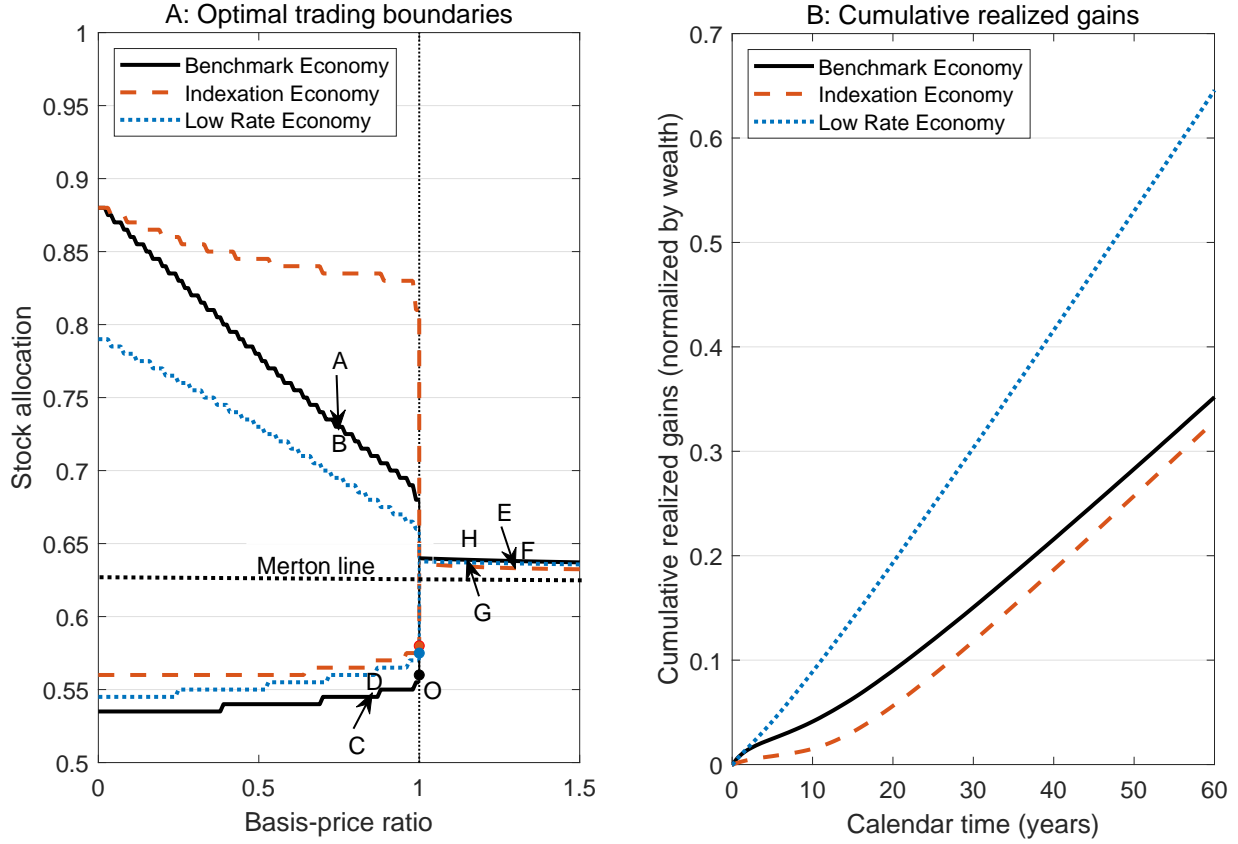


Figure 2: Optimal trading strategies

The left subfigure (Panel A) depicts the optimal trading boundaries, and the right subfigure (Panel B) shows the cumulative realized gains/gross wealth ratio, in three economies considered. The base case parameter values are as follows: pre-tax interest rate $r = 0.041$, ex-dividend expected stock return $\mu_S = 0.083$, pre-tax dividend yield $\delta = 0.034$, stock return volatility $\sigma_S = 0.143$, average inflation rate $\mu_P = 0.034$, inflation volatility $\sigma_P = 0.013$, correlation between the inflation rate and stock returns $\rho = -0.063$, subjective discounting factor $\beta = 0.05$, relative risk aversion coefficient $\gamma = 6$, and ordinary income tax rate $\tau_d = \tau_i = 0.3$. In the Benchmark Economy, the expected growth rate and volatility of the cost basis $\mu_K = \sigma_K = 0$, and the capital gains tax rate $\tau_g = 0.3$; in the Indexation Economy, $\mu_K = \mu_P, \sigma_K = \sigma_P$ and $\tau_g = 0.3$; in the Low Rate Economy, $\mu_K = \sigma_K = 0$ and $\tau_g = 0.15$.

occurs (i.e., $b > 1$), it is optimal for the investor to continuously rebalance her portfolio to maintain an optimal exposure to the stock (e.g., signified by the arrow from point E to point F); wash-sale is not optimal because the investor cannot claim rebates on capital losses.

In Figure 2, the black solid (red dashed and blue dotted, resp.) lines represent the optimal trading boundaries in the Benchmark Economy (Indexation Economy and Low Rate Economy, resp.). Interestingly, Panel A shows that the No-trade region is the widest in the Indexation Economy, suggesting that indexation could induce the investor to hold onto (indexed) gains much longer. By comparison, in the Low Rate Economy, the No-trade region shrinks and the investor realizes gains sooner. This comparison indicates that indexing capital gains and lowering tax rate can have qualitatively different effects on the investor’s tax-deferral incentives.

Panel B of Figure 2 shows the investor’s cumulative realized gains–paper wealth ratio.²⁰ Intuitively, a greater value of this quantity indicates a higher propensity for gains realizations. We note that the curve in the Indexation Economy is the lowest, suggesting that the investor indeed has the weakest incentive to realize gains in this economy. By contrast, the curve in the Low Rate Economy is the highest, confirming that reducing capital gains tax rate increases the investor’s propensity to realize gains.

Why could indexation induce the investor to hold onto gains longer? There are two mechanisms at work. First, with the limited use of losses, deferring gains realizations can provide a particular benefit of making some future losses effectively rebatable (Dai, Liu, Yang, and Zhong (2015)). If capital gains are indexed, experiencing future losses becomes

²⁰ This quantity can be formally expressed as $E \left[\int_0^t 1_{y_s > k_s} \frac{y_s}{x_s + y_s} dD_s \right]$.

more probable as the cost basis tends to inflate over time; further deferring gains realizations allows the investor to exploit this benefit more. Second, as it takes time for the cost basis to grow, the investor needs to hold the stock for a longer period of time to enjoy greater benefit from tax deduction.

It is sometimes concerned that indexing capital gains may unintentionally introduce speculation and short-termism to the stock market (e.g., Gravelle (2018)). However, the above results suggest that indexation may not have such an adverse effect for tax-aware investors.

5.3 Value of Tax-Deferral

In this subsection, we examine the economic value of optimal tax-timing in three economies.

With realization-based capital gains tax, the investor has a tax-deferral option that allows her to save the time value of taxes. Following the standard literature (e.g., Dai, Liu, Yang, and Zhong (2015), Cai, Chen, and Dai (2018), Lei, Li, and Xu (2020)), we measure this option's value, denoted as VTD, as the additional wealth that makes the investor indifferent between never deferring taxes and optimally deferring taxes. Formally, given the initial wealth level W_0 and initial consumption price level p_0 , VTD solves

$$V(W_0, 0, 0, p_0) = V^N((1 + VTD)W_0, 0, 0, p_0), \quad (29)$$

where $V^N(x, y, k, p)$ is the investor's indirect utility function when she never defers gains

realizations, as characterized in Proposition 2.²¹ VTD can also measure the costliness of tax naiveté: the smaller the VTD, the less the investor loses due to tax-inefficient trading.

Table 3 reports VTD under various model parameterizations. Interestingly, although we have shown in Section 5.2 that the investor’s tax-deferral incentive is the strongest in the Indexation Economy, Table 3 suggests that her deferral option’s value in this economy is in fact the lowest. For instance, in the base case, VTD equals 6.56% in the Indexation Economy and 9.18% in the Low Rate Economy. The reason is that in the Indexation Economy, the investor faces heavier tax liabilities under low inflation states where her marginal utilities are lower; as a result, deferring these taxes provides a smaller economic value.

The above result implies that comparing with taxing nominal gains at the long-term tax rate, taxing indexed gains at the ordinary income tax rate could better reduce the costliness of investors’ tax naiveté. This insight is practically important, because it is empirically documented that many individual investors tend to follow tax-inefficient trading strategies (e.g., Odean (1998), Locke and Mann (2005), and Dhar and Zhu (2006)).

We note from Table 3 that VTD increases (decreases) in the average inflation rate (i.e., μ_P) when capital gains are indexed (not indexed). We explain this pattern as follows. When capital gains are indexed, larger value of μ_P implies faster growth of the cost basis; hence, the investor can save more taxes by deferring and VTD increases in μ_P as a result. By comparison, when capital gains are not indexed, larger value of μ_P only implies weaker future purchasing power of the deferred taxes; hence, VTD decreases in μ_P .

²¹ Due to the homogeneity of the value functions, VTD does not depend on W_0 and p_0 .

Table 3: Value of tax-deferral

This table reports the value of tax-deferral (VTD), which is defined as the solution to equation (29). The base case parameter values are as follows: pre-tax interest rate $r = 0.041$, ex-dividend expected stock return $\mu_S = 0.083$, pre-tax dividend yield $\delta = 0.034$, stock return volatility $\sigma_S = 0.143$, average inflation rate $\mu_P = 0.034$, inflation volatility $\sigma_P = 0.013$, correlation between the inflation rate and stock returns $\rho = -0.063$, subjective discounting factor $\beta = 0.05$, relative risk aversion coefficient $\gamma = 6$, and ordinary income tax rate $\tau_d = \tau_i = 0.3$. In the Benchmark Economy, the expected growth rate and volatility of the cost basis $\mu_K = \sigma_K = 0$, and the capital gains tax rate $\tau_g = 0.3$; in the Indexation Economy, $\mu_K = \mu_P, \sigma_K = \sigma_P$ and $\tau_g = 0.3$; in the Low Rate Economy, $\mu_K = \sigma_K = 0$ and $\tau_g = 0.15$. The reported quantities are in percentage.

		Panel A: Benchmark	Panel B: Indexation	Panel C: Low Rate
Baseline		23.62	6.56	9.18
β	-0.01	25.89	6.75	9.98
	+0.01	21.67	6.39	8.50
γ	-1	24.58	7.96	9.76
	+1	23.11	5.58	8.82
δ	$\times 1.1$	22.02	6.46	8.70
	$\times 0.9$	25.35	6.66	9.69
μ_S	$\times 1.1$	22.29	7.11	8.78
	$\times 0.9$	24.87	5.95	9.57
σ_S	$\times 1.2$	23.17	5.92	8.62
	$\times 1.5$	21.13	4.86	7.49
r	$\times 1.2$	23.82	6.62	9.19
	$\times 0.8$	22.58	6.45	8.93
μ_P	$\times 0.9$	20.47	6.85	8.07
	$\times 0.8$	18.00	7.13	7.19
σ_P	$\times 1.2$	23.79	6.54	9.24
	$\times 0.8$	23.47	6.59	9.13
ρ	+0.2	23.32	6.67	9.09
	-0.2	23.90	6.46	9.27

5.4 Welfare Analysis

In this subsection, we present a welfare analysis of our model.

Certainty Equivalent Wealth Gain. Suppose an investor initially lives in the Benchmark Economy. Then, moving to the Indexation Economy will lead to an improvement in her welfare due to the more favorable capital gains taxation rules in this economy. We measure this improvement by the additional wealth that makes the investor indifferent between staying in the Benchmark Economy or moving to the Indexation Economy. Formally, the certainty equivalent wealth gain (CEWG) from moving to the Indexation Economy, denoted as Δ_I , solves the following equation:

$$V(W_0, 0, 0, p_0)_{\mu_K=\mu_P, \sigma_K=\sigma_P} = V((1 + \Delta_I)W_0, 0, 0, p_0)_{\mu_K=\sigma_K=0}. \quad (30)$$

Similarly, the CEWG from moving to the Low Rate Economy, denoted as Δ_L , solves

$$V(W_0, 0, 0, p_0)_{\mu_K=\sigma_K=0, \tau_g=\tau_g(L)} = V((1 + \Delta_L)W_0, 0, 0, p_0)_{\mu_K=\sigma_K=0, \tau_g=\tau_g(S)}. \quad (31)$$

In equation (31), we set $\tau_g(S)$ to the investor's ordinary income tax rate (30% in the base case) and $\tau_g(L)$ to her long-term capital gains tax rate (15% in the base case).

Capital Gains Tax Bills. Capital gains taxes collected from investors constitute an important part of federal tax revenue and can be important for public welfare. Therefore, it is crucial to understand the implications of indexing capital gains or lowering tax rates for the

capital gains tax bills incurred by the investor. For this purpose, we calculate the investor's capital gains tax bills (CGTB) that are incurred over a period of 100 years by 10,000 paths of Monte-Carlo simulations, for both the Indexation Economy and the Low Rate Economy.

5.4.1 Results Analysis

Table 4 summarizes the values of CEWG and CGTB under various model parameterizations. Panel A reports the CEWGs Δ_I and Δ_L , and it suggests that the CEWGs from moving to the Indexation Economy are significantly greater than those from moving to the Low Rate Economy for a wide range of parameter values. For example, in the base case, moving from the Benchmark Economy to the Indexation Economy could lead to a CEWG of 1.28%, while moving to the Low Rate Economy leads to a CEWG of 0.42% only. These results suggest that taxing indexed gains at the investor's ordinary income tax rate is more preferable than taxing her nominal gains at the long-term tax rate.

It can be noted from Table 4 that the difference between CEWGs (i.e., $\Delta_I - \Delta_L$) increases in the stock's return volatility (i.e., σ_S). This is consistent with the idea that indexation is more favorable for riskier assets than a lower rate (e.g., Gravelle (2018)). The intuition is that indexation reduces the expected tax burden by allowing an inflation-dependent exemption, but does not change volatility of the after-tax returns. By contrast, lowering the tax rate would increase the return but also increase the volatility of the after-tax returns. Because the investor is risk averse, this asymmetry is greater for assets with greater return volatilities, making indexation more favorable for these assets.

The most important inflation-related parameter in our model is the average inflation

Table 4: Certainty equivalent wealth gains and capital gains tax bills

This table reports the value of certainty equivalent wealth gain (CEWG) and discounted capital gains tax bills (CGTB) for the Indexation Economy and Low Rate Economy. The base case parameter values are as follows: pre-tax interest rate $r = 0.041$, ex-dividend expected stock return $\mu_S = 0.083$, pre-tax dividend yield $\delta = 0.034$, stock return volatility $\sigma_S = 0.143$, average inflation rate $\mu_P = 0.034$, inflation volatility $\sigma_P = 0.013$, correlation between the inflation rate and stock returns $\rho = -0.063$, subjective discounting factor $\beta = 0.05$, relative risk aversion coefficient $\gamma = 6$, and ordinary income tax rate $\tau_d = \tau_i = 0.3$. In the Benchmark Economy, the expected growth rate and volatility of the cost basis $\mu_K = \sigma_K = 0$, and the capital gains tax rate $\tau_g = 0.3$; in the Indexation Economy, $\mu_K = \mu_P, \sigma_K = \sigma_P$ and $\tau_g = 0.3$; in the Low Rate Economy, $\mu_K = \sigma_K = 0$ and $\tau_g = 0.15$.

		Panel A: CEWG(%)		Panel B: CGTB	
		A1: Indexation	A2: Low Rate	B1: Indexation	B2: Low Rate
Baseline		1.28	0.42	1.34	0.71
β	-0.01	1.15	0.31	1.22	0.72
	+0.01	1.42	0.53	1.41	0.67
γ	-1	1.48	0.64	1.50	0.85
	+1	1.33	0.35	1.20	0.58
δ	$\times 1.1$	0.96	0.27	1.16	0.74
	$\times 0.9$	1.70	0.63	1.37	0.66
μ_S	$\times 1.1$	1.59	0.69	2.96	1.32
	$\times 0.9$	1.15	0.26	0.58	0.37
σ_S	$\times 1.2$	2.05	0.69	0.94	0.40
	$\times 1.5$	3.72	1.62	0.46	0.20
r	$\times 1.2$	1.57	0.62	0.74	0.36
	$\times 0.8$	0.93	0.24	2.09	1.42
μ_P	$\times 0.9$	1.42	0.61	1.47	0.65
	$\times 0.8$	1.50	0.77	1.40	0.60
σ_P	$\times 1.2$	1.28	0.41	1.35	0.69
	$\times 0.8$	1.29	0.43	1.37	0.70
ρ	+0.2	1.33	0.47	1.64	0.80
	-0.2	1.25	0.37	1.14	0.60

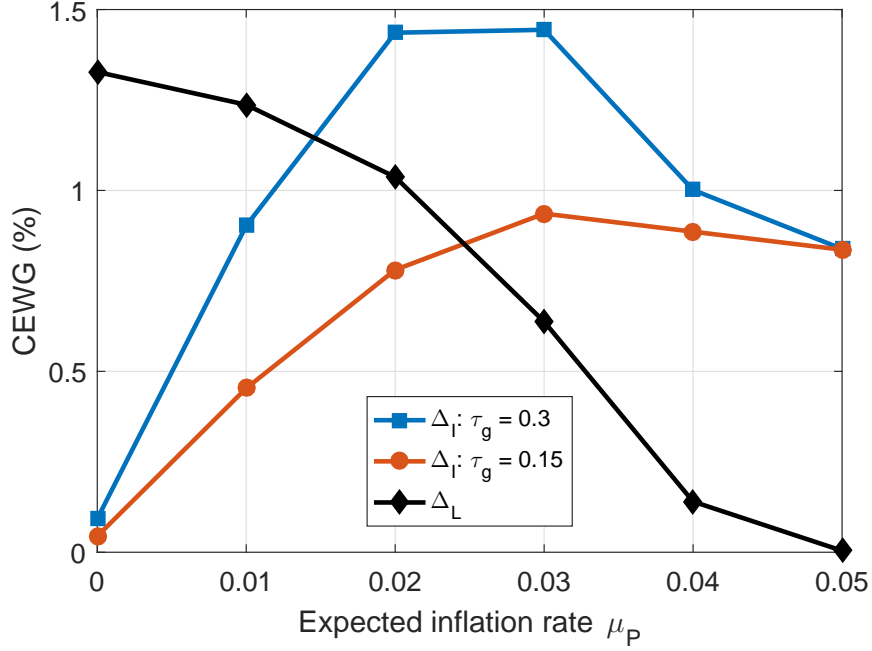


Figure 3: Average inflation rate and certainty equivalent wealth gains

This figure shows the certainty equivalent wealth gain (CEWG) against the average inflation rate. The base case parameter values are as follows: pre-tax interest rate $r = 0.041$, ex-dividend expected stock return $\mu_S = 0.083$, pre-tax dividend yield $\delta = 0.034$, stock return volatility $\sigma_S = 0.143$, average inflation rate $\mu_P = 0.034$, inflation volatility $\sigma_P = 0.013$, correlation between the inflation rate and stock returns $\rho = -0.063$, subjective discounting factor $\beta = 0.05$, relative risk aversion coefficient $\gamma = 6$, and ordinary income tax rate $\tau_d = \tau_i = 0.3$. In the Benchmark Economy, the expected growth rate and volatility of the cost basis $\mu_K = \sigma_K = 0$, and the capital gains tax rate $\tau_g = 0.3$; in the Indexation Economy, $\mu_K = \mu_P, \sigma_K = \sigma_P$ and $\tau_g = 0.3$; in the Low Rate Economy, $\mu_K = \sigma_K = 0$ and $\tau_g = 0.15$.

rate (i.e., μ_P). Interestingly, we find that the relation between Δ_I and μ_P is non-monotone. Figure 3 plots Δ_I against μ_P for two capital gains tax rates, and it reveals a hump-shaped relation between these two quantities. This pattern can be explained as follows. An increase in μ_P has two effects: (1) it saves more taxes because the cost basis grows faster (tax-saving effect), and (2) it reduces the purchasing power of saved taxes because future price level also tends to be higher (purchasing power effect). Due to the limited use of losses, the amount of taxes to be saved is capped from above by the taxes on nominal gains. When the value of μ_P is small, the tax-saving effect dominates and Δ_I increases in μ_P . When the value of μ_P is sufficiently large instead (e.g., larger than a threshold level of about 3%), the purchasing power effect dominates, lowering the economic value of saved taxes. As a result, Δ_I can decrease in μ_P in this case. To further support this argument, Figure 3 also plots CEWG in the Low Rate Economy (i.e., Δ_L), and it suggests a decreasing relation between Δ_L and μ_P . This is because in the Low Rate Economy there is only the purchasing power effect at work, which implies that the economic value of saved taxes decreases in μ_P .

Panel B of Table 4 reports the values of CGTB, with the initial wealth being normalized to 1. Interestingly, we find that the investor in fact incurs more capital gains taxes in the Indexation Economy than she does in the Low Rate Economy. For example, in the base case, the present value of her tax bills equals 0.71 in the Low Rate Economy, and this figure increases to 1.34 in the Indexation Economy. This pattern remains true over a large range of parameter values.²²

²² We have also calculated CEWG and CGTB for an investor with even higher tax rates, and have found even stronger results. For example, for a high income investor with an ordinary income tax rate of 37% and a long-term capital gains tax rate of 20%, her equivalent wealth gain from moving to the Indexation Economy (Low Rate Economy) equals 1.60% (0.38%) of her initial wealth.

A rough calculation can shed light on why the investor incurs more capital gains taxes in the Indexation Economy. In the base case, reducing the capital gains tax rate from 30% to 15% will roughly reduce the expected tax bills by half; by comparison, indexing capital gains will lead to a 41% decrease in annual taxable returns only ($\mu_P/\mu_S = 0.034/0.083 = 41\%$). Therefore, more capital gains taxes are cut for the investor in the Low Rate Economy, despite the resultant welfare improvement is in fact lower.

Discussion. An interesting question is: why could the investor derive a higher utility level in the Indexation Economy where she incurs more capital gains taxes? The reason is that indexation could provide a unique benefit to the investor: it could help balance the investor's effective tax burdens across different states of the world. Specifically, under high inflation states, the investor has high marginal utilities due to low real consumption levels; thus, the tax bills saved by a quickly growing cost basis can be particularly valuable. Under low inflation states, although the investor will incur more tax bills, these taxes are less costly due to lower marginal utilities. Put differently, indexation could provide a partial hedge against the cost of taxes that is interfered with inflation, and it is this hedging effect which enables the investor to derive a higher utility level at the same time of paying more taxes.

To lend more support for the above argument, Figure 4 plots the difference in CEWGs, that is, $\Delta_I - \Delta_L$, against the inflation volatility σ_P . It suggests that the difference increases in σ_P . The intuition is that with a greater inflation volatility, the investor is facing a greater inflation risk; hence, the hedge provided by indexation should be relatively more valuable.

Overall, the above findings indicate that comparing with the practice of taxing nominal gains at the long-term tax rate, taxing indexed gains at the ordinary income tax rate could

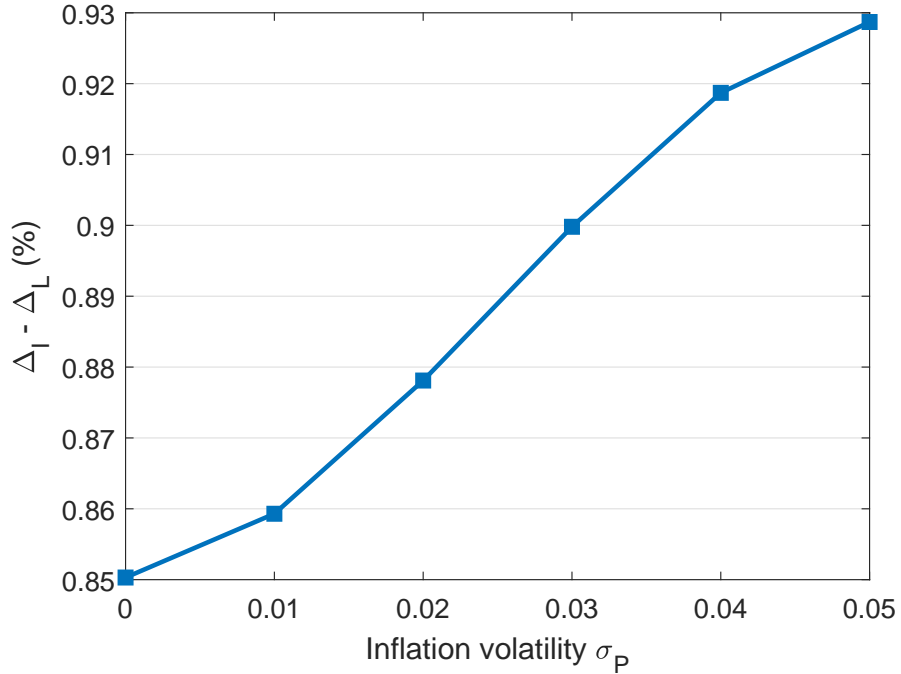


Figure 4: Inflation volatility and difference in certainty equivalent wealth gains

This figure shows the difference in certainty equivalent wealth gains (CEWG), namely $\Delta_I - \Delta_L$, against the inflation volatility. The base case parameter values are as follows: pre-tax interest rate $r = 0.041$, ex-dividend expected stock return $\mu_S = 0.083$, pre-tax dividend yield $\delta = 0.034$, stock return volatility $\sigma_S = 0.143$, average inflation rate $\mu_P = 0.034$, inflation volatility $\sigma_P = 0.013$, correlation between the inflation rate and stock returns $\rho = -0.063$, subjective discounting factor $\beta = 0.05$, relative risk aversion coefficient $\gamma = 6$, and ordinary income tax rate $\tau_d = \tau_i = 0.3$. In the Benchmark Economy, the expected growth rate and volatility of the cost basis $\mu_K = \sigma_K = 0$, and the capital gains tax rate $\tau_g = 0.3$; in the Indexation Economy, $\mu_K = \mu_P, \sigma_K = \sigma_P$ and $\tau_g = 0.3$; in the Low Rate Economy, $\mu_K = \sigma_K = 0$ and $\tau_g = 0.15$.

improve the investor's welfare and increase collected capital gains tax bills simultaneously.²³

5.4.2 Simulations Using Market Return Data

We have also performed simulations of our model in the Indexation Economy and the Low Rate Economy, using the market return data from January 1950 to December 2018 (i.e., the data used to estimate model parameters in the base case). To fix idea, we assume that the ordinary income tax rate and capital gains tax rate are all constant over this testing period.

Figure 5 shows the results obtained from these simulations. Panel A shows the ratio of the real consumption rate in the Indexation Economy to that in the Low Rate Economy, and Panel B shows the cumulative tax bills incurred by the investor (after time discounting). The results suggest that the investor tends to have higher consumption levels and incur more capital gains tax bills when she lives in the Indexation Economy, which is largely consistent with our theoretical results.

5.4.3 Robustness

In this subsection, we show that our main results continue to hold under some alternative scenarios.

Tax Naive Investor. In previous welfare analysis, we have assumed that the investor is able to optimally time her gains realizations. However, it is empirically found that many

²³ We emphasize that our results do not imply that the investor's welfare is unconditionally lower in the Low Rate Economy. Intuitively, the investor's CEWG Δ_L can be continuously improved by reducing τ_g towards zero. However, this is achieved at the cost of continuously decreasing capital gains tax revenue. Our results just suggest that indexation could improve the trade-off of investor welfare and capital gains tax revenue simultaneously.

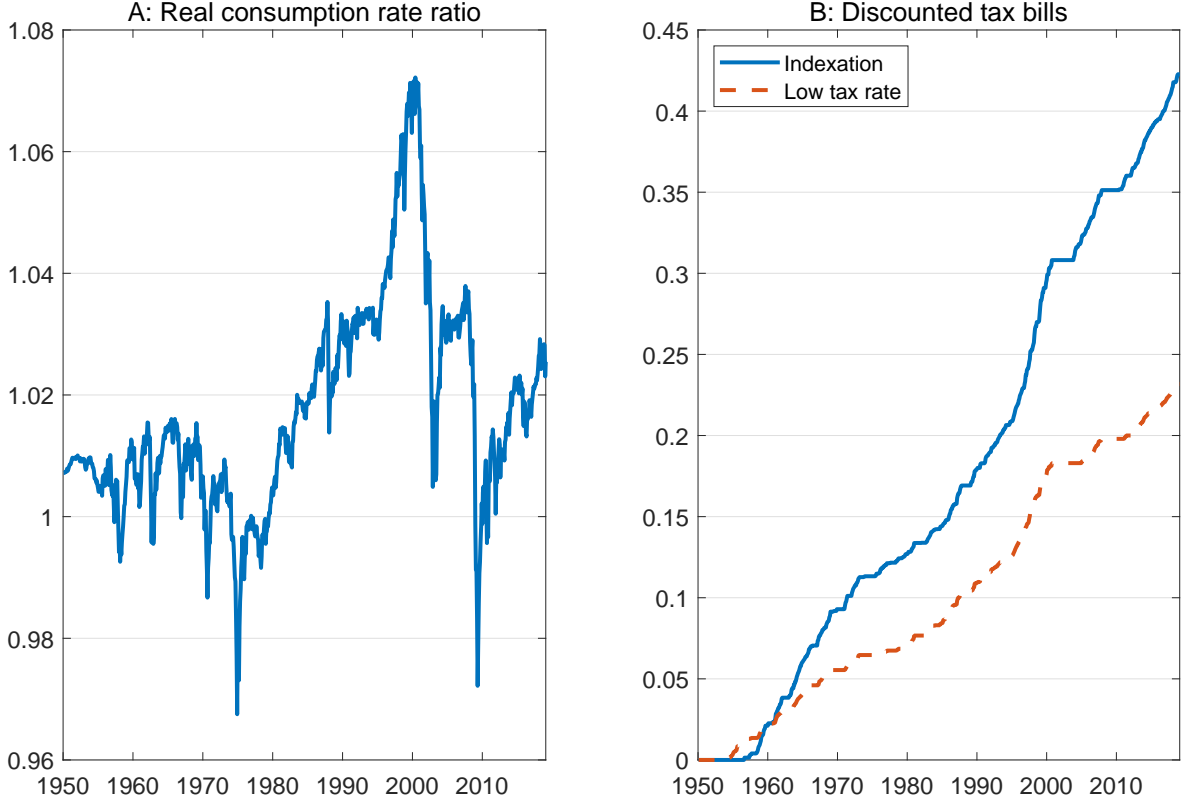


Figure 5: Real consumption rate ratio and discounted tax bills

The left subfigure (Panel A) shows the ratio of the investor's real consumption rates in the Indexation Economy to that in the Low Rate Economy, and the right subfigure (Panel B) shows the discounted capital gains tax bills incurred by the investor, over the testing period from January 1950 to December 2018. Tax rates are assumed to be constant over this period, and the initial wealth level is normalized to 1. The trading strategies are generated using the following base case parameters: pre-tax interest rate $r = 0.041$, ex-dividend expected stock return $\mu_S = 0.083$, pre-tax dividend yield $\delta = 0.034$, stock return volatility $\sigma_S = 0.143$, average inflation rate $\mu_P = 0.034$, inflation volatility $\sigma_P = 0.013$, correlation between the inflation rate and stock returns $\rho = -0.063$, subjective discounting factor $\beta = 0.05$, relative risk aversion coefficient $\gamma = 6$, and ordinary income tax rate $\tau_d = \tau_i = 0.3$. In the Benchmark Economy, the expected growth rate and volatility of the cost basis $\mu_K = \sigma_K = 0$, and the capital gains tax rate $\tau_g = 0.3$; in the Indexation Economy, $\mu_K = \mu_P, \sigma_K = \sigma_P$ and $\tau_g = 0.3$; in the Low Rate Economy, $\mu_K = \sigma_K = 0$ and $\tau_g = 0.15$.

individual investors tend to adopt tax-inefficient trading strategies. Hence, it is important to know whether our main results still hold for a naive investor who cannot optimally defer taxes. For this purpose, we calculate the welfare measures (i.e., CEWG and CGTB) for a naive investor who never defers capital gains realizations.²⁴

Figure 6 depicts the optimal trading strategies of the naive investor in three economies considered.²⁵ In the domain of gains (i.e., with a basis-price ratio $b < 1$), the investor realizes any capital gain immediately and repurchase some stock shares to re-establish an optimal risk exposure, as signified by the arrows from point A to point B then to point C. In the domain of losses (i.e., with $b > 1$), the investor rebalances portfolio continuously so that the stock allocation lies on the curve of optimal exposure, as signified by the arrow from point D to point E.

Table 5 reports the results for CEWG and CGTB. It suggests that in most cases, both the CEWG and CGTB in the Indexation Economy are still greater than those in the Low Rate Economy, indicating our results remain robust for a tax-naive investor. However, the magnitudes of differences in CGTB are generally smaller in this case.

Low Inflation Regime. Empirically, the inflation rate in the United States appeared to drop during the recent years. Thus, it is important to examine whether our results continue to hold under such a low inflation regime. For this purpose, we re-estimate model parameters,

²⁴ For such an investor, the CEWGs in the Indexation Economy or the Low Rate Economy are still calculated from equations (30) and (31) respectively, but with the function $V(x, y, k, p)$ being replaced by $V^N(x, y, k, p)$.

²⁵ Here “optimal trading strategy” refers to the best trading strategy in the family of trading strategies that never defer gains realizations.

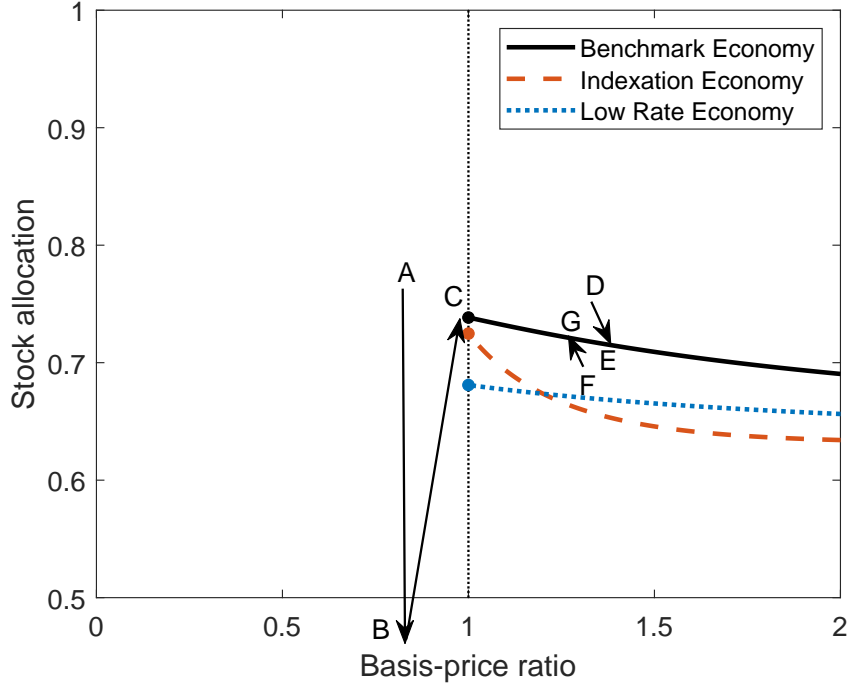


Figure 6: Optimal trading strategies: naive investor

This figure shows the optimal trading strategy of a naive investor who never defers capital gains realization in three economies considered. The base case parameter values are: pre-tax interest rate $r = 0.041$, ex-dividend expected stock return $\mu_S = 0.083$, pre-tax dividend yield $\delta = 0.034$, stock return volatility $\sigma_S = 0.143$, average inflation rate $\mu_P = 0.034$, inflation volatility $\sigma_P = 0.013$, correlation between the inflation rate and stock returns $\rho = -0.063$, subjective discounting factor $\beta = 0.05$, relative risk aversion coefficient $\gamma = 6$, and ordinary income tax rate $\tau_d = \tau_i = 0.3$. In the Benchmark Economy, the expected growth rate and volatility of the cost basis $\mu_K = \sigma_K = 0$, and the capital gains tax rate $\tau_g = 0.3$; in the Indexation Economy, $\mu_K = \mu_P, \sigma_K = \sigma_P$ and $\tau_g = 0.3$; in the Low Rate Economy, $\mu_K = \sigma_K = 0$ and $\tau_g = 0.15$.

Table 5:
Certainty equivalent wealth gains and capital gains tax bills: naive investor

This table reports the value of certainty equivalent wealth gain (CEWG) and discounted capital gains tax bills (CGTB) for a naive investor who never defers capital gains realizations. The base case parameter values are as follows: pre-tax interest rate $r = 0.041$, ex-dividend expected stock return $\mu_S = 0.083$, pre-tax dividend yield $\delta = 0.034$, stock return volatility $\sigma_S = 0.143$, average inflation rate $\mu_P = 0.034$, inflation volatility $\sigma_P = 0.013$, correlation between the inflation rate and stock returns $\rho = -0.063$, subjective discounting factor $\beta = 0.05$, relative risk aversion coefficient $\gamma = 6$, and ordinary income tax rate $\tau_d = \tau_i = 0.3$. In the Benchmark Economy, the expected growth rate and volatility of the cost basis $\mu_K = \sigma_K = 0$, and the capital gains tax rate $\tau_g = 0.3$; in the Indexation Economy, $\mu_K = \mu_P, \sigma_K = \sigma_P$ and $\tau_g = 0.3$; in the Low Rate Economy, $\mu_K = \sigma_K = 0$ and $\tau_g = 0.15$.

		Panel A: CEWG(%)		Panel B: CGTB	
		A1: Indexation	A2: Low Rate	B1: Indexation	B2: Low Rate
Baseline		17.49	13.69	1.49	1.43
β	-0.01	19.30	14.83	1.65	1.58
	+0.01	15.98	12.74	1.36	1.28
γ	-1	17.11	14.23	2.14	2.13
	+1	18.15	13.53	1.14	1.06
δ	$\times 1.1$	15.71	12.55	1.66	1.63
	$\times 0.9$	19.52	14.99	1.34	1.25
μ_S	$\times 1.1$	15.98	13.21	2.60	2.44
	$\times 0.9$	19.22	14.26	0.87	0.86
σ_S	$\times 1.2$	18.66	14.17	0.72	0.65
	$\times 1.5$	19.82	14.53	0.36	0.31
r	$\times 1.2$	17.95	14.11	0.80	0.72
	$\times 0.8$	16.22	12.79	3.10	3.17
μ_P	$\times 0.9$	14.35	12.15	1.36	1.21
	$\times 0.8$	11.81	10.95	1.23	1.02
σ_P	$\times 1.2$	17.68	13.78	1.48	1.43
	$\times 0.8$	17.34	13.62	1.51	1.43
ρ	+0.2	17.15	13.58	1.69	1.59
	-0.2	17.84	13.81	1.33	1.27

using the data from January 1990 to December 2018, and recalculate the welfare measures considered in this subsection. During this period, the average inflation rate (on an annual basis) decreased to 2.39%.²⁶

The corresponding results are reported in Table 6, and they confirm that our main results still hold in this case, for both the sophisticated and naive investors. For example, in the base case of the sophisticated investor, CEWG equals 2.55% (1.56%) in the Indexation Economy (Low Rate Economy), and CGTB equals 1.68 (0.69) in the Indexation Economy (Low Rate Economy). Qualitatively similar results are found for the cases of the naive investor.

Imperfect Indexation. When indexing the investor’s capital gains, it could be the case that the price index used for adjusting the cost basis (e.g., a CPI index) may only partially capture the consumption price level faced by the investor. In this case, indexation is imperfect from the investor’s point of view. To show that our main results remain robust in this case, we slightly extend the baseline model by assuming that the investor’s capital gains are indexed by a price level index \hat{p}_t , which can differ from the price level of her primary consumption goods p_t . This implies that the expected growth rate of the cost basis (i.e., μ_K) may differ from the average inflation rate *faced by the investor* (i.e., μ_P). Similarly, the volatility of the cost basis (i.e., σ_K) may also differ from the inflation volatility (i.e., σ_P).²⁷

²⁶ In this analysis, we re-estimate all the model parameters instead of only changing the average inflation rate parameter because inflation may affect asset prices (e.g., Boons, Duarte, Roon, and Szymanowska (2020)). The parameter values estimated from this sample are as follows: risk-free interest rate $r = 0.031$, average ex-dividend stock return $\mu_S = 0.078$, stock dividend rate $\delta = 0.022$, stock return volatility $\sigma_S = 0.142$, average inflation rate $\mu_P = 2.39\%$, inflation volatility $\sigma_P = 0.0115$, and the correlation between stock return and inflation rate $\rho = -0.0115$.

²⁷ To save space, the details of the model with imperfect indexation are not presented. They are available from the authors upon request.

Table 6: Certainty equivalent wealth gains and capital gains tax bills: low inflation regime

This table reports the value of CEWG and CGTB in economies with a lower inflation rate. The parameter values are calibrated to data from January 1990 to December 2018, as follows: pre-tax interest rate $r = 0.031$, ex-dividend expected stock return $\mu_S = 0.078$, pre-tax dividend yield $\delta = 0.022$, stock return volatility $\sigma_S = 0.142$, average inflation rate $\mu_P = 0.0239$, inflation volatility $\sigma_P = 0.0115$, correlation between the inflation rate and stock returns $\rho = -0.0115$, subjective discounting factor $\beta = 0.05$, and relative risk aversion coefficient $\gamma = 6$; $\tau_d = \tau_i = \tau_g = 0.3$, $\mu_K = \sigma_K = 0$ in the Benchmark Economy, $\tau_d = \tau_i = \tau_g = 0.3$, $\mu_K = 0.0239$, $\sigma_K = 0.0115$ in the Indexation Economy, and $\tau_d = \tau_i = 0.3$, $\tau_g = 0.15$, $\mu_K = \sigma_K = 0$ in the Low Rate Economy.

	Panel A: Sophisticated investor						Panel B: Naive investor					
	Panel A1: CEWG(%)			Panel A2: CGTB			Panel B1: CEWG(%)			Panel B2: CGTB		
	Indexation	Low Rate		Indexation	Low Rate		Indexation	Low Rate		Indexation	Low Rate	
Baseline	2.55	1.56		1.68	0.69		13.09	12.13		1.33	1.06	
β	-0.01	2.51	1.48	1.77	0.74		14.47	13.10		1.47	1.16	
	+0.01	2.58	1.63	1.57	0.64		11.94	11.31		1.20	0.96	
γ	-1	2.81	1.85	2.22	0.90		12.95	12.87		1.80	1.49	
	+1	2.46	1.38	1.35	0.55		13.38	11.69		1.06	0.82	
δ	$\times 1.1$	2.22	1.31	1.79	0.74		12.24	11.50		1.42	1.13	
	$\times 0.9$	2.91	1.85	1.56	0.65		14.02	12.80		1.25	0.98	
μ_S	$\times 1.1$	2.75	1.82	2.99	1.18		12.19	12.00		2.20	1.71	
	$\times 0.9$	2.38	1.32	0.93	0.40		14.02	12.21		0.83	0.66	
σ_S	$\times 1.2$	2.74	1.69	0.92	0.38		12.98	11.76		0.69	0.52	
	$\times 1.5$	3.47	2.35	0.44	0.19		12.43	10.99		0.35	0.26	
r	$\times 1.2$	2.62	1.76	0.97	0.41		13.27	12.40		0.85	0.64	
	$\times 0.8$	2.33	1.32	3.05	1.23		12.49	11.58		2.23	1.87	
μ_P	$\times 0.9$	2.41	1.64	1.57	0.64		11.05	11.16		1.26	0.95	
	$\times 0.8$	2.22	1.70	1.45	0.59		9.28	10.36		1.17	0.84	
σ_P	$\times 1.2$	2.57	1.56	1.71	0.69		13.22	12.18		1.33	1.06	
	$\times 0.8$	2.54	1.57	1.68	0.68		12.99	12.09		1.34	1.05	
ρ	+0.2	2.55	1.62	1.87	0.77		12.86	12.10		1.49	1.17	
	-0.2	2.55	1.51	1.50	0.61		13.32	12.15		1.20	0.96	

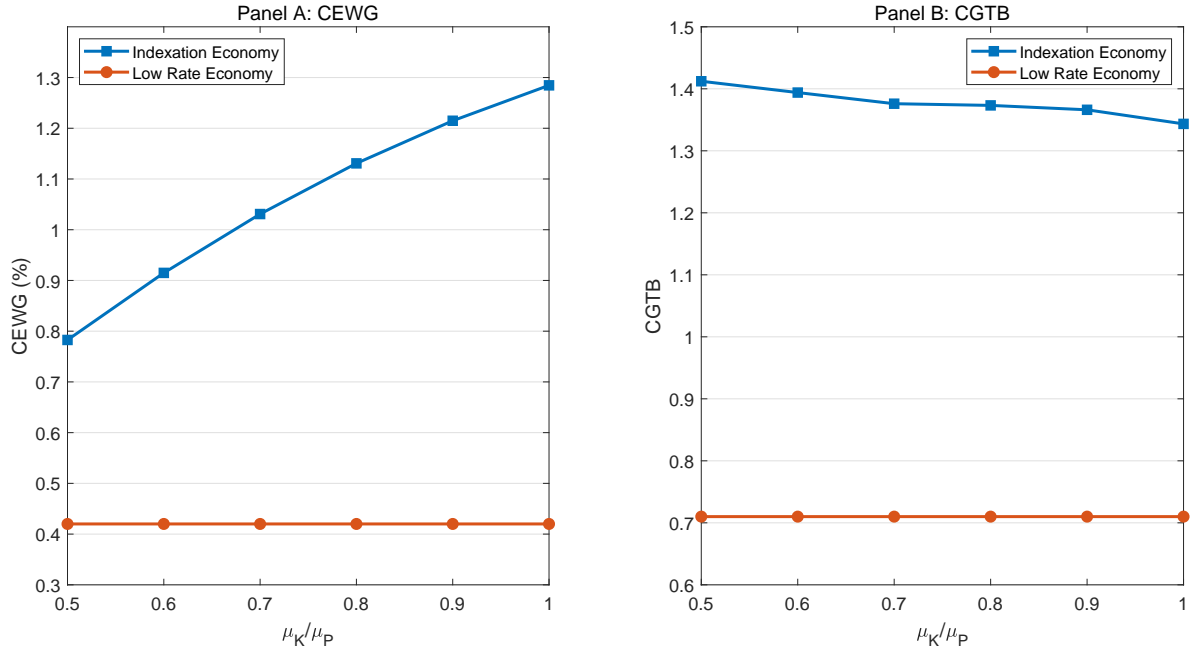


Figure 7: Certainty equivalent wealth gains and capital gains tax bills: imperfect indexation

This figure shows the certainty equivalent wealth gain (CEWG) and expected capital gains tax bills (CGTB) against the ratio μ_K/μ_P . The base case parameter values are as follows: pre-tax interest rate $r = 0.041$, ex-dividend expected stock return $\mu_S = 0.083$, pre-tax dividend yield $\delta = 0.034$, stock return volatility $\sigma_S = 0.143$, average inflation rate $\mu_P = 0.034$, inflation volatility $\sigma_P = 0.013$, correlation between the inflation rate and stock returns $\rho = -0.063$, subjective discounting factor $\beta = 0.05$, relative risk aversion coefficient $\gamma = 6$, and ordinary income tax rate $\tau_d = \tau_i = 0.3$. In the Benchmark Economy, the expected growth rate and volatility of the cost basis $\mu_K = \sigma_K = 0$, and the capital gains tax rate $\tau_g = 0.3$; in the Indexation Economy, $\mu_K = \mu_P, \sigma_K = \sigma_P$ and $\tau_g = 0.3$; in the Low Rate Economy, $\mu_K = \sigma_K = 0$ and $\tau_g = 0.15$.

To fix idea, we focus on the most important case in which $\mu_K \neq \mu_P$.²⁸ Figure 7 plots CEWG and CGTB against the ratio μ_K/μ_P . It suggests that even when the indexation is imperfect, CEWG and CGTB can still be significantly larger in the Indexation Economy than those in the Low Rate Economy. Therefore, our main results still hold in this case.

Labor Income. Our model does not incorporate the investor’s labor income for tractability considerations. In fact, including labor income will introduce another dimension to the optimization problem, making it extremely difficult to solve.

Nonetheless, we can qualitatively discuss the impact of including labor income in our model. On one hand, the growth of labor income over time could act as an important hedge against the inflation risk faced by the investor. On the other hand, we note that incorporating labor income will unlikely invalidate our main insight that indexation could benefit the investor by providing a hedge against the cost of *capital gains taxes* that is interfered with inflation, which enables the investor to derive a higher utility level while paying more capital gains taxes. Therefore, we believe that our main results should not be overturned by the inclusion of labor income.

6 Conclusion

The capital gains indexation proposal suggests to cut taxes by adjusting capital gains for inflation. Using a dynamic tax-timing model with inflation risk and possibly indexed capital

²⁸ We have also perturbed the inflation volatility parameter σ_K , and it turns out that the results are insensitive to changes in σ_K .

gains, we perform a theoretical analysis of this proposal, focusing on its potential implications for stock investors' tax-timing strategies and welfare. We show that with the limited use of losses, indexation would encourage a tax-aware investor to hold onto gains substantially longer, which is qualitatively different from the effect of setting a lower tax rate. Meanwhile, indexation would significantly reduce the costliness of tax naiveté as well. We also show that comparing with the practice of taxing nominal gains at the long-term tax rates, taxing indexed gains at the ordinary income tax rates could improve the investor's welfare and increase capital gains tax revenue simultaneously, and this result holds for both sophisticated and naive investors as characterized by their tax-timing abilities.

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Appendix

The contents of this appendix are as follows. Appendix A collects the proofs of our theoretical results. Appendix B presents the HJB equation after a dimensional reduction and a verification theorem.

A Proofs

Proof of Proposition 1.

Proof. In the absence of capital gains tax, the wealth process $W_t = x_t + y_t$ follows the dynamics

$$dW_t = [r(1 - \tau_i)W_t + (\mu_S + \delta(1 - \tau_d) - r(1 - \tau_i))y_t - C_t]dt + \sigma_S y_t dB_t. \quad (\text{A-1})$$

After choosing W_t as the state variable, the investor's problem can be rewritten as

$$V^0(W, p) = \max_{y_t, C_t} E \left[\int_0^\infty e^{-\beta t} u(C_t/p_t) dt \right], \quad (\text{A-2})$$

subject to equations (A-1) and (2), and the solvency constraint $W_t \geq 0$ for all $t \geq 0$. In this case, the associated HJB equation turns out to be

$$\begin{aligned} \max_{y, C} \left\{ [r(1 - \tau_i)W + (\mu_S + \delta(1 - \tau_d) - r(1 - \tau_i))y - C]V_W^0 + \mu_{FP}V_p^0 \right. \\ \left. + \frac{1}{2}\sigma_S^2 y^2 V_{WW}^0 + \frac{1}{2}\sigma_P^2 p^2 V_{pp}^0 + \rho\sigma_S\sigma_P y p V_{Wp}^0 - \beta V^0 + u(C/p) \right\} = 0. \end{aligned} \quad (\text{A-3})$$

Owing to the homogeneity of the CRRA preference, the value function $V^0(W, p)$ has the functional form as in equation (15). It is then easy to verify that the value function and optimal strategies are indeed given as stated in Proposition 1. \square

Proof of Proposition 2.

Proof. We follow the approach proposed by Lei, Li, and Xu (2020) to derive the value function $V^N(x, y, k, p)$ when the investor never defers realizing capital gains.

(i) When there is a loss (i.e., $k_t > y_t$), the processes $W_t = x_t + y_t$ and $A_t = x_t + k_t$ have the following dynamics

$$dW_t = [r(1 - \tau_i)W_t + (\mu_S + \delta(1 - \tau_d) - r(1 - \tau_i))y_t - C_t]dt + \sigma_S y_t dB_t, \quad (\text{A-4})$$

$$\begin{aligned} dA_t &= [r(1 - \tau_i)W_t + (\delta(1 - \tau_d) - r(1 - \tau_i))y_t - C_t]dt + \mu_K(A_t - W_t + y_t)dt \\ &\quad + \sigma_K(A_t - W_t + y_t)dZ_t. \end{aligned} \quad (\text{A-5})$$

We can choose these two variables as new state variables and rewrite the value function as

$$V^N(W, A, p) = \max_{y_t, C_t} E \left[\int_0^\infty e^{-\beta t} u(C_t/p_t) dt \right] \quad (\text{A-6})$$

(with a slight abuse of notation, we still denote this value function as V^N). The associated

HJB equation is

$$\begin{aligned}
\max_{y,C} \left\{ [r(1-\tau_i)W + (\mu_S + \delta(1-\tau_d) - r(1-\tau_i))y - C]V_W^N + \frac{1}{2}\sigma_S^2 y^2 V_{WW}^N \right. \\
+ [r(1-\tau_i)W + (\delta(1-\tau_d) - r(1-\tau_i))y + \mu_K(A-W+y) - C]V_A^N \\
+ \frac{1}{2}\sigma_K^2 (A-W+y)^2 V_{AA}^N + \mu_{PP}V_p^N + \frac{1}{2}\sigma_{PP}^2 V_{pp}^N \\
+ \rho\sigma_S\sigma_{Pyp}V_{Wp}^N + \rho\sigma_S\sigma_K y(A-W+y)V_{WA}^N \\
\left. + \sigma_K\sigma_{PP}(A-W+y)V_{Ap}^N - \beta V^N + u(C/p) \right\} = 0 \tag{A-7}
\end{aligned}$$

for $A > W > 0$. Due to the homogeneity property, we make the following transformation:

$$V^N(W, A, p) = p^{\gamma-1} \frac{W^{1-\gamma}}{1-\gamma} e^{(1-\gamma)\Psi^N(\zeta)}, \tag{A-8}$$

where $\zeta = \frac{W}{A} \in [0, 1)$; then, direct calculations show that the function $\Psi^N(\zeta)$ satisfies

$$\begin{aligned}
\max_{z,c} \left\{ \left[\frac{1}{2}\sigma_S^2 z^2 + \frac{1}{2}\sigma_K^2 (1-\zeta+z\zeta)^2 - \rho\sigma_S\sigma_K z(1-\zeta+z\zeta) \right] \zeta^2 [\Psi_{\zeta\zeta}^N + (1-\gamma)(\Psi_{\zeta}^N)^2] \right. \\
+ \left[(1-\gamma)\sigma_S^2 z^2 + r(1-\tau_i) + (\mu_S - r(1-\tau_i))z - r(1-\tau_i)(1-z)\zeta + \delta(1-\tau_d)z(1-\zeta) \right. \\
+ \left. \sigma_K^2 (1-\zeta+z\zeta)^2 - \tilde{\mu}_K(1-\zeta+z\zeta) + (\gamma-2)\rho\sigma_S\sigma_K z(1-\zeta+z\zeta) \right] \zeta \Psi_{\zeta}^N \\
- [1 + (1-\zeta)\zeta \Psi_{\zeta}^N] c + e^{-(1-\gamma)\Psi^N} u(c) \\
\left. - \frac{1}{2}\gamma\sigma_S^2 z^2 + [\mu_S - r(1-\tau_i) + \delta(1-\tau_d)]z + r(1-\tau_i) - \frac{\beta}{1-\gamma} \right\} = 0, \tag{A-9}
\end{aligned}$$

with the boundary condition

$$-\Psi_{\zeta}^N = \tau_g, \tag{A-10}$$

at $\zeta = 1$ (cf. equation (A-16) below), where

$$\tilde{\mu}_K = \mu_K + \sigma_K \sigma_P (\gamma - 1). \quad (\text{A-11})$$

The optimal strategy $(z, c) = (\frac{y}{W}, \frac{C}{W})$ can be obtained by solving the first-order conditions of (A-9).

(ii) When there is a gain (i.e., $y_t > k_t$), because the investor does not defer its realization, the optimal trading strategy can be expressed as follows:

$$(x_t, y_t, k_t) \rightarrow (W_t, 0, 0) \rightarrow ((1 - \pi_*)W_t, \pi_*W_t, \pi_*W_t), \quad (\text{A-12})$$

where $W_t = x_t + y_t - \tau_g(y_t - k_t)$. Then, we have

$$V^N(x, y, k, p) = V^N((1 - \pi_*)W, \pi_*W, \pi_*W, p) \quad (\text{A-13})$$

for all $\pi_* \geq 0$. Again, exploiting the homogeneity property, we can rewrite the value function as follows:

$$V^N(x, y, k, p) = p^{\gamma-1} \frac{(x+y)^{1-\gamma}}{1-\gamma} e^{(1-\gamma)\psi^N(z,b)}, \quad (\text{A-14})$$

where $z = \frac{y}{x+y}$ and $b = \frac{k}{y}$. Then, from (A-13), we have equation (19), or equivalently

$$\frac{\psi^N(z, 1) - \psi^N(z, b)}{1 - b} = \frac{-\log(1 - \tau_g z(1 - b))}{1 - b}, \quad (\text{A-15})$$

where $\psi^N(z, 1) \equiv \Psi^N(1)$. Therefore, by sending $b \rightarrow 1$, we obtain the boundary condition

$$\frac{\partial \psi^N(z, b)}{\partial b} \Big|_{b=1} = \tau_g z. \quad (\text{A-16})$$

By rewriting $\psi^N(z, b) = \Psi^N(\zeta) = \Psi^N\left(\frac{1}{1-z+bz}\right)$ for $b \geq 1$ and assuming the first order derivative with respect to b is continuous, we can obtain the boundary condition (A-10).

This completes our derivation. \square

B Dimensional Reduction and Verification Theorem

HJB Equation after Dimensional Reduction. Applying the chain rule to calculate the partial derivatives of $V(x, y, k, p)$ through the partial derivatives of $\Psi(z, b)$, we can derive that $\Psi(z, b)$ satisfies the following equation:

$$\max \left\{ \mathcal{L}_1 \Psi - \frac{1}{2} \gamma \sigma_S^2 z^2 + [\tilde{\mu}_S - r(1 - \tau_i) + \delta(1 - \tau_d)]z + r(1 - \tau_i) - \frac{\beta}{1 - \gamma}, \mathcal{B}_1 \Psi, \mathcal{S}_1 \Psi \right\} = 0, \quad (\text{A-17})$$

where

$$\begin{aligned} \mathcal{L}_1 \Psi &= \frac{1}{2} \sigma_S^2 z^2 (1 - z)^2 [\Psi_{zz} + (1 - \gamma) \Psi_z^2] + \frac{1}{2} (\sigma_S^2 + \sigma_K^2 - 2\rho \sigma_S \sigma_K) b^2 [\Psi_{bb} + (1 - \gamma) \Psi_b^2] \\ &\quad + (\rho \sigma_S \sigma_K - \sigma_S^2) z (1 - z) b [\Psi_{zb} + (1 - \gamma) \Psi_z \Psi_b] \\ &\quad + [(\tilde{\mu}_S - r(1 - \tau_i) - \gamma \sigma_S^2 z)(1 - z) - \delta(1 - \tau_d)z] z \Psi_z \\ &\quad + [\sigma_S^2 - \tilde{\mu}_S - (1 - \gamma) \sigma_S^2 z + \tilde{\mu}_K - \rho \sigma_S \sigma_K + \rho \sigma_S \sigma_K (1 - \gamma)z] b \Psi_b \\ &\quad + \frac{\gamma}{1 - \gamma} (e^\Psi (1 - z \Psi_z))^{\gamma-1/\gamma}, \end{aligned} \quad (\text{A-18})$$

$$\mathcal{B}_1\Psi = z\Psi_z + (1-b)\Psi_b, \quad (\text{A-19})$$

and

$$\mathcal{S}_1\Psi = -\tau_g z(1-b)^+ - [1 - \tau_g z(1-b)^+]z\Psi_z + (b-1)^+\Psi_b. \quad (\text{A-20})$$

Then, the Sell region, Buy region, and No-trade region can be characterized through Ψ as follows:

$$SR = \{(z, b) : \mathcal{S}_1\Psi = 0\}, \quad (\text{A-21})$$

$$BR = \{(z, b) : \mathcal{B}_1\Psi = 0\}, \quad (\text{A-22})$$

and

$$NTR = \{(z, b) : \mathcal{S}_1\Psi < 0, \mathcal{B}_1\Psi < 0\}. \quad (\text{A-23})$$

The penalty method, combined with finite differences scheme, is used to solve equation (A-17) (see, e.g., Bian, Chen, Dai, and Qian (2019)).

When there is a capital loss (i.e., $b > 1$), continuous trading is optimal. This is because we simultaneously have

$$\mathcal{S}V = yV_x - yV_y - yV_k \leq 0, \quad (\text{A-24})$$

and

$$\mathcal{B}V = -V_x + V_y + V_k \leq 0. \quad (\text{A-25})$$

Hence, it must be the case that

$$V_x - V_y - V_k = 0, \quad (\text{A-26})$$

which implies continuous trading. In this case, we denote the optimal allocation to the stock, as a function of b , by $\pi^*(b)$.²⁹

Verification Theorem. Next, we provide a verification theorem.

Proposition 3. *Let $\Psi(z, b)$ be a solution to the HJB equation (A-17) satisfying certain regularity conditions. Denote $\partial B = \overline{NTR} \cap BR$ and $\partial S = \overline{NTR} \cap SR$. Define*

$$V(x, y, k, p) = \frac{1}{1 - \gamma} (x + y)^{1-\gamma} p^{\gamma-1} e^{(1-\gamma)\Psi(\frac{y}{x+y}, \frac{k}{y})}. \quad (\text{A-27})$$

Assume that the process $(V(x_t, y_t, k_t, p_t))_{t \geq 0}$ is uniformly integrable for any admissible trading and consumption strategy. Then, $V(x_0, y_0, k_0, p_0)$ coincides with the investor's value function, and the optimal strategies are given by

(i) *Optimal trading strategies: if $b > 1$, then it is optimal to trade continuously to maintain the proportion $\pi^*(b)$ of wealth in the stock; otherwise, the optimal selling strategy is*

$$D_t^* = \int_0^t 1_{\{(\frac{y_s^*}{x_s^* + y_s^*}, \frac{k_s^*}{y_s^*}) \in \partial S\}} dD_s^*, \quad (\text{A-28})$$

²⁹The numerical algorithm of finding the solution is provided by Dai, Liu, Yang, and Zhong (2015).

and the optimal buying strategy is

$$I_t^* = \int_0^t 1_{\left\{\left(\frac{y_s^*}{x_s^* + y_s^*}, \frac{k_s^*}{y_s^*}\right) \in \partial B\right\}} dI_s^*. \quad (\text{A-29})$$

(ii) *Optimal consumption strategy:*

$$C_t^* = (x_t^* + y_t^*) e^{\frac{\gamma-1}{\gamma} \Psi\left(\frac{y_t^*}{x_t^* + y_t^*}, \frac{k_t^*}{y_t^*}\right)} \left(1 - \frac{y_t^*}{x_t^* + y_t^*} \Psi_z\left(\frac{y_t^*}{x_t^* + y_t^*}, \frac{k_t^*}{y_t^*}\right)\right)^{-1/\gamma}. \quad (\text{A-30})$$

Proof. The proof is similar to that in Davis and Norman (1990) and Dai, Liu, Yang, and Zhong (2015). Here, we provide only the main steps for the proof.

For any $(C, I, D) \in \mathcal{A}(x, y, k, p)$, we denote by (x_t, y_t, k_t, p_t) the resultant processes satisfying (2)–(6) with the initial endowment $(x_0, y_0, k_0, p_0) = (x, y, k, p)$. We define

$$N_t = \int_0^t e^{-\beta s} u(C_s/p_s) ds + e^{-\beta t} V(x_t, y_t, k_t, p_t). \quad (\text{A-31})$$

An application of the generalized Itô's formula yields

$$\begin{aligned} N_t &= N_0 + \int_0^t e^{-\beta s} (\mathcal{L}^C V - \beta V) ds + \int_0^t e^{-\beta s} (\mathcal{S}V) dD_s^c + \int_0^t e^{-\beta s} (\mathcal{B}V) dI_s^c \\ &+ \sum_{0 \leq s \leq t} e^{-\beta s} \left[V(x_{s-} + f(0, y_{s-}, k_{s-}) \Delta D_s, y_{s-} - y_{s-} \Delta D_s, \right. \\ &\quad \left. k_{s-} - (k_{s-} - (k_{s-} - y_{s-})^+) \Delta D_s, p_{s-}) - V(x_{s-}, y_{s-}, k_{s-}, p_{s-}) \right] \\ &+ \sum_{0 \leq s \leq t} e^{-\beta s} \left[V(x_{s-} - \Delta I_s, y_{s-} + \Delta I_s, k_{s-} + \Delta I_s, p_{s-}) - V(x_{s-}, y_{s-}, k_{s-}, p_{s-}) \right] \\ &+ \int_0^t e^{-\beta s} \sigma_S y_s V_y dB_s + \int_0^t e^{-\beta s} \sigma_K k_s V_k dZ_s + \int_0^t e^{-\beta s} \sigma_P p_s V_p dZ_s, \end{aligned} \quad (\text{A-32})$$

where the operators \mathcal{L}^C , \mathcal{S} , and \mathcal{B} are given by (22)-(24), I_t^c and D_t^c are the continuous parts of I_t and D_t respectively, and $\Delta I_s = I_s - I_{s-}$ and $\Delta D_s = D_s - D_{s-} \in (0, 1]$ are the discrete jumps at time s . From the mean value theorem, there exists $0 \leq d \leq \Delta D_s \leq 1$ such that the fifth term equals

$$\begin{aligned}
& \sum_{0 \leq s \leq t} e^{-\beta s} \left[f(0, y_{s-}, k_{s-}) V_x - y_{s-} V_y - (k_{s-} - (k_{s-} - y_{s-})^+) V_k \right] \Delta D_s \\
&= \sum_{0 \leq s \leq t} e^{-\beta s} \frac{f(0, y_d, k_d) V_x - y_d V_y - (k_d - (k_d - y_d)^+) V_k}{1 - d} \Delta D_s \\
&\leq 0,
\end{aligned} \tag{A-33}$$

where V_x , V_y , and V_k are evaluated at (x_d, y_d, k_d, p_{s-}) with

$$x_d = x_{s-} + f(0, y_{s-}, k_{s-})d, \tag{A-34}$$

$$y_d = y_{s-} - y_{s-}d, \tag{A-35}$$

$$k_d = k_{s-} - (k_{s-} - (k_{s-} - y_{s-})^+)d. \tag{A-36}$$

Using a similar argument, the sixth term is also nonpositive.

For equation (A-32), we have (i) the second to fourth terms are nonpositive for any admissible strategy (C, I, D) and are zero for the proposed one (C^*, I^*, D^*) since V satisfies the HJB equation (21), $dD_s^c \geq 0$, and $dI_s^c \geq 0$; (ii) the fifth and sixth terms are nonpositive for any admissible strategy and are zero for the proposed one under the above argument; and (iii) the seventh to ninth terms are martingales for any admissible strategy because of the boundedness of V_y , V_k , and V_p . These observations show that N_t is a martingale under

the proposed strategy and a supermartingale for any admissible strategy. Then, we have $N_0 \geq E[N_t]$, that is,

$$V(x_0, y_0, k_0, p_0) \geq E \left[\int_0^t e^{-\beta s} u(C_s/p_s) ds + e^{-\beta t} V(x_t, y_t, k_t, p_t) \right] \quad (\text{A-37})$$

for any admissible strategy with equality for the proposed one. Taking $t \rightarrow \infty$ and using the transversality condition, we have

$$\lim_{t \rightarrow \infty} E \left[e^{-\beta t} V(x_t, y_t, k_t, p_t) \right] = 0, \quad (\text{A-38})$$

which follows from applying Itô's formula to $e^{-\beta t} V(x_t, y_t, k_t, p_t)$, the HJB equation, and the boundness of Ψ inside the NTR , and thus we complete the proof. \square