Stock Prices, Changes in Liquidity, and Liquidity Premia

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Abstract

This paper develops a present-value framework that factors in expectations of future market illiquidity. In our framework, an implied liquidity premium is a function of prices, dividends, and, illiquidity costs. We find that the liquidity premium for the CRSP market portfolio is significantly priced over short horizons, but its long-horizon effect is not significant. This finding implies that unexpected illiquidity news—a main cause of the liquidity premium—is so transient that even its big variation in the first place could not build up over horizons toward a big price change. We reconcile our findings with some theoretical debate over the importance of the liquidity premium on asset pricing. In sum, market liquidity risk is basically second-order.

JEL Classifications: C12, C32, G12.

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1. Introduction

A perfect market assumes no trade impediments. Standard asset pricing theory (e.g., the Capital Asset Pricing Model) says that all securities are commonly affected by a systematic risk factor. In financial markets, therefore, investors want to be compensated for holding risky securities, which is determined *solely* by the common systematic risk (i.e., market beta).

In reality, however, we encounter various kinds of market imperfections that can bring other sources of premia, e.g., transaction costs.[1] Due to the risk of incurring high costs in liquidating portfolios in the future, investors demand a *liquidity premium* to compensate for potential losses, which can result in lowering prices today.

The starting point of this article is to acknowledge that it is hard to detect the liquidity premium empirically. The big obstacle of estimating the hard-to-detect *actual* premium motivates us to come up with another way of inference. In this paper, we challenge to identify the *implied* market liquidity premium caused by market liquidity risk,[2] which is inferred from a new present-value relationship we propose. With the implied premium in place, our purpose is to examine whether the liquidity premium is a main source of price variation.

To the point, we go on to tackle some theoretical debate over the importance of the liquidity premium on asset pricing. On the one side, Amihud and Mendelson (1986) and Lynch and Tan (2011) claim that asset returns carry a substantial size of the liquidity premium. On the other side, Constantinides (1986) and Vayanos (1998) argue that the liquidity premium should only be second-order because investors want to reduce the trading frequency in the presence of transaction costs.

To understand this debate clearly, suppose that market liquidity risk is so "second-order"

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[1] Vayanos and Wang (2013) provide an extensive review for market imperfections.

[2] Apart from market liquidity risk, investors with leveraged positions (e.g., a hedge fund) can want funding liquidity risk compensation. This funding liquidity risk can interact with the market liquidity risk, which jointly affects required returns (Pedersen, 2015). In this paper, our focus is on market liquidity risk.

that its cumulative effect on prices *cannot* last for a long time. This *transient* feature allows risk-averse investors to adjust more myopic demand than intertemporal hedging demand arising from the desire against adverse changes in *persistent* shocks. Therefore, a deep understanding of the liquidity premium is essential for investors to especially forming long-term investment decisions.

This paper presents compelling evidence reconciling the aforementioned theoretical debate. It turns out that market liquidity risk just matters in the short run. In the long-run, however, investors do *not* seem to mind it seriously since the liquidity risk is *not* expected to last for a long time. The implication is that if you are planning to sell the market portfolio somewhere in the *far* future, in fact, the liquidity risk is *not* relevant to you. Hence, you hardly ask for a market liquidity risk premium because market liquidity risk is basically second-order.

To explain these ideas precisely, Section 2 develops a *new* present-value framework that incorporates illiquidity costs beyond the conventional price–dividend one (henceforth, “illiquidity costs”, “negative dividends”, and “(il)liquidity” are interchangeably used for convenience). As a result, we devise log liquidity-adjusted price-dividend ratio (hereafter, *pdl* ratio), which embodies expected dividend growth, illiquidity growth, returns, and liquidity premia over the indefinite future. Hence, the *implied* liquidity premium can be inferred as residuals, provided that the other variables discussed above are observable *ex-post*. This is why the liquidity premium can be expressed as a function of prices, dividends, and costs.

One thing to note is that we leverage *price*-based illiquidity as a mirror image of (positive) dividends, which are nonstationary, I(1). This premise is the most challenging because the conventional studies have usually stuck to *return*-based illiquidity, I(0).[3] Given the fact that excess price volatility relative to dividends is evident in favor of return predictability (Cochrane,

[3] See Amihud and Mendelson (1986), Amihud (2002), Jones (2002), Pástor and Stambaugh (2003), Acharya and Pedersen (2005), Bekaert et al. (2007), Korajczyk and Sadka (2008), Ben-Rephael et al (2015), and among others.

2008), it makes sense in our present-value context that excess illiquidity volatility relative to prices and dividends can give rise to liquidity-premium forecastability. When market liquidity risk turns out to be second-order, for example, it is worth mentioning that the implied liquidity premium should be *rarely* forecastable. This predictive analysis can thus be a simple way of examining whether the liquidity premium is a crucial source of asset prices.

One big hurdle of conducting empirical analysis is that market illiquidity is hard to define and measure. To handle this, Section 3 uses the Amihud's (2002) illiquidity measure to estimate the *pdl* ratios (we will elaborate on a full exposition of our proxy choice later). By using annual Center for Research in Security Prices (CRSP) index data from 1926 to 2017, Figure 1 graphs the *pdl* ratios along with the market price–dividend ratios (henceforth, *pd* ratio).

[INSERT FIGURE 1 HERE]

The wedge between the two series offers a great deal of intuition that market illiquidity provides a good estimate of the *trend* in prices. The rationale behind this is that the *pdl* ratio looks like the trend-adjusted *pd* ratio by our cost proxy. It is important to note that such intuition is key in understanding our main results in the rest of the paper.

Section 4 departs from simple predictive regressions. In particular, we hypothesize a *constant-expected-liquidity-premium* model by restricting expected liquidity premia to be constant through time. As a result, the short-term hypothesis is strongly rejected at the conventional levels, whereas the long-term hypothesis does *not*. The two hypothesis results suggest that even a big shock in the first place *cannot* lead to a big price change in the end. Stated differently, investors take great care of unexpected illiquidity news in the first place, but do *not* likely care much in the long-run. This dynamic change can support the idea that market liquidity risk is essentially second-order.

Section 5 conducts impulse-response functions to explore why investors react to market illiquidity separately over both horizons. The main answer is due to the *transient* nature of market illiquidity because the excess illiquidity volatility can produce liquidity premia inherent in prices, as mentioned before. By design, we identify a *pure* illiquidity shock that generates expected liquidity premia moving forward with *no* changes in the other information sources. As a result, we confirm transient evidence that prices, dividends, and illiquidity *stay put* at the point of the shock onward. If the excess illiquidity volatility were by far persistent, it is clear that the three variables would *not* stay there.

The transient nature gives us a solid understanding of market illiquidity. The conventional norm is that shocks to illiquidity are pretty persistent. Hence, it seems natural that one illiquidity rise can lead to (a) a rise in expected illiquidity and then (b) a rise in expected returns, which lowers current prices as a result. As for the market portfolio at least, we contend that shocks to market illiquidity might lower prices immediately, *little* relying on (a) and (b). The reason boils down to the central intuition that market illiquidity provides the natural price trend. The technical interpretation for this is that market illiquidity is nearly a random walk so that its cyclical variation is almost a white noise (or i.i.d.). This interpretation is also supported by the lack of illiquidity-growth forecastability conducted in Section 4. Therefore, the *pure* illiquidity shock can give stay-put evidence.

We want to emphasize two points discerning the extant literature. First, we provide a restrictive structure under which the identification problem on the *unique* price–dividend–cost relationship is alleviated. In contrast, Jones (2002) analyzes the *ad-hoc* interplay between the *pd* ratios and illiquidity, which could induce misleading outcomes. Second, present-value logic says that all information sources are *interconnected* with each other. However, Amihud (2002), Jones (2002), and Bekaert et al. (2007) assume that market illiquidity is *independent* of aggregate dividends. Relatedly, the conventional price–dividend relationship shows that return predictability is evident due to the *lack* of dividend growth forecastability (Cochrane, 2008).

In this context, our price–dividend–cost relationship can pinpoint such an interconnection by addressing, for example, "how stronger liquidity-premium forecastability is than the others?", whereas the conventional studies do not.

Finally, Section 6 concludes the paper. Admittedly, our conclusion relies on which proxies are used. To mitigate this concern, we also use the log quarterly bid-ask spread in Appendix B, but our conclusion is invariant. Indeed, market liquidity risk is essentially second-order.

2. Theoretical framework

In principle, asset prices should equal expected discounted cash flows. Motivated by this, illiquidity costs—referred to as negative dividends (Jones, 2002; Acharya and Pedersen, 2005)—can be another priced factor in a present-value context.

First, we build upon the cross-sectional identity that stock prices and dividends are linked to illiquidity costs. This linkage can be rewritten as a natural log of *net return*, r^*_{t+1}, measured at the end of time $t+1$:

$$r^*_{t+1} = \log\left(\frac{P_{t+1}+D_{t+1}-C_{t+1}}{P_t}\right) = \log\left(1+\frac{D_{t+1}}{P_{t+1}}-\frac{C_{t+1}}{P_{t+1}}\right) + p_{t+1} - p_t, \qquad (1)$$

where P_t is the real price of a stock, measured at the end of time period t; D_t is the real dividend during period t; and C_t denotes the illiquidity cost during period t. Here, we assume that C_t is nonstationary, and use the convention that logs of variables are denoted by lowercase letters, i.e., $p_t \equiv \log(P_t)$.

We then rewrite log net return (1) as

$$r^*_{t+1} = \log(1+\exp(dp_{t+1}) - \exp(cp_{t+1})) + p_{t+1} - p_t, \qquad (2)$$

where $dp_t \equiv d_t - p_t$ is the log dividend-price ratio with sample mean of $\overline{dp}$, and $cp_t \equiv c_t - p_t$ is the log liquidity-price ratio with sample mean of $\overline{cp}$. In log net return (2), the first term on the right-hand-side (RHS) is a nonlinear function of dp_{t+1} and cp_{t+1}. The two ratios will be linearized to derive a new price–dividend–cost relationship.

Second, we apply a first-order Taylor expansion to the nonlinear function above:

$$\log(1 + \exp(dp_{t+1}) - \exp(cp_{t+1})) \approx$$
$$k_l + (\rho_l - 1)p_{t+1} + \rho_l(1/\rho - 1)d_{t+1} - \rho_l(1/\rho - 1/\rho_l)c_{t+1}, \quad (3)$$

where ρ_l and ρ are log-linear discount factors ($\rho_l > \rho$), and k_l is a log-linear coefficient (see Appendix A for more technical details). Substituting (3) into (2) yields the linear difference equation of the log net return:

$$r^*_{t+1} \approx k_l + \rho_l \cdot p_{t+1} + \rho_l(1/\rho - 1)d_{t+1} - \rho_l(1/\rho - 1/\rho_l)c_{t+1} - p_t. \quad (4)$$

Third, we solve difference equation (4) forward with a terminal condition that rules out rational bubbles: $\lim_{j\to\infty}(\rho_l)^j p_{t+1+j} = 0$. We then obtain the following present-value identity without a constant term $k_l/(1-\rho_l)$ in the form of log prices:

$$p_t \approx \sum_{j=0}^{\infty}(\rho_l)^j \left[\rho_l\left[\left(\frac{1}{\rho} - 1\right)d_{t+1+j} - \left(\frac{1}{\rho} - \frac{1}{\rho_l}\right)c_{t+1+j}\right] - r^*_{t+1+j}\right]$$
$$\approx \sum_{j=0}^{\infty}(\rho_l)^j \left[\rho_l\left[\left(\frac{1}{\rho} - 1\right)d_{t+1+j} - \left(\frac{1}{\rho} - \frac{1}{\rho_l}\right)c_{t+1+j}\right] - (r_{t+1+j} - lp_{t+1+j})\right], \quad (5)$$

provided that the state variables on the RHS are observable *ex-post*. In non-stationary present-value form (5), we further impose an ex-post liquidity premium lp_t such that the following accounting identity holds:

$$r^*_t = r_t - lp_t. \quad (6)$$

This new accounting identity means that investors can gain liquidity-adjusted profit r^*_t at the expense of the liquidity premium; large liquidity premium lp_t reduces net profit r^*_t.

Fourth, we take an expectation $E[\cdot|\mathcal{H}_t]$ (or $E_t[\cdot]$) conditional on a new information set $\mathcal{H}_t$

available at time t. This corresponds to the following *ex-ante* (expected) present-value identity:

$$pd_t^l \approx E\left[\sum_{j=0}^{\infty}(\rho_l)^j\left[\beta_1 \Delta d_{t+1+j} - \beta_2 \Delta c_{t+1+j} - r_{t+1+j} + lp_{t+1+j}\right] \middle| \mathcal{H}_t\right], \quad (7)$$

where Δ denotes the first difference operator (e.g., $\Delta d_t = d_t - d_{t-1}$), $\beta_1 = \frac{\rho_l(1-\rho)}{\rho(1-\rho_l)} = \frac{1}{1-\bar{C}/\bar{D}} > 1$ because $\rho_l > \rho$, $\beta_2 = \frac{\rho_l - \rho}{\rho(1-\rho_l)} = \frac{\bar{C}/\bar{D}}{1-\bar{C}/\bar{D}} > 0$, and $pd_t^l \equiv p_t - \beta_1 d_t + \beta_2 c_t$ is the log *liquidity-adjusted price–dividend ratio*. Importantly, present-value identity (7) provides a restrictive structure to identify the unique cointegration relationship as below.

PROPOSITION 1: Cointegration restriction

If log prices share a common trend with log dividends and log illiquidity costs, their linear combination must satisfy

$$1 - \beta_1 + \beta_2 = 0,$$

given that expected dividend growth, illiquidity growth, real returns, and liquidity premia are stationary.

Proof: It is straightforward to show Proposition 1 from $\beta_1 = \frac{1}{1-\bar{C}/\bar{D}}$ and $\beta_2 = \frac{\bar{C}/\bar{D}}{1-\bar{C}/\bar{D}}$.

As stressed before, one key premise is that the illiquidity cost can function as a negative dividend (Jones 2002; Acharya and Pedersen 2005). Hence, the total weights of positive and negative dividends can add up to one: $\beta_1 - \beta_2 = 1$. The cointegrating vector between log prices and log total dividends thus becomes $[1, -(\beta_1 - \beta_2)]' = [1, -1]'$, as in price–dividend ratio $pd_t \equiv p_t - d_t$.

Suppose that Proposition 1 holds with data. Then high prices relative to total dividends (i.e., pd_t^l) signal high expected dividend growth, low expected illiquidity growth, low expected returns, and high expected liquidity premia in the future. At a technical level, the inclusion of negative dividends gives rise to two additional expectations of (a) illiquidity growth and (b) liquidity premia. Notably, these additional sources have an offsetting effect on prices.

To ease of grasp, let us interpret the costs due to *illiquidity* as c_t and inversely the 'gains' due to *liquidity* as $-c_t$. In this sense, a price rise indicates a *rise* in expected 'liquidity' (i.e., $-c_t - E_t\left[\sum_{j=0}^{\infty}(\rho_l)^j \beta_2 \Delta c_{t+1+j}\right]$). At the same moment, the price rise also signals a *decline* in expected 'liquidity' premia (i.e., $-E_t\left[\sum_{j=0}^{\infty}(\rho_l)^j(-lp_{t+1+j})\right]$), implying lower compensation for bearing market liquidity risk in the future. We will use this conversion with single quotation marks when interpreting our results for the sake of easy interpretation (e.g., high liquidity is easier to interpret than low illiquidity).

In sum, our present-value identity provides a unified framework to link the conventional literature (e.g., Campbell and Shiller, 1988; Amihud and Mendelson, 1986). Interested readers are referred to Appendix A.

3. Data and Estimation

3.1. A proxy for stock liquidity

We have one serious obstacle: stock liquidity is hard to define and measure. In fact, the "legitimate" illiquidity costs defined in Section 2 comprise several components (Jones, 2002; Vayanos and Wang, 2013): the brokerage cost, the bid-ask spread, the price impact, the opportunity cost, and so forth. In practice, it is almost impossible to combine all components above. Although the bid-ask spread is popularly used, it alone is *not* the representative and also does *not* cover a sufficiently long interval.[4] Indeed, numerous liquidity measures used in the extant literature are *literally* proxies, *not* the legitimate costs by definition.

[4] The U.S. markets transaction data are only available since 1983 (Goyenko et al., 2009).

To handle this problem, we want to narrow down the scope of cost c_t to the annual value of Amihud's (2002) measure c_t^* . This proxy has a couple of merits. First, Amihud (2002) shows that c_t^* is an effective measure of the price impact referred to as the concept of Kyle's (1985) lambda. Second, Hasbrouck (2009) shows that c_t^* is *most strongly* correlated with the price impact coefficient of high-frequency Trade and Quote (TAQ) data. Third, Goyenko et al. (2009) show that c_t^* provides a good measure of the high-frequency data.

Concretely, we construct the annual Amihud measure C_t^* from the daily CRSP data from 1926 to 2017. First, we single out ordinary common shares listed on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and the Nasdaq Stock Market (NASDAQ). Moreover, each firm i with more than 200 days of transactions in year t must have the last trading price at the end of the year. Second, we average out the daily Amihud measures for each filtered share in year t in the way of

$$C_t^i \equiv \frac{1}{N_t^i} \sum_{d=1}^{N_t^i} \frac{\left|R_{d,t}^i\right|}{DVOL_{d,t}^i} \times 10^6,$$

where N_t^i is the number of trading days in year t, $|R_{d,t}^i|$ is the absolute daily rate of return on day d in year t, and $DVOL_{d,t}^i$ is dollar volume (i.e., trading volume × price). Note that a high value of c_t^i means high *illiquidity*, which occurs when high price variation goes along with low trading volume. Third, we winsorize all individual cross-section measures in year t over the range of [1%, 99%] to mitigate the effect of outliers.

One thing to note is that when aggregating all the cross-section measures into market-wide illiquidity C_t^*, we use the median in year t throughout the paper:

$$C_t^* = \text{median}(C_t^i) \tag{8}$$

for all the filtered firms i. Although most of the conventional literature has tended to use the arithmetic average, we contend that a median reflects a better central tendency for cross-section data than does an average, especially when the data likely suffer from lots of extreme outliers. To see this, Figure 2 plots the natural logarithms of the cross-section median and average in year t.

[INSERT FIGURE 2 HERE]

Both median and average tend to move in tandem (Figure 2). The apparent downward slope, primarily driven by an increase in dollar volume (denominator), can point to an improvement in liquidity over time due to technological innovations, and changes in regulation and stock market participation (Chordia et al., 2000; Hasbrouck and Seppi, 2001; Jones, 2002; Chordia et al., 2008; Lettau and Nieuwerburgh, 2008). Some spikes are associated with market turmoil, for example, in the recent financial crisis.

Due to the increase in dollar volume over past decades, both series appear to be nonstationary.[5] A large volume of the literature (e.g., Amihud and Mendelson, 1986; Chordia et al., 2001; Jones 2002; Acharya and Pedersen, 2005; among other cross-sectional studies) tends to use either the proportional transaction cost proxies or the first difference in a *return* measure context, possibly leading to information loss. Doing so even manifests nonstationary nature inherent in the liquidity data. Hence, we want to keep up the intrinsic feature in a *price* measure context to accord with our cost definition (Section 2).

Next, let us turn to the difference between the two series. Since the early 1990s, their gap has been widening in the way that the means are larger than the medians. We give great attention to the period when prices move substantially relative to dividends. The gap can widen because highly illiquid outliers of small-size firms increase in number. In other words, small- and medium-cap firms since then likely have a massive influence on market illiquidity. This

[5] The first-order autocorrelation of the average is about 0.95, while that of the median is 1.01.

finding accords well with that of Ben-Rephael et al. (2015), who show that liquidity is significantly priced among NASDAQ stocks in the recent period.

Such a trend-like pattern also has supportable evidence. Acharya and Pedersen (2005) document that the Amihud measure itself is *not* stationary. Hasbrouck (2009) also reveals that the distributions of the TAQ and CRSP/Gibbs measures do *not* seem to be stationary. Ben-Rephael et al. (2015) illustrate several liquidity measures including the Amihud proxy, which also exhibit the observed trend pattern.

3.2. Liquidity-adjusted price–dividend ratios

Direct calculation of *pdl* ratio pd_t^l through Proposition 1 is a difficult task for the following reasons. First, we have to resort to price impact proxy c_t^* , *not* actual legitimate cost c_t. Second, even though c_t^* is assumed to be the perfect proxy, its unit differs from log price p_t and log dividend d_t, because c_t^* is measured in percent per dollar.

To overcome this, we assume that liquidity measure C_t^* is proportional to illiquidity cost C_t: $C_t^* = A \cdot C_t$ as in Bekaert et al. (2007). The logarithm permits $c_t^* = \log C_t^*$ to divide into c_t and proportional constant $a = \log A$: constant a has nothing to do with any stochastic behavior. We then put a restriction on $\hat{\beta}_2 = \hat{\beta}_1 - 1$ through Proposition 1 and estimate $pd_t^l = p_t - \hat{\beta}_1 d_t + \hat{\beta}_2 c_t^*$ through OLS in the Engle-Granger way of

$$p_t = \hat{\beta}_0 + \hat{\beta}_1 d_t - \hat{\beta}_2 c_t^* + \epsilon_t,$$

where $\hat{\beta}_0$ is an intercept; $\hat{\beta}_1$ and $\hat{\beta}_2$ are estimates of β_1 and β_2, respectively; and ϵ_t denotes the cointegrating-equation error. Thus, pd_t^l can be expressed as $\hat{\beta}_0 + \epsilon_t$. In the rest of the paper, note that a hat notation is used to distinguish such preliminary estimation above from our main analysis made in Sections 4 and 5.

More precisely, we use P_t from the annual CRSP value-weighted index from 1926 to 2017 at the end (December) of year t. We also recover dividend D_t from the CRSP value-weighted returns with and without dividends via

$$D_t = \frac{D_t}{P_t} \times P_t = \left(\frac{R_t}{Rx_t} - 1\right) \times P_t,$$

where R_t denotes CRSP value-weighted gross return with dividends, and Rx_t denotes the CRSP value-weighted return without dividends (see Appendix A.2 of Cochrane (2011) for details). All three series (e.g., P_t, D_t, and C_t^*) are deflated by the Consumer Price Index (CPIIND) in December of year t.[6] Another merit of using the annual frequency is that we can avoid strong seasonality in dividend payments.

With these series in hand, we generate the estimates of the shared trend as

$$p_t = \hat{\beta}_0 + \hat{\beta}_1 \cdot d_t - \hat{\beta}_2 \cdot c_t^* = 3.156 + 1.084 \cdot d_t - 0.084 \cdot c_t^*.^{7} \quad (9)$$

All three estimates are significant at the 5% level under the Newey and West's (1987) *t*-statics. The estimates of $\hat{\beta}_1 = \frac{1}{1-\overline{C^*}/\overline{D}}$ and $\hat{\beta}_2 = \frac{\overline{C^*}/\overline{D}}{1-\overline{C^*}/\overline{D}}$ (Section 2) suggest that the historical amount of negative dividends (contingent on price impact c_t^*) is roughly 7.7% of positive dividends on average: $\bar{C}/\overline{D} = \hat{\beta}_2/\hat{\beta}_1$ during our sample.

There are three reasons why Proposition 1 must hold in estimating cointegrating regression (9). First, doing so ensures that *pdl* ratio pd_t^l is a *unique* representation of present-value identity (7). Second, this unique representation also alleviates a probable concern for log-linear approximation errors. If Proposition 1 does not hold, additional model-misspecification term

[6] In fact, Amihud measure c_t^* is not perfectly inflation-adjusted. To attain this, one should apply the daily inflation adjustment to individual instruments' measures, even though CPIIND in CRSP is released on a monthly basis. One can use interpolation to obtain daily CPIINDs with monthly CPIINDs, but even doing so does not eschew data processing. For simplicity, we ignore the precise construction in the paper.

[7] We notice that the estimation result is quite robust. In particular, dynamic least squares yield $p_t = 3.182 + 1.072 \cdot d_t - 0.088 \cdot c_t^*$, and fully-modified least squares deliver $p_t = 3.208 + 1.055 \cdot d_t - 0.096 \cdot c_t^*$.

not displayed in present-value identity (7) must come out. Third, when estimating *pd* ratio $pd_t = p_t - d_t$ in practice, even log price p_t and log dividend d_t do *not* have the theoretical trend relationship of $[1, -1]$ as well.

The estimates of $\hat{\beta}_1 = \frac{\hat{\rho}_l(1-\rho)}{\rho(1-\hat{\rho}_l)} = 1.084$ and $\hat{\beta}_1 = \frac{\hat{\rho}_l - \rho}{\rho(1-\hat{\rho}_l)} = 0.084$ also allow for guessing a discount factor $\hat{\rho}_l$ as the estimate of ρ_l. Specifically, we first calculate $\rho = 1/(1 + \exp(\overline{dp})) \approx 0.966$ based on the Campbell and Shiller's present-value identity and then back $\hat{\rho}_l \approx 0.969$ out of the two beta formulas. The fact of $\hat{\rho}_l > \rho$ also grants the validity of present-value identity (7) (Section 2). To sum up, our final present-value representation is

$$pd_t^l \approx E\left[\sum_{j=0}^{\infty} (\hat{\rho}_l)^j \left[\hat{\beta}_1 \Delta d_{t+1+j} - \hat{\beta}_2 \Delta c_{t+1+j}^* - r_{t+1+j} + lp_{t+1+j}\right] \middle| \mathcal{H}_t\right]. \quad (10)$$

Table 1 reports summary statistics for the variables used in the rest of our analysis.

[INSERT TABLE 1 HERE]

The summary statistics show that pd_t^l can embody distinct information from pd_t as displayed in Figure 1. In Panel A, the standard deviation of pd_t^l (0.269) is less volatile than that of pd_t (0.433). In Panel B, the correlation between pd_t^l and pd_t is 0.621. In Panel C, pd_t^l gives more stationary evidence than pd_t, although pd_t^l delivers somewhat controversial interpretations for unit root.[8]

4. Empirical analysis

Predictive analysis by present-value identity (10) is another way of representing how much of each information source accounts for *pdl* ratio pd_t^l. The stronger predictability is, the more

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[8] In particular, pd_t^l appears to be nonstationary in the Phillips-Perron (PP) test but stationary in the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test. However, it is well known that unit root tests such as the PP test lack the power to reject the null hypothesis (Campbell, 2003).

share each information source has to explain the volatility of pd_t^l. Now, let us dissect the volatility of pd_t^l in detail below.

4.1. Simple forecasting regression

We start from four kinds of simple forecasting regressions:

$$\Delta d_{t+1} = a_d + b_d \cdot pd_t^l + \varepsilon_{t+1}^d, \tag{11}$$

$$\Delta c_{t+1}^* = a_{c^*} + b_{c^*} \cdot pd_t^l + \varepsilon_{t+1}^{c^*}, \tag{12}$$

$$r_{t+1} = a_r + b_r \cdot pd_t^l + \varepsilon_{t+1}^r, \tag{13}$$

$$pd_{t+1}^l = a_\phi + \phi \cdot pd_t^l + \varepsilon_{t+1}^{pdl}, \tag{14}$$

where ε_{t+1}^d denotes a dividend shock, $\varepsilon_{t+1}^{c^*}$ denotes an illiquidity shock, ε_{t+1}^r denotes a return shock, and ε_{t+1}^{pdl} denotes a *pdl* shock. The first four rows of Table 2 present estimation results in regressions (11)–(14), respectively. For robustness, the results on *pd* ratio pd_t are reported in the last three rows of Table 2.

[INSERT TABLE 2 HERE]

Overall, short-term forecastable evidence is *not* clear. As mentioned in Stambaugh (1999), it could be argued that our regressor pd_t^l on the RHS is still persistent (Panel C, Table 1), so its inherent near unit root might cause biased estimates (b_x) and *t*-statistics. Simply put, the autocorrelations ϕ of pd_t^l in regression (14) (0.866; 4th row) and pd_t (0.942; 7th row) do not differ from each other at the conventional levels.

Besides, dividend-growth regression coefficient b_d is *not* statistically significant, and R^2 (1.1%) is also quite low (1st row). Moreover, the sign of b_d turns out to be negative against the positive sign by present-value logic (10). However, the forecasting regression of dividend growth Δd_{t+1} on *pd* ratio pd_t (5th row) also delivers the opposite direction as shown in

Cochrane (2008). Similarly, log illiquidity growth Δc_{t+1}^{*} exhibits the opposite sign of $b_{c^*} =$ 0.126 with the R^2 of 0.68% (2nd row).

One key finding is that one-period return coefficient b_r of the *pdl* ratio (3rd row) is *more than* twice as large as that of the *pd* ratio (6th row) in an absolute sense. In particular, our return coefficient $b_r = 0.205$ can be strongly supported by the *t*-statistic of -3.214, despite the small R^2 of 7.77% and the standard deviation 5.5% of the fitted value (i.e., $\sigma(a_r + b_r \cdot pd_t^l)$). In contrast, the return regression on pd_t yields $b_r = -0.085$ with the *t*-statistic of -1.894, the R^2 of 3.49%, and the standard deviation 3.69%. This conclusion is analogous to that of Cochrane (2008).

Why does pd_t^l look like a better return predictor than pd_t? The main answer comes from the central intuition that market illiquidity displayed in Figure 2 can adjust a price trend. To see this , Figure 3 graphs one-period real return r_{t+1}, the one-period return forecast on pd_t^l (i.e., $a_r + b_r \cdot pd_t^l$; see (13)), and the one-period return forecast on pd_t.

[INSERT FIGURE 3 HERE]

Clearly, the return forecast of pd_t^l (i.e., *pdl* forecast) better helps trace the return behavior than does the *pd* forecast. To highlight the illiquidity effect solely, we also plot the return forecast on the scaled *pd* ratio given by $p_t - \hat{\beta}_1 d_t$, but it cannot outperform the *pdl* forecast.

Does it mean that pd_t^l forecasts more returns than does pd_t? No, because the two ratios underlie different information sets, especially with respect to market illiquidity. Also, unexpected illiquidity news can give rise to the additional liquidity premium not covered by the conventional information set (see Appendix A). From the next section, we will devote special attention to the inference of the implied liquidity premium by present-value logic.

4.2. Short-term liquidity premium

We start off by restating present-value identity (10) in a short period:

$$lp_{t+1} = -\hat{\rho}_l \cdot pd^l_{t+1} - \hat{\beta}_1 \Delta d_{t+1} + \hat{\beta}_2 \Delta c^*_{t+1} + r_{t+1} + pd^l_t. \tag{15}$$

Here, we assume no serious approximation errors forced by $1 - \hat{\beta}_1 + \hat{\beta}_2 = 0$ (Proposition 1) and then link forecasting regressions (11)–(14) to *implied* liquidity premium lp_{t+1} in (15).

Following Cochrane (2008), we regress both sides of (15) on *pdl* ratio pd^l_t. In brief, the LHS of (15) can be expresses as

$$lp_{t+1} = a_{lp} + b_{lp} \cdot pd^l_t + \varepsilon^{lp}_{t+1}, \tag{16}$$

where ε^{lp}_{t+1} denotes a liquidity-premium shock. The RHS respectively yields the following forecast and error identities:

$$b_{lp} = 1 - \hat{\rho}_l \cdot \phi - \hat{\beta}_1 \cdot b_d + \hat{\beta}_2 \cdot b_{c^*} + b_r, \tag{17}$$

$$\varepsilon^{lp}_{t+1} = -\hat{\rho}_l \cdot \varepsilon^{pdl}_{t+1} - \hat{\beta}_1 \cdot \varepsilon^d_{t+1} + \hat{\beta}_2 \cdot \varepsilon^{c^*}_{t+1} + \varepsilon^r_{t+1}. \tag{18}$$

The two identities in (17)–(18) make it clear that the implied liquidity premium can be expressed as a function of prices, dividends, and costs.[9]

With the regression coefficients in Table 2 in hand, forecast identity (17) produces $b_{lp} = 0.027$, and error identity (18) delivers the 0.4% standard deviation: $\sigma(\varepsilon^{lp}) \approx 0.42\%$. From the '*liquidity*' perspective (Section 2), the converted coefficient (i.e., $-b_{lp} = -0.027$) implies that investors need *lower* compensation, when observing that prices are high relative to total (positive plus negative) dividends (i.e., a *pdl* rise).

[9] One might wonder why returns are omitted, but they can be computed from other sources: prices and dividends.

To help better understand this, suppose that investors observe that prices are low relative to total dividends (i.e., a *pdl* drop) in the event of (a) market turmoil. To make matters worse, supposedly, we further assume that (b) market liquidity dries up. In this turmoil (a), investors want a high risk-adjusted premium, thereby corresponding to a rise in expected returns: $E_t[r_{t+1}] \approx b_r \cdot pd_t^l = 0.205$, where $pd_t^l = -1$ for example. In addition, the liquidity dry-up (b) allows investors to demand additional compensation. In sum, expected net returns should be far higher than expected returns: $E_t[r_{t+1}^*] = E_t[r_{t+1} - lp_{t+1}] \approx (b_r - b_{lp}) \cdot pd_t^l = 0.205 + 0.027 = 0.232$ by accounting identity (6). Such higher premium induces prices to drop more sharply when the liquidity dry-up deteriorates the market turmoil.

In fact, the unusual signs of b_d and b_{c^*} (Subsection 4.1) might raise attention because they distort the innate attributes of the implied liquidity premium. In Appendix B where we use the quarterly bid-ask spread available after 1983, however, we confirm the correct signs of log dividend growth and spread growth. Despite the different sample, time frequencies, and proxies, the liquidity-premium coefficient in Appendix B turns out to be positive: $b_{lp} = 0.005$.

Now, we go on to address the empirical question, "Is the liquidity premium of the CRSP market portfolio predictable?" Putting all regression results into perspective, our null hypothesis, the *constant-expected-liquidity-premium model,* has the following form:

$$\mathrm{H}_0\colon b_{lp} = 1 - \hat{\rho}_l \cdot \phi - \hat{\beta}_1 \cdot b_d + \hat{\beta}_2 \cdot b_{c^*} + b_r = 0. \tag{19}$$

As a result, linear Wald test (19) delivers that $b_{lp} = 0.027$ is substantially far away from the null at the conventional levels; the χ^2-statistic is over 200. The rejection of the hypothesis points out that liquidity premium lp_{t+1} is strongly predictable by *pdl* ratio pd_t^l over short horizons. It should thus be priced in a short period, implying that investors take great care of unexpected illiquidity news in the first place.

Does this strong short-run evidence mechanically give a substantive contribution of long-run liquidity premia to prices? In other words, do investors want compensation for taking market liquidity risk in the long-run? We are *not* aware of the concrete answer yet, which essentially hinges on the long-run predictive results, as will be discussed in the next section.

4.3. Long-term liquidity premium

Following Cochrane (2008), we first divide both sides of short-term forecast identity (17) by $1 - \hat{\rho}_l \cdot \phi$ and then rearrange it as

$$\hat{\beta}_1 \cdot b_d^{lr} - \hat{\beta}_2 \cdot b_{c^*}^{lr} - b_r^{lr} + b_{lp}^{lr} = 1, \tag{20}$$

where $b_d^{lr} = b_d/(1 - \hat{\rho}_l \cdot \phi)$, $b_{c^*}^{lr} = b_{c^*}/(1 - \hat{\rho}_l \cdot \phi)$, $b_r^{lr} = b_r/(1 - \hat{\rho}_l \cdot \phi)$, and $b_{lp}^{lr} = b_{lp}/(1 - \hat{\rho}_l \cdot \phi)$. In basic, each component of long-term forecast identity (20) stands for the variance fraction of a set of information $[\Delta d_t, \Delta c_t^*, r_t, lp_t]$ that adds up to the whole variance of *pdl* ratio pd_t^l (100%). For example, we rewrite the following ex-post identity:

$$pd_t^l = \sum_{j=0}^{\infty} (\hat{\rho}_l)^j \left[\hat{\beta}_1 \Delta d_{t+1+j} - \hat{\beta}_2 \Delta c_{t+1+j}^* - r_{t+1+j} + lp_{t+1+j}\right]. \tag{21}$$

For example, $b_{lp}^{lr} = \frac{\mathrm{Cov}\left(\sum_{j=0}^{\infty} (\hat{\rho}_l)^j lp_{t+1+j},\ pd_t^l\right)}{\mathrm{Var}\left(pd_t^l\right)}$ refers to the regression coefficient of long-run liquidity premia $\sum_{j=0}^{\infty} (\hat{\rho}_l)^j\ lp_{t+1+j}$ on pd_t^l in (21). An easy way of computing such long-run estimates is to *infer* them from the results of the simple forecasting regressions (Table 2). Table 3 reports the first three long-run estimates of $\hat{\beta}_1 \cdot b_d^{lr}$, $\hat{\beta}_2 \cdot b_{c^*}^{lr}$, and b_r^{lr} and standard errors calculated from the standard delta method.

[INSERT TABLE 3 HERE]

First, we examine long-run dividend growth forecast $\hat{\beta}_1 \cdot b_d^{lr} = -0.377$. As mentioned before, its sign is unintuitive, which is totally misled by short-term dividend estimate $b_d = -0.056$ (Table 2). The *t*-statistic of $\hat{\beta}_1 \cdot b_d^{lr} = -0.377$ (-0.958) also ensures that the subsequent dividend variation *barely* accounts for the *pdl* volatility. In the similar sense, long-run illiquidity growth forecast $\hat{\beta}_2 \cdot b_{c^*}^{lr} = 0.066$ is of *little* economic significance; the *t*-statistic is 0.168.

Next, we turn to long-run return estimate $b_r^{lr} = -1.276$ with the *t*-statistic of -2.932.[10] This number gives a strong indication for return forecastability. When using simple excess return forecast b_{er} where $er_{t+1} = a_{er} + b_{er} \cdot pd_t^l + \varepsilon_{t+1}^{er}$, $b_{er}^{lr} = -1.262$ with the *t*-statistic (-2.785) is in favor of return predictability (last column).[11]

Those findings demonstrate that *almost all* variation in *pdl* ratios corresponds to changes in expected returns, which is analogous to that of the *pd* ratios as shown in Cochrane (2008). The reason is that market illiquidity just provides a natural trend in prices. As seen in Figure 1, both ratios seem to share almost the same *cyclical* variations even with the trend adjustment in effect, implying that expected-return news is the most dominant among the others. We also infer from this finding that market illiquidity (i.e., $\hat{\beta}_2 \cdot b_{c^*}^{lr}$) cannot fully help attenuate the excess volatility (i.e., b_r^{lr}) inherent in prices. If its unexpected news were by far first-order, the excess volatility not justified by subsequent changes in (positive) dividends would be reduced in part.

Therefore, market liquidity risk should be second-order. This reasoning can also be tested through the following questions, "Are long-run liquidity premia for the CRSP market portfolio

[10] Note that all the information sources are not orthogonal. That is why the return estimate over 100% can arise.

[11] To calculate $b_{er}^{lr} = -1.262$, we use the following identity:

$$pd_t^l = E_t\left[\sum_{j=0}^{\infty} (\hat{\rho}_l)^j \{\hat{\beta}_1 \Delta d_{t+1+j} - \hat{\beta}_2 \Delta c_{t+1+j}^* - er_{t+1+j} - r_{t+1+j}^f + lp_{t+1+j}\}\right],$$

where r_t^f is the three-month T bill rate such that $er_t = r_t - r_t^f$.

a main source of price variation?" Do investors care about such premia a lot from a long-term perspective? Algebraically, long-term forecast identity (20) yields $b_{lp}^{lr} = 1 - \left(\hat{\beta}_1 \cdot b_d^{lr} - \hat{\beta}_2 \cdot b_{c^*}^{lr} - b_r^{lr}\right) = 0.167$, meaning that about 16.7% of the price (i.e., *pdl*) volatility comes from changes in expected liquidity premia over the indefinite future.

To assess its economic significance, we hypothesize the constant-expected-liquidity-premium model as

$$\mathrm{H}_0\colon b_{lp}^{lr} = 1 - \hat{\beta}_1 \cdot b_d^{lr} + \hat{\beta}_2 \cdot b_{c^*}^{lr} + b_r^{lr} = 0. \tag{22}$$

More precisely, we adopt the non-linear Wald test statistic in (22):

$$\lambda'\left(\partial\lambda/\partial\gamma'\, \theta\, \partial\lambda/\partial\gamma\right)^{-1}\lambda,$$

where $\lambda = 1 - \hat{\beta}_1 \cdot b_d^{lr} + \hat{\beta}_2 \cdot b_{c^*}^{lr} + b_r^{lr}$ is defined as the scalar of deviations imposed by (22); $\gamma = [b_d, b_{c^*}, b_r, \phi]'$ is the vector of the short-term estimates; θ is the estimated variance-covariance matrix in regressions (11)–(14); and $\partial\lambda/\partial\gamma$ is the partial derivative with respect to estimate vector γ. The Wald-test statistic follows a χ^2 distribution with degrees of freedom n equal to the number of observable variables: $n = 4$ in our case. As a result, the χ^2-statistic for $\mathrm{H}_0\colon b_{lp}^{lr} = 0$ is about 2.09, which is close to the null at the conventional levels: the *p*-value is about 72.0%.

In sum, the hypothesis result above shows that the long-term liquidity premium is *not* a main source of price variation. Stated intuitively, investors do *not* seem to care about market liquidity risk in the long-run, even though they consider its unexpected news with great care in the first place (Subsection 4.2).

5. Impulse-response functions

Why do investors respond to market illiquidity differently through time? To answer this, we go on to investigate the dynamic behavior of expected liquidity premia over time. Specifically, we display impulse-response functions proposed by Cochrane (2011) with reference to a set of error shocks, $[\varepsilon^{pdl}, \varepsilon^{d}, \varepsilon^{c^*}, \varepsilon^{r}, \varepsilon^{lp}]$.

5.1. Error standard deviations and correlations

Before jumping into impulse-response functions, it is important to scrutinize the empirical relationship between the error shocks, $[\varepsilon^{pdl}, \varepsilon^{d}, \varepsilon^{c^*}, \varepsilon^{r}, \varepsilon^{lp}]'$. Panel A of Table 4 presents error standard deviations on the diagonal and correlations on the off-diagonal for the error shocks.

[INSERT TABLE 4 HERE]

We find two outstanding observations. First, *pdl* shock ε^{pdl} is strongly correlated with return shock ε^{r}: $\text{Corr}(\varepsilon^{pdl}, \varepsilon^{r}) \approx 50.3\%$. This feature accords well with that of *pd* ratios (Cochrane, 2008); our sample reveals $\text{Corr}(\varepsilon^{pd}, \varepsilon^{r}) \approx 68.4\%$. Of course, ε^{pdl} never comes out independently without dividend shock ε^{d} and illiquidity shock ε^{c^*}: $\text{Corr}(\varepsilon^{pdl}, \varepsilon^{d}) \approx -31.0\%$ and $\text{Corr}(\varepsilon^{pdl}, \varepsilon^{c^*}) \approx -22.5\%$. Putting all correlation sizes into perspective, it is apparent that return shock ε^{r} is a more dominant factor tied to *pdl* shock ε^{pdl} than the other shocks. Second, liquidity-premium shock ε^{lp} is almost perfectly and positively correlated with *pdl* shock ε^{pdl}. The strong correlation reveals that the *pdl* ratio could be a good proxy for expected liquidity premia, especially over short horizons (Subsection 4.2). For robustness check, we also use the quarterly bid-ask spreads in Appendix B and also find the strong correlation (0.932).

5.2 Impulse-response functions

Recall that the implied liquidity premium can be expressed as a function of prices, dividends, and costs by present-value logic. Among these, we draw great attention to an illiquidity-driven shock in the rest of the analysis. This is because unexpected illiquidity news (i.e., the excess illiquidity volatility) is a main cause of the liquidity premia.

To understand this simply, let us express the gross net return:

$$R_{t+1}^{*} = R_{t+1} - LP_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t} - \frac{C_{t+1}}{P_t}. \tag{23}$$

As can be seen in (23), in principle, a rise in market illiquidity (C_{t+1} ↑) with no other changes primarily raises ex-post liquidity premia (LP_{t+1} ↑).

Specifically, our goal is to examine whether *pure* illiquidity shock ε^{c^*} is so transient that investors could become insensitive to market illiquidity in the long-run. To conduct the examination, we design the illiquidity shock with no current changes in *pdl* ratio, dividend, and return:

$$\varepsilon^{pdl} = 0, \qquad \varepsilon^{d} = 0, \qquad \varepsilon^{c^*} = 1, \qquad \varepsilon^{r} = 0, \qquad \varepsilon^{lp} = \hat{\beta}_2. \tag{24}$$

Such a shock identification follows a first-order VAR system:[12]

$$\begin{bmatrix} pd_{t+1}^{l} \\ \Delta d_{t+1} \\ \Delta c_{t+1}^{*} \\ r_{t+1} \\ lp_{t+1} \end{bmatrix} = \begin{bmatrix} \phi \\ b_d \\ b_{c^*} \\ b_r \\ 1-\hat{\rho}_l \cdot \phi - \hat{\beta}_1 \cdot b_d + \hat{\beta}_2 \cdot b_{c^*} + b_r \end{bmatrix} \cdot pd_t^{l} + \begin{bmatrix} \varepsilon_{t+1}^{pdl} \\ \varepsilon_{t+1}^{d} \\ \varepsilon_{t+1}^{c^*} \\ \varepsilon_{t+1}^{r} \\ -\hat{\rho}_l \cdot \varepsilon_{t+1}^{pdl} - \hat{\beta}_1 \cdot \varepsilon_{t+1}^{d} + \hat{\beta}_2 \cdot \varepsilon_{t+1}^{c^*} + \varepsilon_{t+1}^{r} \end{bmatrix}. \tag{25}$$

[12] One can add lags into a higher-order VAR system, but we want to keep the parsimonious VAR in the interest of brevity. We admit that such a shock definition might be oversimplification, but it also has a merit of facilitating the analysis simply.

We also express a cumulative response of prices as $p_t = \sum_{j=1}^{t} \Delta p_j$ and represent the others as $\hat{\beta}_1 d_t = \sum_{j=1}^{t} \hat{\beta}_1 \Delta d_j$, and $\hat{\beta}_2 c_t^* = \sum_{j=1}^{t} \hat{\beta}_2 \Delta c_j^*$ in the spirit of present-value identity (21).[13] Figure 4 plots the impulse-response functions in (24).

[INSERT FIGURE 4 HERE]

You find easily that the pure illiquidity shock is essentially *transient* (Figure 4). Concretely, the illiquidity shock ($\varepsilon^{c^*} = 1$) in (23) leads to a price drop, no dividend move, and an illiquidity rise at the point of the shock (time 1; 2nd panel). At this point onward, they stay *there* with no subsequent moves, highlighting the transient nature of market illiquidity. In sum, even a big but *transient* change in illiquidity (strong short-term evidence in Subsection 4.2) *cannot* build up over time toward a big price change (week long-term evidence in Subsection 4.3).

The conventional norm is that a positive shock to illiquidity is pretty *persistent*. This persistent nature can correspond to (i) a rise in expected illiquidity, (ii) a rise in expected returns, which in turn leads to a price drop today (e.g., Acharya and Pedersen (2005)). When the illiquidity shock is basically transient, however, the first two channels (i) and (ii) *hardly* work. The reason crystalizes again that market illiquidity just provides a price trend; equivalently, its cyclical variation *cannot* affect prices a lot. Another way of interpreting this technically is that it is *nearly* a random walk so that its cyclical change is *almost* a white noise (or i.i.d.) independent of the other-variable moves going on in principle.

Importantly, the whole story above seems similar to that of the *pd* ratios (Cochrane, 2011) despite factoring in market illiquidity additionally. Evidence is that $b_r^{lr} = \frac{b_r}{1-\hat{\rho}_l \phi} = -1.276$ is

[13] we write a change in prices with first-order VAR (25) as

$$\Delta p_{t+1} = pd_{t+1}^l + \hat{\beta}_1 \Delta d_{t+1} - \hat{\beta}_2 \Delta c_{t+1}^* - pd_t^l$$
$$= \underbrace{(\phi + \hat{\beta}_1 b_d - \hat{\beta}_2 b_{c^*} - 1)}_{= b_p} \cdot pd_t^l + \underbrace{(\varepsilon_{t+1}^{pdl} + \hat{\beta}_1 \varepsilon_{t+1}^d - \hat{\beta}_2 \varepsilon_{t+1}^{c^*})}_{= \varepsilon_{t+1}^p},$$

where $b_p = \phi + \hat{\beta}_1 b_d - \hat{\beta}_2 b_{c^*} - 1$, and $\varepsilon_{t+1}^p = \varepsilon_{t+1}^{pdl} + \hat{\beta}_1 \varepsilon_{t+1}^d - \hat{\beta}_2 \varepsilon_{t+1}^{c^*}$ denotes a price shock.

compelling in terms of the size and significance (Subsection 4.3). This result comes from the persistent feature that even a small expected return change can cumulate over time toward a huge price variation in the end. At its core, the fact that illiquidity shock ε^{c^*} as additional source should be *transient* is why there is hardly a big difference between the two ratios. If the illiquidity shock were substantially *persistent* as long as return shock ε^{r}, it would help reduce the excess volatility not justified by subsequent changes in dividends.

6. Conclusion

This study sheds new light on the long-horizon interplay between market illiquidity and prices. The key finding is that market liquidity risk is so transient that its cumulative effect on prices could *not* last for a long time. Stated differently, investors tend to overreact to unexpected illiquidity news in the first place, but *not* to consider it with great care in forming long-term portfolio decisions. In other words, the financial market is sufficiently resilient to market liquidity risk so that prices bounce back quickly from the investors' perspective.

One might doubt that the trend-adjusted effect of market illiquidity seems quite large (Figure 1). But keep in mind that our whole story must be narrated within our information set. If further incorporating another source (e.g., share repurchase), it is *not* surprising that the explanatory share of information sources to account for financial-valuation ratios (e.g., *pdl* ratios) should change *quantitatively*. Nevertheless, we contend that the trend implication of market illiquidity should be intact *qualitatively*, provided that market illiquidity provides a good estimate of the trend in prices, as shown in Figure 1.

In fact, improvement in liquidity can have two offsetting effects on liquidity premia. One the one side, it can lead to low compensation, as mentioned in Section 2. On the other side, it can also raise trading frequency, corresponding to large effective transaction costs and high compensation as a result. Although we cannot present effective-cost evidence here, the conclusion that market liquidity risk is basically second-order suggests that investors should also be reluctant to entail the risk of the effective costs.

We also admit that small- and medium-cap. securities are of great importance for liquidity pricing. But it is worthwhile that we open a new venue of liquidity pricing in the time-series. We believe that our price–dividend–cost relationship is readily applicable to such small- and medium-cap. securities, which deserves for future work.

The last thing to remind is that the CRSP dividend series involves all distribution payments to investors, including cash mergers, liquidations, actual dividends, and so forth (Cochrane, 2008). Hence, one can further dissect such an inclusive series to find out which individual component is a primary driver of price variation. One of the good candidates may be share repurchases and issuances, as shown in Larrain and Yogo (2008). We also leave them for future research.

APPENDIX A: Derivation of log-linear approximation

For an arbitrary nonlinear function $f(x, y)$, it can be approximated around $\bar{x}$ and $\bar{y}$ as

$$f(x,y) \approx f(\bar{x},\bar{y}) + \frac{\partial f}{\partial x}\bigg|_{x=\bar{x},y=\bar{y}} (x-\bar{x}) + \frac{\partial f}{\partial y}\bigg|_{x=\bar{x},y=\bar{y}} (y-\bar{y}).$$

Using this first-order Taylor expansion yields Eq. (3):

$$\log(1+\exp(dp_{t+1}) - \exp(cp_{t+1})) = -\log(\rho_l) + \frac{\exp(\overline{dp})}{1+\exp(\overline{dp})-\exp(\overline{cp})}\left(dp_{t+1} - \overline{dp}\right).$$
$$-\frac{\exp(\overline{cp})}{1+\exp(\overline{dp})-\exp(\overline{cp})}(cp_{t+1} - \overline{cp}),$$

where $\rho_l = \frac{1}{1+\exp(\overline{dp})-\exp(\overline{cp})} = \frac{\bar{P}}{\bar{P}+\bar{D}-\bar{C}} > \rho = \frac{1}{1+\exp(\overline{dp})} = \frac{\bar{P}}{\bar{P}+\bar{D}}$, $\frac{\exp(\overline{dp})}{1+\exp(\overline{dp})-\exp(\overline{cp})} = \rho_l(1/\rho - 1)$, $\frac{\exp(\overline{cp})}{1+\exp(\overline{dp})-\exp(\overline{cp})} = \rho_l(1/\rho - 1/\rho_l)$, $\overline{dp} = \log(1/\rho - 1)$, $\overline{cp} = \log(1/\rho - 1/\rho_l)$,

and

$$k_l = -\log(\rho_l) - \frac{\exp(\overline{dp})}{1+\exp(\overline{dp})-\exp(\overline{cp})} \cdot \overline{dp} + \frac{\exp(\overline{cp})}{1+\exp(\overline{dp})-\exp(\overline{cp})} \cdot \overline{cp}.$$

$$= -\log(\rho_l) - \rho_l(1/\rho - 1)\log(1/\rho - 1) + \rho_l(1/\rho - 1/\rho_l)\log(1/\rho - 1/\rho_l).$$

Substituting the Taylor expansion into Eq. (2) delivers the linear difference equation of the log net return:

$$r_{t+1}^* \approx k_l + \rho_l \cdot p_{t+1} + \rho_l(1/\rho - 1)d_{t+1} - \rho_l(1/\rho - 1/\rho_l)c_{t+1} - p_t.$$

Proposition 1 carries

$$r_{t+1}^* \approx k_l + \rho_l \cdot pd_{t+1}^l + \beta_1 \Delta d_{t+1} + \beta_2 \Delta c_{t+1} - pd_t^l.$$

When illiquidity is *not* a source of price variation, $\rho_l = \rho$ and $k_l = k$, where $k = -\log(\rho) - (1-\rho)\log(1/\rho - 1)$ is the log-linear coefficient of log real return (Campbell and Shiller, 1988). In this case, Eq. (4) corresponds to the conventional difference equation of the real return:

$$r_{t+1} \approx k + \rho \cdot p_{t+1} + (1-\rho)d_{t+1} - p_t = k + \rho \cdot pd_{t+1} + \Delta d_{t+1} - pd_t,$$

where $pd_t \equiv p_t - d_t$ is log price–dividend ratio.

Some may argue that our framework requires stationary dp_{t+1} and cp_{t+1} in Eq. (3) to apply the first-order Taylor expansion. If Proposition 1 holds in the real data, however, we contend that the stationary property for dp_{t+1} and cp_{t+1} is *not* necessary. For example, suppose that $\exp(z)$ is given by $\exp(x) - \exp(y)$. An arbitrary nonlinear function $f(z) = \log(1 + \exp(z))$ can be approximated by the first-order Taylor expansion around $\bar{z}$:

$$f(z) \approx \log(1 + \exp(\bar{z})) + \left.\frac{\partial f}{\partial z}\right|_{z=\bar{z}} (z - \bar{z}).$$

When x and y are restricted by $1 - \beta_1 + \beta_2 = 0$ and approximation errors are *not* serious enough to affect our derivation, Eq. (3) can be rewritten with the one state variable:

$$-\log(\rho_l) - \frac{\exp(\overline{dp}) - \exp(\overline{cp})}{1+\exp(\overline{dp}) - \exp(\overline{cp})}\left(pd_{t+1}^{l} - \overline{pd^{l}}\right),$$

where $\frac{\exp(\overline{dp}) - \exp(\overline{cp})}{1+\exp(\overline{dp}) - \exp(\overline{cp})} = 1 - \rho_l$, $\overline{pd^{l}} = -\beta_1 \overline{dp} + \beta_2 \overline{cp}$, and $-\log(\rho_l) + (1-\rho_l)\overline{pd^{l}} = k_l$. This formula leads to the same representation as Eq. (7).

Eq. (7) represents a unified framework to link the conventional studies as well. First, we allow for negative dividends, which Campbell and Shiller (1988) do not consider. For example, suppose that liquidity is *not* a source of price variation. In this case, $\beta_1 = 1$ and $\beta_2 = 0$ as a result of $\rho_l = \rho$ and $k_l = k$. This assumption corresponds to the conventional Campbell and Shiller's (1988) present-value identity:

$$pd_t \approx \frac{k}{1-\rho} + E\left[\sum_{j=0}^{\infty} \rho^{j}\left[\Delta d_{t+1+j} - r_{t+1+j}\right] \middle| \Omega_t\right],$$

where Ω_t is the conventional information set generated by the joint history of Δd_t and r_t.

Second, we consider that investment opportunities vary over time, whereas Amihud and Mendelson (1986) do not. Constant variations in expected returns and liquidity premia can allow our present-value identity to be analogous to Amihud and Mendelson's (1986) framework over a holding period. Our framework also supports the conclusion of Amihud and Mendelson (1986), who show that prices decrease with the expected relative bid-ask spreads.

APPENDIX B: Robustness test

For robustness, we use log bid-ask spread s_t displayed in Figure B.1 as another liquidity proxy. To measure price-based illiquidity, we do *not* normalize bid-ask spreads by price levels. Also, it should be noted that the bid-ask spread is available since 1983. Hence, we proceed with the median of all the *quarterly* cross-section bid-ask spreads to enlarge the number of samples until the fourth quarter of 2017. The construction is almost the same as stated in Subsection 3.1 with the exception of *one* condition that each security must have at least 40 trading days over a quarter.

[INSERT FIGURE B.1 HERE]

Proposition 1 allows for estimating *pdl* ratio $pd_t^l = p_t - \hat{\beta}_1 d_t + \hat{\beta}_2 s_t$ displayed in Figure B.2:

$$p_t = 3.291 + 1.060 \cdot d_t - 0.060 \cdot s_t. \tag{B.1}$$

All of them are significant at the conventional levels. These numbers give a sense that the amount of bid-ask spreads is about 5.7% ($= \hat{\beta}_2/\hat{\beta}_1$) of that of dividends over the sample; the autocorrelation of s_t is about one in line with the price-based illiquidity.

[INSERT FIGURE B.2 HERE]

Figure B.2 also shows clearly that the bid-ask spread provides a natural price trend, which ensures that the resulting implications are almost similar. Therefore, we do not repeat the details to conserve space and instead present a couple of key points.

Our present-value identity has the form:

$$pd_t^l \approx E\left[\sum_{j=0}^{\infty}(\hat{\rho}_l)^j\left[\hat{\beta}_1 \Delta d_{t+1+j} - \hat{\beta}_2 \Delta s_{t+1+j} - r_{t+1+j} + lp_{t+1+j}\right]\middle|\mathcal{H}_t\right]. \tag{B-2}$$

We set $\hat{\rho}_l$ to 0.995. To obtain this number, we first compute the annual value of $\hat{\rho} = 1/(1 + \exp(\overline{dp})) = 0.977$ from 1983 to 2017 and then transform it into the quarterly value of $\hat{\rho} = \sqrt[4]{0.977} = 0.994$. With the resulting $\hat{\rho} = 0.994$ in place, we can back the value of $\hat{\rho}_l$ out of the beta formulas in Proposition 1.

[INSERT TABLE B.1 HERE]

Table B.1 calculates the results of simple predictive regressions. Likewise, short-term forecastable evidence is not reliable because the autocorrelation of pd_t^l is about 0.91 close to one. Noteworthy are the signs of $b_d = 0.044$ and $b_s = -0.076$, which show correct signs as shown in Eq. (B-2). The forecast identity brings us to compute liquidity-premium coefficient b_{lp} as

$$b_{lp} = 1 - \hat{\rho}_l \cdot \phi - \hat{\beta}_1 \cdot b_d + \hat{\beta}_2 \cdot b_s + b_r = 0.005.$$

The linear Wald test for $\mathrm{H}_0\colon b_{lp} = 0$ says that $b_{lp} = 0.005$ is far away from the null at the conventional levels, confirming that the liquidity premium should be priced on a short-term basis. Next, we move on to the long-run forecasting estimates reported in Table B.2, all of which are calculated based on the short-run coefficients.

[INSERT TABLE B.2 HERE]

One impressive thing is that dividends present a dominant share to explain the price volatility: $\hat{\beta}_1 \cdot b_d^{lr} = 47.2\%$. This share is even larger than $b_r = -42.9\%$ and $b_{er} = -42.7\%$ in an absolute sense. Given that the CRSP dividends contain all payments to shareholders (Cochrane, 2008), the popularity of share repurchases since 1983 might cause such a result; the enactment of Rule 10b-18 in November 1982 has permitted firms to buy back a predetermined fraction of their shares under strict supervision. Nevertheless, the shares of $b_r = -42.9\%$ and $b_{er} = -42.7\%$ are evident in favor of return predictability. With b_r in place, we calculate the long-term liquidity-premium estimate as $b_{lp}^{lr} = 1 - \hat{\beta}_1 \cdot b_d^{lr} + \hat{\beta}_2 \cdot b_s^{lr} + b_r^{lr} = 0.052$. The non-linear Wald test for $\mathrm{H}_0\colon b_{lp}^{lr} = 0$ shows that its time variability is *not* economically significant; the *p*-value is about 92.0%. Likewise, the resulting liquidity premium is *not* significantly priced in the long-run because market illiquidity has to do with the price trend, *not* with its cyclical fluctuations.

[INSERT TABLE B.3 HERE]

Let us see the error correlations and standard deviations reported in Table B.3 and compare them with those of Table 4. The common feature is that liquidity-premium shock ε^{lp} exhibits a strong correlation with *pdl* shock ε^{pdl} (0.932). Conversely, we discover in Table B.3 that *pdl* shock ε^{pdl} and dividend shock ε^{d} are strongly and negatively correlated (−0.800), indicating that the *pdl* shock can be a large fraction of expected cash flow news since 1983, whereas Table 4 illustrates that a *pdl* move is almost expected return news. Even so, it is still intact that market liquidity risk is essentially second-order.

[INSERT FIGURE B.3 HERE]

To see this, illiquidity shock ε^{s} is designed to be *solely* accompanied by liquidity-premium shock ε^{lp}:

$$\varepsilon^{pdl} = 0, \qquad \varepsilon^{d} = , \qquad \varepsilon^{s} = 1, \qquad \varepsilon^{r} = 0, \qquad \varepsilon^{lp} = \hat{\beta}_2 = 0.06. \tag{B-3}$$

In Figure B.3, you confirm stay-put moves shown in Figure 4.

APPENDIX C: Over-identifying restriction on a GMM

In Appendix C, all endogenous variables are redefined as deviations from their sample means. This change is justifiable because constant terms do not affect any stochastic behavior. Doing so has another advantage of increasing one degree of freedom (see Hypothesis C).

Our empirical question here is, "How much expected net return r^* accounts for expected return r?" To conduct the investigation, unlike the previous ex-post identity of $r_t^* = r_t - lp_t$, we replace net return $r_t^* = r_t - lp_t$ with actual real return r_t by $r_t^* = \omega \cdot r_t$, where ω is the proportion of the net return to the real return. This proportion can be estimated through tests of over-identifying restriction on a GMM.

HYPOTHESIS C: $\mathbf{H_0}$: $\boldsymbol{E}[\boldsymbol{v}_{t+1}|\boldsymbol{\mathcal{H}}_t] = \boldsymbol{E}[\boldsymbol{v}_{t+1} \cdot \boldsymbol{I}_t^{\mathcal{H}}] = \mathbf{0}$

We have the following forward-looking moment equation by $r_t^* = \omega \cdot r_t$:

$$pd_t^l = \hat{\rho}_l \cdot pd_{t+1}^l + \hat{\beta}_1 \Delta d_{t+1} - \hat{\beta}_2 \Delta c_{t+1}^* - \omega \cdot r_{t+1} + v_{t+1}.$$

By redefining the variables discussed above, we do not need to estimate the intercept in the forward moment equation above. Residual v_{t+1} should be orthogonal to instrumental variables $I_t^{\mathcal{H}}$ in information set $\mathcal{H}_t = [pd_t^l, \Delta d_t, \Delta c_t^*, r_t]'$:

$$E\left[\left[pd_t^l - \{\hat{\rho}_l \cdot pd_{t+1}^l + \hat{\beta}_1 \Delta d_{t+1} - \hat{\beta}_2 \Delta c_{t+1}^* - \omega \cdot r_{t+1}\}\right] \cdot I_t^{\mathcal{H}}\right] = 0.$$

Table C.1 presents the results of over-identifying restriction on a GMM. Note that the number of the instrumental variables (simply, rank) can be up to five due to $\mathcal{H}_t = [pd_t^l, \Delta d_t, \Delta c_t^*, r_t]'$ including a constant. To conserve space, we report three simple cases (a) just-identification with rank 1, $\mathcal{H}_t = [pd_t^l]'$, (b) over-identification with rank 2, $\mathcal{H}_t = [pd_t^l]'$ and a constant, and (c) over-identification with full rank 5.

[INSERT TABLE C.1 HERE]

As a result, the proportion ω spans from about 1.118 (rank 5) to 1.131 (rank 1), all of which are statistically different from one. This finding gives another evidence in favor of liquidity-premium predictability over short horizons. Otherwise, ω should not be far from one. The number ω such that $E_t[r_{t+1}^*] = \omega E_t[r_{t+1}]$ also suggests that the one-period expected return hovers around 88% (=1/1.131) – 89% (=1/1.118) of the one-period net return from 1926 to 2017 for the market portfolio. The remaining 10-11% thus comes from the variability of the liquidity premium by present-value logic.

The GMM results can pertain to our main findings in Subsection 4.2. The fact that $b_r = -0.205$ and $b_{lp} = 0.027$ can lead roughly to $b_{r^*} \approx b_r - b_{lp} = -0.232$ by accounting identity (6). With these numbers in place, the resulting proportion ω is about 88% (= $0.205/0.232$), implying that about 12% ($= 1 - \omega$) of the net return is accounted for by the liquidity premium.

APPENDIX D: Responses to other shocks

In the main article, we have dealt only with the illiquidity-driven shock that generates expected liquidity premia. This shock identification can be understandable because market illiquidity itself is a *direct* source of (*ex-post*) liquidity premia:

$$R^*_{t+1} = R_{t+1} - LP_{t+1} = \frac{\overbrace{P_{t+1}+D_{t+1}}^{\substack{indirect\\ \text{sources}}}}{P_t} - \frac{\overbrace{C_{t+1}}^{\substack{direct\\ \text{source}}}}{P_t}. \tag{D-1}$$

Notably, we have two other liquidity-premium sources, when it comes to the fact that the implied liquidity premium is a function of prices, dividends, and costs by present-value logic. As seen in gross net return (23), price- and dividend-driven shocks *cannot directly* affect ex-post liquidity premia. Rather, both shocks are going to have an *indirect* effect on *expected* liquidity premia, as will be explained in Appendix D. For brevity, we do not repeat the technical details here, interested readers are referred to Section 5.

First, the price-driven shock is formulated as

$$\varepsilon^{pdl} = 1, \quad \varepsilon^{d} = 0, \quad \varepsilon^{c^*} = 0, \quad \varepsilon^{r} = \hat{\rho}_l, \quad \varepsilon^{lp} = 0. \tag{D-2}$$

Clearly, a *pdl* rise ($\varepsilon^{pdl} = 1$) with no changes in positive and negative dividends ($\varepsilon^{d} = \varepsilon^{c^*} = 0$) should be accompanied by a price rise solely ($\varepsilon^{p} = \varepsilon^{pdl} + \hat{\beta}_1\varepsilon^{d} - \hat{\beta}_2\varepsilon^{c^*} = 1$; see also footnote 13). Unlike illiquidity-driven shock (24), the price rise ($\varepsilon^{p} = 1$) mechanically corresponds to a return rise ($\varepsilon^{r} = \hat{\rho}_l$), *not* a liquidity-premium change at the point of the shock occurance. Through VAR (25), however, the price-driven shock can *indirectly* affect expected liquidity premia beyond time 1. Figure D.1 shows the impulse-response functions in (D-1).

[INSERT FIGURE D.1 HERE]

Consistent with our predictive results (Table 3), it is apparent that *pdl* shock ε^{pdl} is almost expected-return news. The reason for this is that the sum of subsequent expected-return moves amounts to $\sum {\rho_l}^j \, r_{t+1+j} = -1.28$. Note that this number can be over (minus) one since the error shocks, $[\varepsilon^{pdl}, \varepsilon^{d}, \varepsilon^{c^*}, \varepsilon^{r}, \varepsilon^{lp}]$, are *not* orthogonalized. Another impressive result is that the liquidity-premium moves look persistent until time 20 (20 years) such that the sum of the subsequent liquidity premia amounts to $\sum {\rho_l}^j \, lp_{t+1+j} = 0.17$. About 20% looks quite large, but its economic significance is *little* evident, as shown in Subsection 4.3.

Second, the dividend-driven shock has a form as

$$\varepsilon^{pdl} = 0, \qquad \varepsilon^{d} = 1, \qquad \varepsilon^{c^*} = 0, \qquad \varepsilon^{r} = \hat{\beta}_1, \qquad \varepsilon^{lp} = 0. \tag{D-3}$$

Similarly, the dividend-driven shock can also affect expected liquidity premia indirectly. We plot this shock identification in Figure D.2.

[INSERT FIGURE D.2 HERE]

As a result, we find that the dividend-driven shock has nothing to do with expected liquidity premia. Specifically, the dividend-driven shock comes with a huge price rise, but all variables displayed in second panel stay there at the shock point onward.

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Table 1. Summary statistics

Panel A: Means and standard deviations from 1927 to 2017

	Δd_t	Δc_t^*	r_t	er_t	pd_t	pd_t^l
Mean	0.054	−0.081	0.065	0.057	3.365	3.156
Standard deviation	0.143	0.410	0.197	0.199	0.433	0.269

Panel B: Contemporaneous correlations from 1927 to 2017

	Δd_t	Δc_t^*	r_t	er_t	pd_t	pd_t^l
Δd_t	1.000					
Δc_t^*	**−0.344**	1.000				
r_t	**0.652**	**−0.613**	1.000			
er_t	**0.649**	**−0.671**	**0.980**	1.000		
pd_t	−0.115	**−0.214**	0.054	0.038	1.000	
pd_t^l	**−0.246**	−0.041	0.002	−0.013	**0.621**	1.000

Panel C: Unit root tests from 1927 to 2017

	Δd_t	Δc_t^*	r_t	er_t	pd_t	pd_t^l
PP	**−11.314**	**−5.925**	**−9.261**	**−8.852**	−1.593	−2.503
KPSS	0.055	0.313	0.056	0.069	**0.869**	0.067

NOTE: Δd_t is log real dividend growth, Δc_t^* is log illiquidity growth contingent on Amihud (2002) price impact proxy c_t^* (see Figure 2), r_t is log real return, er_t is log excess return, pd_t is the conventional price–dividend ratio, and $pd_t^l = p_t - \hat{\beta}_1 d_t + \hat{\beta}_2 c_t^*$ is the liquidity-adjusted price–dividend ratio, where $\hat{\beta}_1 = 1.084$ and $\hat{\beta}_2 = 0.084$ in Eq. (9). The PP test has the null that a variable has a unit root, and the KPSS test has the null that a variable is stationary. We allow for an intercept and select an optimal lag in each test equation, based on the SC. We report adjusted *t*-statistics in the PP test and Lagrange Multiplier (LM) statistics in the KPSS test, respectively. Significant statistics at the 5% level are highlighted in bold face.

Table 2. Forecasting regressions

Regression	b_x or ϕ (S.E.)	t-statistic	R^2 (%)	$\sigma(bx)$ (%)
$\Delta d_{t+1} = a_d + b_d \cdot pd_t^l + \varepsilon_{t+1}^d$	−0.056 (0.062)	−0.899	1.10	1.50
$\Delta c_{t+1}^* = a_{c^*} + b_{c^*} \cdot pd_t^l + \varepsilon_{t+1}^{c^*}$	0.126 (0.123)	1.023	0.68	3.39
$r_{t+1} = a_r + b_r \cdot pd_t^l + \varepsilon_{t+1}^r$	−0.205 (0.064)	−3.214	7.77	5.50
$pd_{t+1}^l = a_\phi + \phi \cdot pd_t^l + \varepsilon_{t+1}^{pdl}$	0.866 (0.056)	15.394	74.73	23.25
$\Delta d_{t+1} = a_d + b_d \cdot pd_t + \varepsilon_{t+1}^d$	−0.027 (0.038)	−0.724	0.69	1.19
$r_{t+1} = a_r + b_r \cdot pd_t + \varepsilon_{t+1}^r$	−0.085 (0.045)	−1.894	3.49	3.69
$pd_{t+1} = a_\phi + \phi \cdot pd_t + \varepsilon_{t+1}^{pd}$	0.942 (0.039)	24.106	88.31	40.69

NOTE: We report slope estimate b_x or ϕ with respect to each forecasting regression (1st column) for state variable x and its standard errors in parentheses. The *t*-statistic estimated from a GMM method is corrected for heteroscedasticity.

Table 3. Long-run implications

	$\hat{\beta}_1 \cdot b_d^{lr}$	$\hat{\beta}_2 \cdot b_{c^*}^{lr}$	b_r^{lr}	b_{er}^{lr}
Long-run estimate	−0.377	0.066	−1.276	−1.262
(S.E.)	(0.394)	(0.968)	(0.435)	(0.453)
t-statistic	−0.958	0.168	−2.932	−2.785

NOTE: We report the long-run slope coefficient calculated as $b_x^{lr} = b_x/(1 - \hat{\rho}_l \phi)$ in long-run forecast identity (20). Here, b_x and ϕ is the one-period regression coefficient in Table 2, and $\hat{\rho}_l$ is set to 0.969. We use $\hat{\beta}_1 = 1.084$ and $\hat{\beta}_2 = 0.084$ in Eq. (9). The *t*-statistic is calculated through the standard delta method.

Table 4. Error standard deviations and correlations

	ε^{d}	$\varepsilon^{c^{*}}$	ε^{r}	ε^{pdl}	ε^{lp}
ε^{d}	14.2	−33.8	65.2	−31.0	−31.0
$\varepsilon^{c^{*}}$	−33.8	40.8	−61.6	−22.5	−22.5
ε^{r}	65.2	−61.6	18.9	50.3	50.3
ε^{pdl}	−31.0	−22.5	50.3	13.5	100
ε^{lp}	−31.0	−22.5	50.3	100	0.4

NOTE: Each number stands for error standard deviations on the diagonal (%) and correlation on the off-diagonal (%). The error shocks are dividend shock ε^{d}, illiquidity shock $\varepsilon^{c^{*}}$ subject to Amihud's (2002) illiquidity proxy in Eq. (8), return shock ε^{r}, *pdl* shock ε^{pdl}, and implied liquidity-premium shock ε^{lp} by error identity (18).

Table B. 1. Forecasting regressions and error standard deviations

Regression	b_x or ϕ (S.E.)	t-statistic	R^2 (%)	$\sigma(bx)$ (%)
$\Delta d_{t+1} = a_d + b_d \cdot pd_t^l + \varepsilon_{t+1}^d$	0.044 (0.030)	1.448	1.45	1.4
$\Delta s_{t+1} = a_s + b_s \cdot pd_t^l + \varepsilon_{t+1}^s$	−0.076 (0.030)	−2.546	5.75	2.5
$r_{t+1} = a_r + b_r \cdot pd_t^l + \varepsilon_{t+1}^r$	−0.042 (0.024)	−1.791	2.81	1.4
$pd_{t+1}^l = a_\phi + \phi \cdot pd_t^l + \varepsilon_{t+1}^{pdl}$	0.906 (0.035)	26.019	83.71	29.5

NOTE: We report slope estimate b_x or ϕ with respect to each forecasting regression (1st column) for state variable x and its standard errors in parentheses. The *t*-statistic estimated from a GMM method is corrected for heteroscedasticity.

Table B. 2. Long-run implications

	$\hat{\beta}_1 \cdot b_d^{lr}$	$\hat{\beta}_2 \cdot b_s^{lr}$	b_r^{lr}	b_{er}^{lr}
Long-run estimate	0.472	−0.047	−0.429	−0.427
(S.E.)	(0.223)	(0.209)	(0.209)	(0.204)
t-statistic	2.120	−0.209	−2.056	−2.091

NOTE: We report the long-run slope coefficient calculated as $b_x^{lr} = b_x/(1-\hat{\rho}_l\phi)$ in long-run forecast identity (20). Here, b_x and ϕ is the one-period regression coefficient in Table B. 1, and $\hat{\rho}_l$ is set to 0.995. We use $\hat{\beta}_1 = 1.060$ and $\hat{\beta}_2 = 0.060$ in Eq. (B-1). The *t*-statistic is calculated through the standard delta method.

Table B. 3. Error standard deviations and correlations

	ε^d	ε^s	ε^r	ε^{pdl}	ε^{lp}
ε^d	11.8	11.0	24.9	−80.0	−62.5
ε^s	11.0	10.0	−9.2	−11.5	−39.8
ε^r	24.9	−9.2	8.1	37.9	56.4
ε^{pdl}	−80.0	−11.5	37.9	13.0	93.2
ε^{lp}	−62.5	−39.8	56.4	93.2	0.1

NOTE: Each number stands for error standard deviations on the diagonal (%) and correlation on the off-diagonal (%). The error shocks are dividend shock ε^d, illiquidity shock ε^s subject to bid-ask spread proxy in Eq. (B-1), return shock ε^r, *pdl* shock ε^{pdl}, and implied liquidity-premium shock ε^{lp} by error identity: $\varepsilon^{lp} = -\hat{\rho}_l \cdot \varepsilon^{pdl} - \hat{\beta}_1 \cdot \varepsilon^d + \hat{\beta}_2 \cdot \varepsilon^s + \varepsilon^r$, where $\hat{\rho}_l = 0.995$, $\hat{\beta}_1 = 1.060$, and $\hat{\beta}_2 = 0.060$.

Table C. 1. GMM test

$I_t^{\mathcal{H}}$ (rank, identification)	Implications of GMM	Significance level for J test
pd_t^l (1, just)	$pd_t^l = \hat{\rho}_l \cdot pd_{t+1}^l + \hat{\beta}_1 \Delta d_{t+1} - \hat{\beta}_2 \Delta c_{t+1}^* - 1.131 \cdot r_{t+1} + \nu_{t+1}$ (0.049)	–
pd_t^l and constant (2, over)	$pd_t^l = \hat{\rho}_l \cdot pd_{t+1}^l + \hat{\beta}_1 \Delta d_{t+1} - \hat{\beta}_2 \Delta c_{t+1}^* - 1.126 \cdot r_{t+1} + \nu_{t+1}$ (0.043)	0.830
$[pd_t^l, \Delta d, \Delta c_t^*, r_t]'$ and constant (5, over)	$pd_t^l = \hat{\rho}_l \cdot pd_{t+1}^l + \hat{\beta}_1 \Delta d_{t+1} - \hat{\beta}_2 \Delta c_{t+1}^* - 1.118 \cdot r_{t+1} + \nu_{t+1}$ (0.034)	0.583

NOTE*:* The table reports the results of over-identifying restrictions on the GMM through continuous updating. Here, $I_t^{\mathcal{H}}$ denotes the instrumental variable, and the rank is the number of the instrumental variables. The last column presents *p*-values for H_0: $E\left[\nu_{t+1} \cdot I_t^{\mathcal{H}}\right] = 0$ in terms of the *J*-statistics.

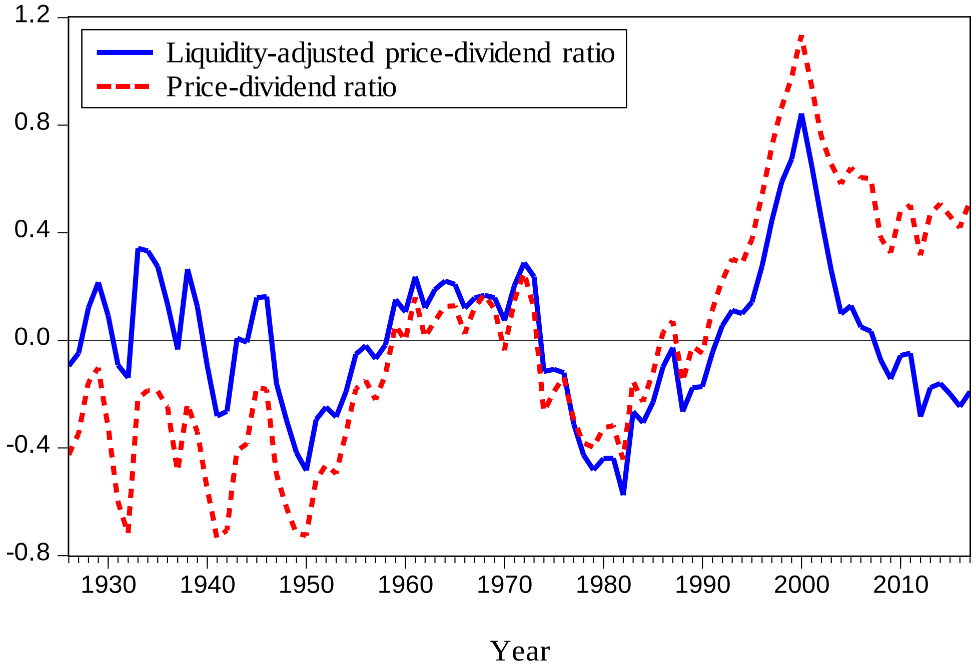

Figure 1. Price–dividend and liquidity-adjusted price–dividend ratios. The two ratios are plotted from 1926 to 2017. They are redefined as deviations from their sample means.

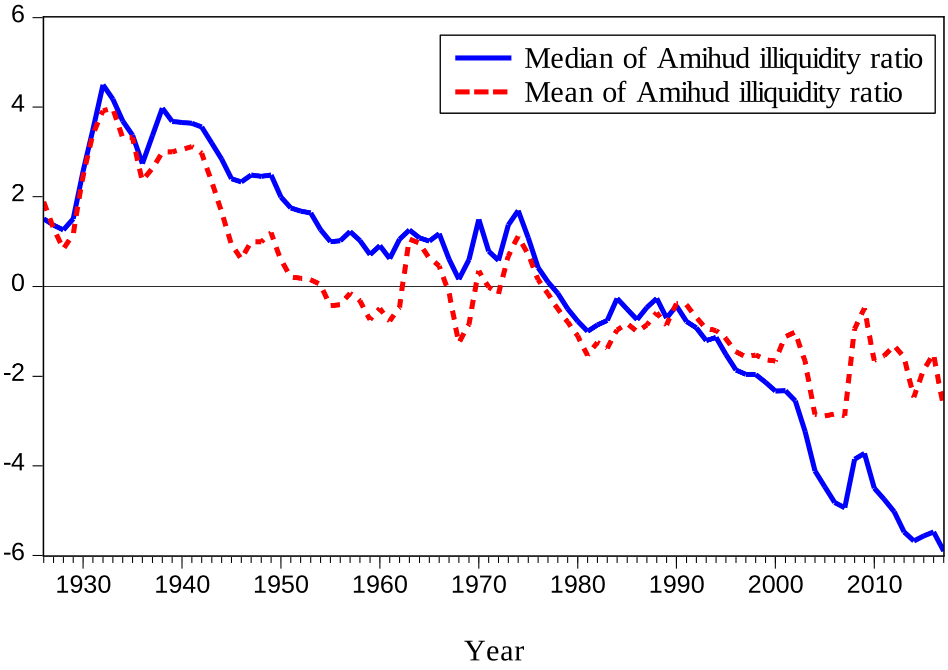

Figure 2. Log Amihud (2002) illiquidity ratio c_t^*. See Subsection 3.1 for calculation. In particular, c_t^* is deflated the CPI provided by CRSP at the end of month in year t. We also redefine c_t^* as deviations from its sample mean from 1926 to 2017.

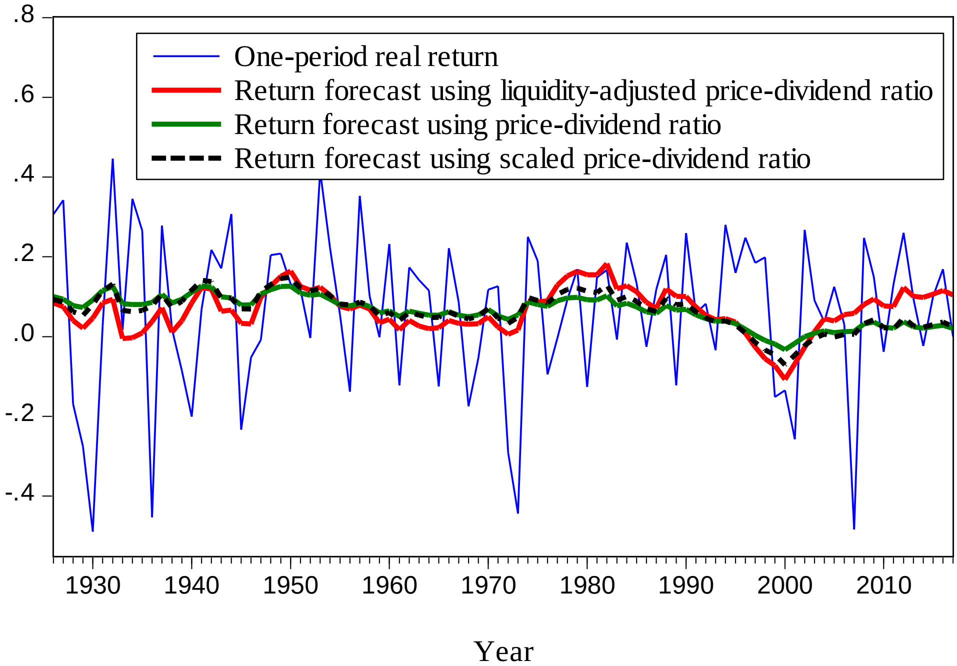

Figure 3. Forecast and one-period ex-post returns. The figure plots one-period real return r_{t+1} and the fitted regression values of simple forecasting regressions in Subsection 4.1 on *pdl* ratio $pd_t^l = p_t - \hat{\beta}_1 d_t + \hat{\beta}_2\, c_t^*$, *pd* ratio $pd_t = p_t - d_t$, and scaled *pd* ratio $p_t - \hat{\beta}_1 d_t$: i.e., $a_x + b_x \times$ regressor.

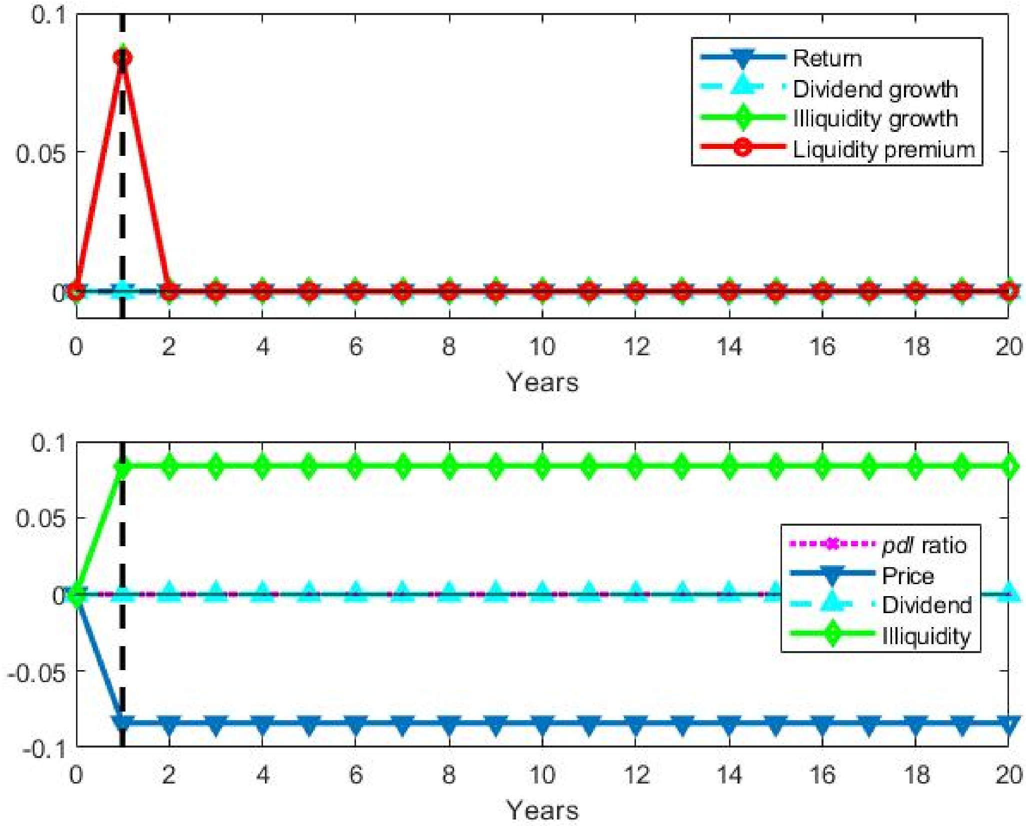

Figure 4. Impulse response to *illiquidity* shock with no moves in *pdl* ratio, dividend, and return. The impulse-response functions are plotted based on the first-order VAR (25). The first panel uses the five error shocks, $[\varepsilon^{pdl}, \varepsilon^{d}, \varepsilon^{c^*}, \varepsilon^{r}, \varepsilon^{lp}]$, where liquidity-premium shock ε^{lp} is inferred from error identity (18): $\varepsilon^{lp} = -\hat{\rho}_l \cdot \varepsilon^{pdl} - \hat{\beta}_1 \cdot \varepsilon^{d} + \hat{\beta}_2 \cdot \varepsilon^{c^*} + \varepsilon^{r}$, where $\hat{\rho}_l = 0.969$, $\hat{\beta}_1 = 1.084$, and $\hat{\beta}_2 = 0.084$. The second panel further uses the price shock $\varepsilon^{p} = \varepsilon^{pdl} + \hat{\beta}_1 \cdot \varepsilon^{d} - \hat{\beta}_2 \cdot \varepsilon^{c^*}$ in footnote 15. We identify the *illiquidity* shock as $\left[\varepsilon^{pdl}, \varepsilon^{d}, \varepsilon^{c^*}, \varepsilon^{r}, \varepsilon^{lp}\right] = [0,0,1,0,\hat{\beta}_2]$ in Eq. (24). The vertical dashed line represents the starting time of the shock.

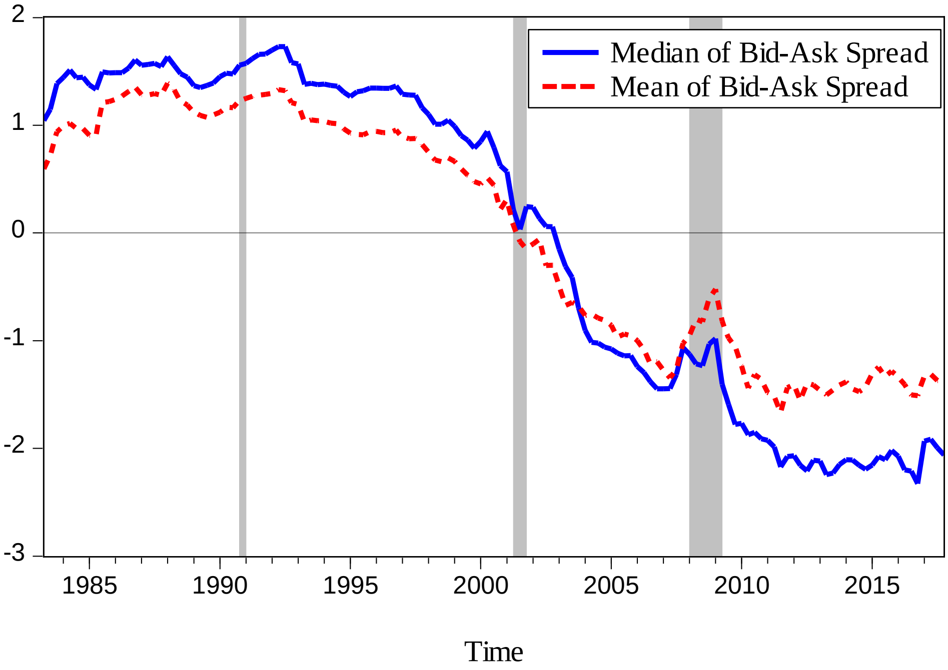

Figure B. 1 Log bid-ask spread s_t. We plot both median and mean of all the cross-sectional spreads from the first quarter of 1983 (1983Q1) to the fourth quarter of 2017 (2017Q4). Both of them are redefined as deviations from its sample means and deflated the CPI provided by CRSP at the end of month in quarter t. Shaded areas represent the NBER recessions for the period following the peak through the trough.

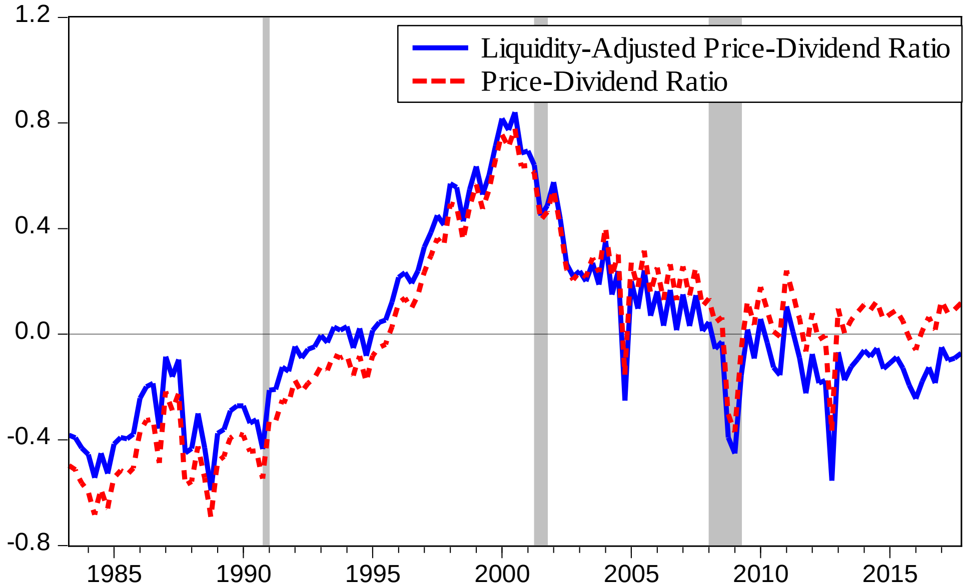

Figure B. 2 Price–dividend and liquidity-adjusted price–dividend ratios. The two ratios are plotted over the period from 1983Q1 to 2017Q4. They are redefined as deviations from their sample means. Shaded areas represent the NBER recessions for the period following the peak through the trough.

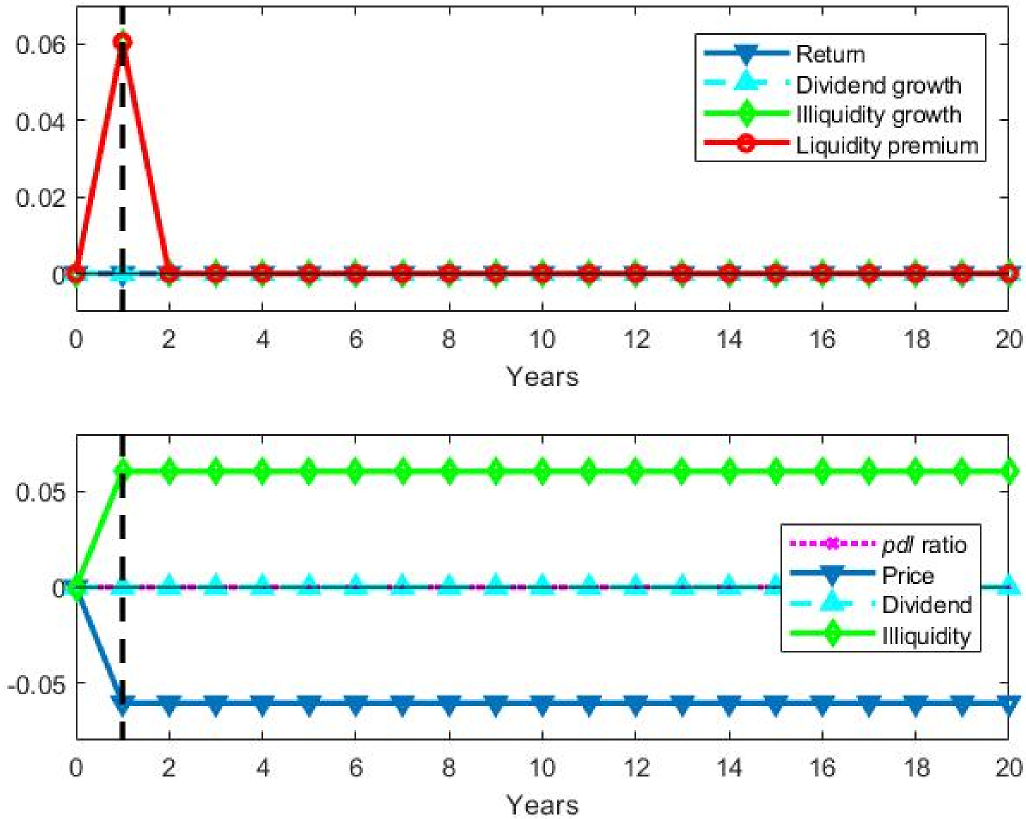

Figure B. 3. Impulse response to *illiquidity* shock with no moves in *pdl* ratio, dividend, and return. The impulse-response functions are plotted based on the similar form of the first-order VAR (25). The first panel uses the five error shocks, $[\varepsilon^{pdl}, \varepsilon^{d}, \varepsilon^{s}, \varepsilon^{r}, \varepsilon^{lp}]$, where liquidity-premium shock ε^{lp} is inferred from the error identity: $\varepsilon^{lp} = -\hat{\rho}_l \cdot \varepsilon^{pdl} - \hat{\beta}_1 \cdot \varepsilon^{d} + \hat{\beta}_2 \cdot \varepsilon^{s} + \varepsilon^{r}$, where $\hat{\rho}_l = 0.995$, $\hat{\beta}_1 = 1.060$, and $\hat{\beta}_2 = 0.060$ in Eq. (B-1). The second panel further uses the price shock: $\varepsilon^{p} = \varepsilon^{pdl} + \hat{\beta}_1 \cdot \varepsilon^{d} - \hat{\beta}_2 \cdot \varepsilon^{s}$. We identify the *pdl* shock as $[\varepsilon^{pdl}, \varepsilon^{d}, \varepsilon^{s}, \varepsilon^{r}, \varepsilon^{lp}] = [0,0,1,0,\hat{\beta}_2]$. The vertical dashed line represents the starting time of the shock.

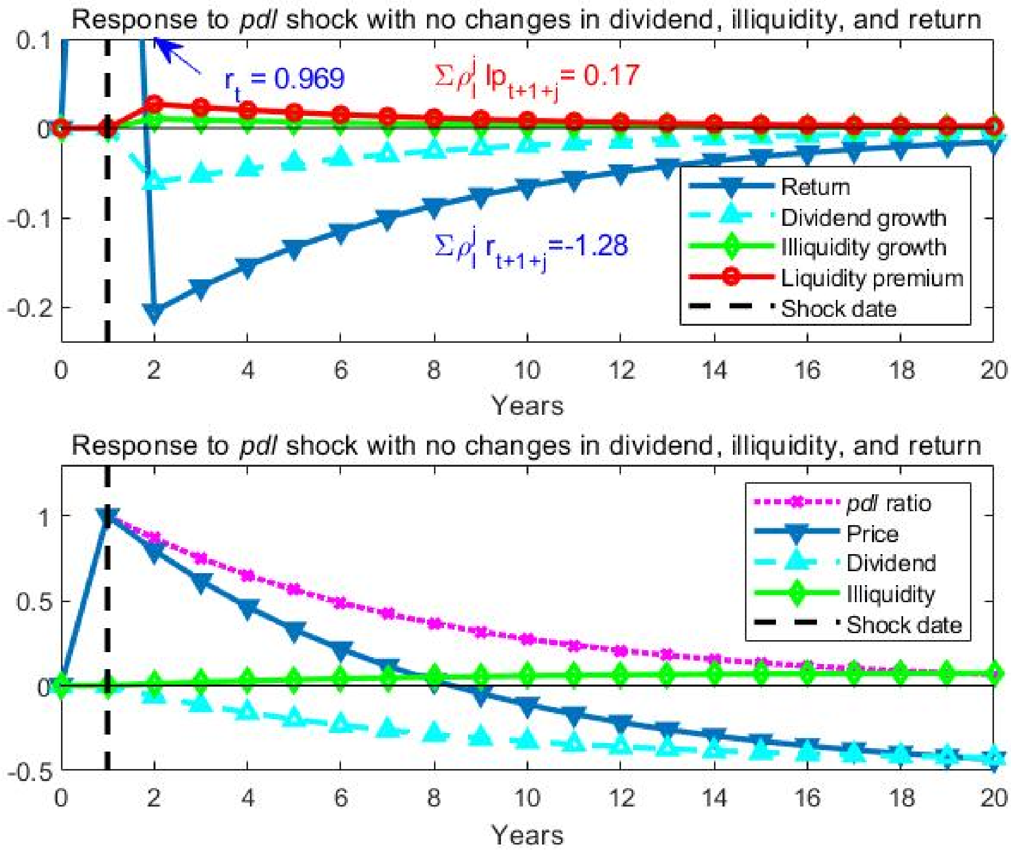

Figure D. 1. Impulse response to *price* shock with no moves in dividend, illiquidity, and return. The impulse-response functions are plotted based on the similar form of the first-order VAR (25). The first panel uses the five error shocks, $[\varepsilon^{pdl}, \varepsilon^{d}, \varepsilon^{c^*}, \varepsilon^{r}, \varepsilon^{lp}]$, where liquidity-premium shock ε^{lp} is inferred from error identity (18): $\varepsilon^{lp} = -\hat{\rho}_l \cdot \varepsilon^{pdl} - \hat{\beta}_1 \cdot \varepsilon^{d} + \hat{\beta}_2 \cdot \varepsilon^{c^*} + \varepsilon^{r}$, where $\hat{\rho}_l = 0.969$, $\hat{\beta}_1 = 1.084$, and $\hat{\beta}_2 = 0.084$. The second panel further uses the price shock $\varepsilon^{p} = \varepsilon^{pdl} + \hat{\beta}_1 \cdot \varepsilon^{d} - \hat{\beta}_2 \cdot \varepsilon^{c^*}$ in footnote 15. We identify the *illiquidity* shock as $\left[\varepsilon^{pdl}, \varepsilon^{d}, \varepsilon^{c^*}, \varepsilon^{r}, \varepsilon^{lp}\right] = [1,0,0,\hat{\rho}_l, 0]$ in Eq. (D-2). The vertical dashed line represents the starting time of the shock

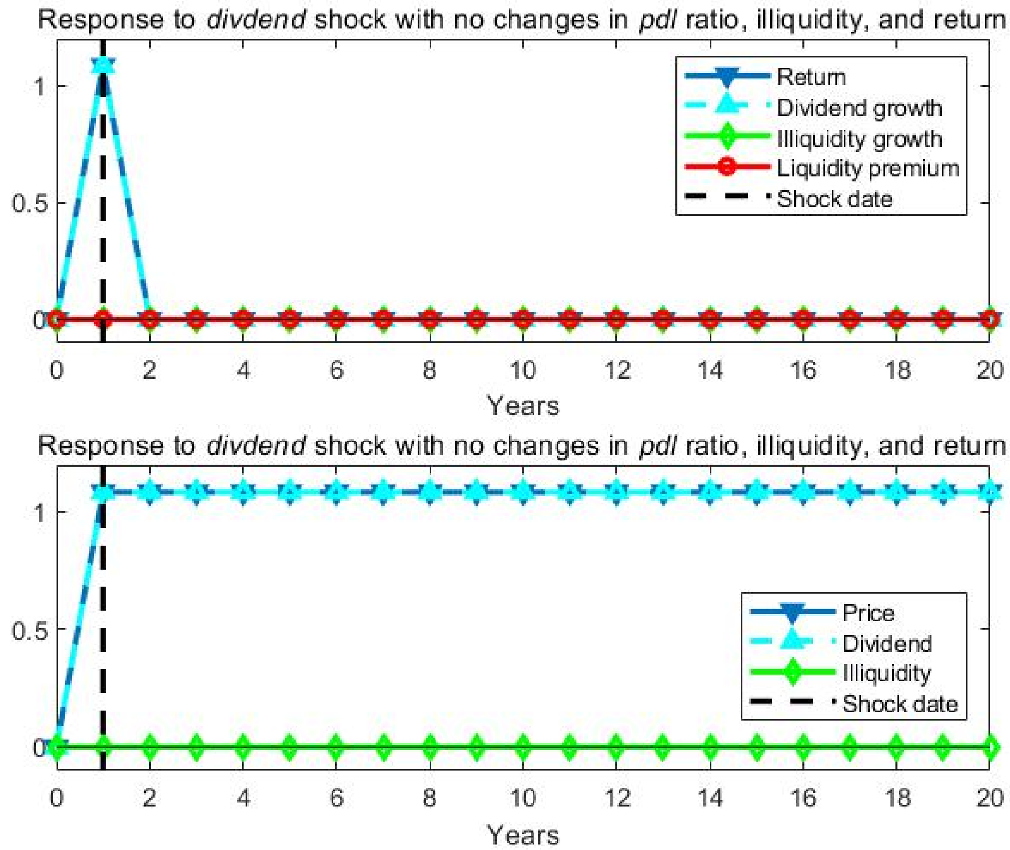

Figure D. 2. Impulse response to *dividend* shock with no moves in dividend, illiquidity, and return. The impulse-response functions are plotted based on the similar form of the first-order VAR (25). The first panel uses the five error shocks, $[\varepsilon^{pdl}, \varepsilon^{d}, \varepsilon^{c^*}, \varepsilon^{r}, \varepsilon^{lp}]$, where liquidity-premium shock ε^{lp} is inferred from error identity (18): $\varepsilon^{lp} = -\hat{\rho}_l \cdot \varepsilon^{pdl} - \hat{\beta}_1 \cdot \varepsilon^{d} + \hat{\beta}_2 \cdot \varepsilon^{c^*} + \varepsilon^{r}$, where $\hat{\rho}_l = 0.969$, $\hat{\beta}_1 = 1.084$, and $\hat{\beta}_2 = 0.084$. The second panel further uses the price shock $\varepsilon^{p} = \varepsilon^{pdl} + \hat{\beta}_1 \cdot \varepsilon^{d} - \hat{\beta}_2 \cdot \varepsilon^{c^*}$ in footnote 15. We identify the *illiquidity* shock as $\left[\varepsilon^{pdl}, \varepsilon^{d}, \varepsilon^{c^*}, \varepsilon^{r}, \varepsilon^{lp}\right] = [0,1,0,\hat{\beta}_1, 0]$ in Eq. (D-3). The vertical dashed line represents the starting time of the shock