

Does Corporate Investment Respond to the Time-Varying Cost of Capital? Empirical Evidence

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Abstract

I test whether firms take the time-varying cost of capital into account in capital budgeting decisions. This test requires a measure of the conditional cost of equity, and I estimate it nonparametrically using individual equity option prices. I find that corporate investment responds negatively to the option-implied measures of the cost of equity and the weighted average costs of capital. Furthermore, the analysis based on these new measures reveals that empirical investment is equally responsive to the cost of capital and the expected future cash flow, as the theory predicts. This finding suggests that firm managers correctly update their discount rates over time despite the failure of the conventional frameworks – Capital Asset Pricing Model and multi-factor models – in this regard.

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1. Introduction

Determining the cost of capital is essential to firms' decisions on capital budgeting. Across time, this cost of capital is likely to vary, as manifested by recent asset-pricing studies [see the survey by [Cochrane \(2011\)](#)]. Facing a variation in a firm's cost of capital, the firm's optimal response, in theory, is straightforward. Its capital investment should react negatively because a rise in the discount rate lowers the present value of future cash flows from the capital installation. However, when investing in practice, do firms consider a cost-of-capital variation, as the theory predicts?

Testing the empirical validity of this theoretical idea has been challenging. In particular, the challenge lies in the measurement of a conditional cost of equity. De facto standard approaches, the Capital Asset Pricing Model or other factor models, are of little help in detecting variations in the cost through time [[Fama and French \(1997\)](#); [Welch and Goyal \(2008\)](#)]. In alternative attempts, some prior studies have instead used proxy variables such as the credit spreads on corporate bonds and realized stock returns after firms' investment. However, the literature has found only mixed results across different choices of the proxy; the association between the cost of capital and capital investment is found to be negative in [Lamont \(2000\)](#) and [Gilchrist and Zakrajsek \(2012\)](#) but insignificant in [Abel and Blanchard \(1986\)](#) and [Arif and Lee \(2014\)](#).

In this study, I propose a direct measurement of the cost of equity, utilizing the market prices of individual equity options. Using this new measure that overcomes the limitations discussed above, this study establishes the empirical link that capital investment does respond to temporal changes in the cost of capital. Furthermore, through decomposing the marginal value of capital, I reveal that the empirical investment is equally responsive to the cost of capital and the expected future cash flows, consistent with the theory.

In these analyses of capital investment, the option-implied measure of cost of equity has the potential to detect cost variations well. Due to the security nature of equity options, their current prices inherently reflect both the physical probabilities of future stock prices and investors' risk aversion. Utilizing this information embedded in a particular firm's option prices on a certain date, I can determine the firm's cost of equity conditional on the date. This forward-looking approach relies only on recent prices, thus able to capture changes in the return-relevant information better than the conventional approaches. In the existing factor models and an accompanying backward-looking

estimation, the recent information tends to be diluted by historical data.

A crucial finding of this study is that capital investment responds negatively to the option-implied cost of equity, as the theory predicts. The association is statistically significant at the 1% level after I control for other determinants of capital investment noted in the literature – firm size, Tobin’s q , leverage ratio, risk-free rate, corporate bond yields, and cash flow-to-asset ratio. The economic magnitude of the cost-of-equity impact is comparable to that of Tobin’s q ; an increase in the cost of equity by one standard deviation lowers a typical firm’s investment by 4.5% (e.g., the annual investment-capital ratio changes from 0.211 to 0.201), whereas an one-standard-deviation increase in Tobin’s q raises investments by 6.6%. This finding is intriguing because, as shown below, the option-implied measures explain the cost-of-equity variations that cannot be captured by the CAPM or multi-factor models. It appears that in updating the discount rate, firm managers make adjustments beyond what the conventional frameworks guide, and they do so correctly.

Moreover, I document that the investment is also negatively associated with the “unlevered” cost of capital, the weighted average of cost of debt and the option-implied cost of equity. This result suggests that firms indeed update the discount rates in response to changes in fundamental business risk, apart from the influence of the financing structure. Besides, I confirm that the investment’s response to the cost of capital is not spuriously driven by the other forces that may cause investment swings; the literature has noted financial constraints², investors’ irrational sentiments, and the real-option value of projects as potential sources of a cyclicity in investment. After controlling for these alternative forces, I find that the cost of capital continues to be a strong predictor of investment.

Beyond the directional aspect of the association, I take a step toward examining the quantitative significance of the cost of capital, as compared to that of the expected future productivity (or cash flow) as determinants of investment. Intuitively, these two quantities are major components of the marginal q , the expected present value of marginal profits to capital. Formally, based on the model structure of the neoclassical investment framework, I derive the decomposition of the marginal q into the cost-of-capital component and the productivity component. In particular, I provide the two components in measurable forms, obtainable from the option-implied cost of capital and the future

²The two forces, cost of capital and financial constraints, are distinct in nature. A shift in the cost of capital changes the fair value of investment projects, whereas financial constraints restrict firms’ spending irrespective of the project values. For example, a firm with sufficient internal cash would not be financially constrained. Still, it may optimally choose not to undertake an investment project if the discount rate is high and the project NPV is negative.

productivity implicit in the stock's forward prices. The theory then predicts that a unit increase in the two components should change investment by equal amounts in the absolute value.

The key empirical finding is as follows. The slopes in the regression of the investment on the two components of the marginal q are of similar absolute magnitudes; the coefficient on the cost-of-capital component ranges from -1.030 to -0.898, while the coefficient on the productivity component is from 1.373 to 1.403. Consistently, the null hypothesis that two coefficients are equal in the absolute value is accepted with p-values 0.34 - 0.56 of the Wald test. These findings uncover that a unit decrease in the cost-of-capital component raises firms' investment as much as a unit increase in productivity component does, supporting the quantitative prediction of the theory. Interestingly, I find that the option-implied cost of equity plays a crucial role here. Alternative identification of the cost-of-capital component derived from the conventional measures of the expected return, including CAPM, a multi-factor model, a firm-characteristics-based model, results in far lower economic significance of the cost-of-capital component with the slope ranging from -0.126 to -0.037. As a result, the null hypothesis of equal sensitivity is rejected.

A question remains: how can firm managers recognize the variations in the cost of capital implicit in the option prices? Despite the empirical validity of the option-implied measures, it is not likely that the required estimation has been conducted in corporate practice. Probably, managers invest as if they are aware of the cost variations through their subjective adjustment, which takes place in capital budgeting decisions [[Graham et al. \(2015\)](#) and [Jagannathan et al. \(2016\)](#)]. To explore the underlying mechanism of forming this subjective adjustment, I first identify particular firms with a good response to the cost of equity. I categorize these firms into "responders" and compare their characteristics to those of "non-responders" (i.e., the remaining firms). The logit regressions find that the responders have a higher tendency to finance through equity. Also, these firms tend to be older and more dependent on overall external finance. This finding suggests that the learning-by-doing mechanism may be at work as noted by [Berk et al. \(2004\)](#); firms recognize the time variations in the cost of capital through experience. In particular, direct exposures to equity financing help firms better identify the cost fluctuations.

Discussing the option-implied measure of the cost of equity in more detail, the underpinning of this measurement is the Euler equation. This equation helps to identify the return-relevant information from the option prices. For robustness, I employ two distinct approaches to extracting

the relevant information from individual equity options to determine stocks' expected returns. The first approach recovers the implicit information on the physical probabilities of future stock price and the stochastic discount factor nonparametrically as in [Ross \(2015\)](#). Performing this recovery from option prices observed in a particular month leads to the determination of the cost of equity conditional on that month. The second approach is to compute the risk-neutral variance of stock returns, which can be easily obtained from option prices without a parametric assumption. [Martin and Wagner \(2019\)](#) shows that this risk-neutral variance is, in theory, a lower bound for the expected return and, simultaneously, an empirically strong predictor of subsequent returns.

I apply these approaches to the US non-financial and non-utilities companies constituting the S&P 500 Index and estimate the monthly cost of equity for each firm. I provide evidence confirming the empirical validity of the two option-implied measures. First, each measure significantly predicts subsequent stock returns in excess of the risk-free rate and also foretells the return amount. Second, they are well suited for out-of-sample forecasts. Predicting the one-year returns with these option-implied measures results in an out-of-sample R^2 of 17.5%. This performance is far superior to that of the conventional measures of cost of equity; the out-of-sample R^2 of the CAPM, the Fama-French five-factor model, and characteristics-based model by [Lewellen \(2015\)](#) are 9.2% or even negative.³ Third, these two measures are highly correlated, although they are from different estimation methods. In the regression of the recovery-based measure on the lower-bound measure, R^2 is as high as 73%, suggesting that the identification of the expected return from option prices is reasonably robust to different specifications.

Literature Review The topic of estimating the cost of capital and testing its validity has long been studied in the literature, including [Fama and French \(1997\)](#), [Gebhardt et al. \(2001\)](#), and [Welch and Goyal \(2008\)](#). I contribute to the literature by introducing the option-implied measure of the cost of capital. This forward-looking measure addresses the concern raised by [Fama and French \(1997\)](#) that the estimates in factor models tend to be unstable, hampering the model's ability to update the cost of capital properly. Likewise, [Welch and Goyal \(2008\)](#) note the poor out-of-sample predictability of return models based on firm characteristics. The option-implied measure far outperforms the

³A negative R^2 means that a forecast model underperforms the simple model that predicts return with the five-year historical mean.

conventional models in this out-of-sample forecast. Meanwhile, [Gebhardt et al. \(2001\)](#) introduce a forward-looking measure derived from accounting data. The option-implied measure, in contrast, relies on market price data and thus avoids noise in accounting reports.

The empirical link between cost of capital and firms' investment has been studied in [Abel and Blanchard \(1986\)](#), [Lamont \(2000\)](#), [Lettau and Ludvigson \(2002\)](#), and [Arif and Lee \(2014\)](#). These prior studies have used different proxies for the cost of capital and found mixed results; the association is negative in [Lamont \(2000\)](#) and [Lettau and Ludvigson \(2002\)](#), but insignificant in [Abel and Blanchard \(1986\)](#) and [Arif and Lee \(2014\)](#). Instead of the proxies, I directly measure the cost of capital using the Euler equation and provide evidence supporting the negative impact. Moreover, I uncover that empirical investment is equally responsive to the cost of capital and the expected future cash flow.

The prior studies by [Berk et al. \(1999\)](#) and [Zhang \(2005\)](#) have shown how capital investment affects the firm's risk profiles and realized stock returns. Taking an alternative route, I let the option prices speak of the risk profiles, determine the required return at the time of investment, and then examine the response of investment. This approach is to better assess firms' investment decisions with emphasis on whether or not they react to news on the discount rate. Empirically, [Anderson and Garcia-Feijoo \(2006\)](#), [Liu et al. \(2009\)](#), and [Hou et al. \(2015\)](#) have documented the cross-sectional association between investment and the required return. I complement the findings by revealing the time-series association.

Finally, this study is also related to the literature on recovering option-implied information [[Ait-Sahalia and Lo \(2000\)](#); [Backus et al. \(2011\)](#); [Ross \(2015\)](#); [Martin and Wagner \(2019\)](#)]. I explore the implications of the option-implied information for corporate finance decisions. Also, I testify to the reliability of using option-implicit information to identify the expected return; the returns recovered through two different frameworks, [Ross \(2015\)](#)'s recovery and [Martin and Wagner \(2019\)](#)'s lower bound, are highly correlated.

The remainder of this paper is organized as follows. Section 2 describes the theoretical relation between cost of capital and investment and the methodology for estimating the firm-level cost of equity. Section 3 describes the main findings on the relationships between the cost of capital and investment. Section 4 concludes.

2. Model

2.1. Theoretical Relation between Cost of Capital and Capital Investment

This section presents the theoretical relation between the cost of capital and capital investment. I model the investment decision following the neoclassical framework as in [Liu et al. \(2009\)](#) and [Kogan and Papanikolaou \(2012\)](#). Consider a firm that operates in a stochastic economy. The firm employs a production technology with constant returns to scale and the output is $\chi_t K_t^\alpha L_t^{1-\alpha}$, where χ_t is the productivity shock, K_t is the capital stock, L_t is the labor input, and $\alpha > 0$ is the constant capital share.

The capital stock accumulates according to $K_{t+1} = (1 - \delta)K_t + I_t$, where I_t is investment, and δ is the constant depreciation rate. Undertaking the investment, the firm incurs purchasing and adjustment costs, and the total cost is $\phi(I_t/K_t) K_t$. $\phi(\cdot)$ is a convex function as in the literature. The labor is costlessly adjustable each time.

Suppose that the financial markets are complete and frictionless. Due to the absence of the frictions, the firm's financing choice is irrelevant to the firm's value, so I assume for simplicity that the firm is entirely financed with equity. The labor input is chosen to maximize the single-period output. Then, the date- t payout of the firm is

$$\max_{L_t} \left\{ \chi_t K_t^\alpha L_t^{1-\alpha} - \phi\left(\frac{I_t}{K_t}\right) K_t - W_t L_t \right\} \equiv A_t K_t - \phi\left(\frac{I_t}{K_t}\right) K_t \quad (1)$$

where W_t is the wage and A_t is the augmented productivity shock⁴ that reflects the optimal choice of the labor.

The firm chooses capital investment to maximize the market value

$$V(A_t, K_t) = \max_{\{I_s\}_{s=t}^{\infty}} \mathbb{E}_t \left[\sum_{s=t}^{\infty} M_s \left(A_s K_s - \phi\left(\frac{I_s}{K_s}\right) K_s \right) \right], \quad (2)$$

where M_s is the stochastic discount factor that is exogenously given for ease of exposition. For the

⁴The solution to the first order condition for L_t leads to the result $A_t = \alpha \left(\frac{1-\alpha}{W_t}\right)^{(1-\alpha)/\alpha} \chi_t^{1/\alpha}$.

recursive structure, the firm value can be expressed as the Bellman equation:

$$V(A_t, K_t) = \max_{I_t} \left\{ A_t K_t - \phi \left(\frac{I_t}{K_t} \right) K_t + \mathbb{E}_t [M_{t+1} V(A_{t+1}, K_{t+1})] \right\}. \quad (3)$$

The first-order condition of the optimal investment implies that $\mathbb{E}_t [M_{t+1} R_{I,t+1}] = 1$, where $R_{I,t+1}$ is the investment return, defined as

$$R_{I,t+1} = \frac{\frac{\partial V_{t+1}}{\partial K_{t+1}}}{\phi' \left(\frac{I_t^*}{K_t} \right)} \quad (4)$$

with the optimally chosen investment I_t^* and $V_{t+1} = V(A_{t+1}, K_{t+1})$.

The above Euler equation alludes to the connection between the investment return and financial asset returns on the firm. I formally prove in [Appendix A](#) that the investment return is identical to the stock return of this all-equity-financed firm, as noted in [Cochrane \(1991\)](#) and [Liu et al. \(2009\)](#). In practice, firms often use both equity and debt financing, and the financial return can be identified by the weighted average of returns on stocks and bonds according to the Modigliani-Miller theorem. Then, it follows that the date- t weighted average costs of capital (WACC_t), the required return, should be equal to the expected return on investment:

$$\text{WACC}_t = \mathbb{E}_t [R_{I,t+1}] = \mathbb{E}_t \left[\frac{\frac{\partial V_{t+1}}{\partial K_{t+1}}}{\phi' \left(\frac{I_t^*}{K_t} \right)} \right]. \quad (5)$$

The investment return in equation (5) can be further simplified. Because the production and investment technology exhibit constant returns to scale, the maximized firm value from the Bellman equation takes the form of $H(A_t)K_t$, where $H(A_t) > 0$ [[Abel and Eberly \(1994\)](#); [Kogan and Papanikolaou \(2012\)](#)]. Then, the derivative $\partial V_{t+1}/\partial K_{t+1}$ is equal to $H(A_{t+1})$, so equation (5) is re-expressed as follows:

$$\phi' \left(\frac{I_t^*}{K_t} \right) = \frac{\mathbb{E}_t [H(A_{t+1})]}{\text{WACC}_t}. \quad (6)$$

The right-hand side of equation (6) can be interpreted as the marginal q , or the expected present value of marginal profits to investment, in the q -theory of investment. Importantly, this equation reveals the negative relationship between the cost of capital and optimal investment. A rise in the cost of capital (discount rate) lowers the present value of marginal profits to capital, all other things being equal. Thus, to achieve the equilibrium, the firm should decrease investment considering the

convex costs of investment. This theoretical prediction implies that if the cost of capital fluctuates over time as the asset pricing studies suggest, the firm's investment should respond negatively.

Beyond the qualitative aspect, the second goal of this study is to quantitatively assess the sensitivity of investment to the cost of capital. In particular, motivated by equation (6), I contrast it with the investment sensitivity to productivity (cash-flow) component, $\mathbb{E}_t[H(A_{t+1})]$. For this analysis, a parametric structure on the investment-adjustment costs is imposed: $\phi(I/K) = (I/K)^\eta$, where $\eta > 1$ as in [Jermann \(1998\)](#). The following lemma presents the decomposition of the marginal q .

Lemma 2.1. *The log of optimal investment satisfies the following relation:*

$$\log\left(\frac{I_t^*}{K_t}\right) \approx \kappa_0 + \underbrace{\frac{1}{\eta-1} \log\left(1 + \frac{1}{H(\bar{A})} (\mathbb{E}_t(A_{t+1}) - \bar{A})\right)}_{\equiv L(A)_t} - \frac{1}{\eta-1} \underbrace{\log(WACC_t)}_{\equiv L(R)_t}, \quad (7)$$

where \bar{A} is the unconditional mean of the productivity and κ_0 is a constant.

The proof is provided in [Appendix B](#). Briefly speaking, taking the logarithm of both sides of equation (6) leads to the decomposition into the productivity component and the cost-of-capital component. Applying the Taylor expansion with respect to A_{t+1} , I then obtain the productivity component $L(A)_t$ that can be measured directly from the data⁵, once the expected future productivity $\mathbb{E}_t(A_{t+1})$ is provided. Notice that the cost-of-capital component $L(R)_t$ can also be easily obtained if one has the estimate of cost of capital.

The equation (7) provides the theoretical framework to assess the investment sensitivity. A unit increase in productivity component $L(A)_t$ and a unit decrease in cost-of-capital component $L(R)_t$ should raise the investment by equal amounts. This decomposition is in the same spirit as [Abel and Blanchard \(1986\)](#) and [Lettau and Ludvigson \(2002\)](#). Distinct from the prior studies, however, the above decomposition expresses the two components of the marginal q in measurable forms and do not require deep structural assumptions about the economy, thereby robust to model misspecification.

⁵ $H(A_t)$ is easily obtained from data. It is equal to the ratio of firm's market value to book value because the firm value satisfying the Bellman equation (3) takes the form of $V_t = H(A_t)K_t$.

2.2. Estimating Firm-Level Cost of Capital

Having established the theoretical relationship between the cost of capital and investment, this section discusses the identification of the cost of capital from the financial markets. In the frictionless economy as in the Modigliani-Miller theorem, the cost of capital for a firm is the weighted average of its cost of equity and cost of debt. Empirically, it has been challenging to determine the cost of equity. Here, I describe two new approaches that use option prices to estimate the firm-level cost of equity. As shown below, these approaches enable one to identify the conditional cost of equity for a certain month, utilizing only the recent observations of individual option prices in that month.

The theoretical underpinning of this measurement is the Euler equation. In equilibrium, the expected return on a stock from date t to T in excess of the risk-free return is

$$\mathbb{E}_t [R_T] - R_{f,t} = -R_{f,t} \text{Cov}_t (M_T, R_T) \quad (8)$$

where $R_{f,t}$ is the risk-free return, and M_T is the stochastic discount factor (SDF).

According to the Euler equation, one way to pin down the expected return is to try to obtain the joint distribution of future stock return and the SDF (or, the projection of the SDF onto the space spanned by the stock return). I aim to determine this joint distribution for each firm using the target firm's equity options and the option-implied state prices, or the prices of the Arrow-Debrue securities. This approach requires the decomposition of the state prices into the SDF and the physical probabilities and I recover these quantities employing [Ross \(2015\)](#)'s recovery theorem (recovery-based measure).

The second approach does not aim to fully recover the joint distribution. Instead, [Martin and Wagner \(2019\)](#) show that the Euler equation itself relates the lower bound for the expected return to the risk-neutral variance of stock returns, under certain regularity conditions.⁶ Although this measure is theoretically designed as a lower bound, they show that the bound is empirically tight and forecasts subsequent returns well. I replicate their measurement as a second way to estimate option-implied expected return (lower-bound measure).

⁶See [Martin and Wagner \(2019\)](#) and [Martin \(2017\)](#) for details.

2.2.1. The Recovery-Based Measure of Cost of Equity

Consider a stock k with current price S_t on date t . Suppose that its future price on date T will be one of a finite number of prices (S_1, \dots, S_N) .⁷

Given the support of future stock prices, the first step is to calculate the state price, or the price of Arrow-Debrue security that pays only at a certain stock price on date T . [Breedon and Litzenberger \(1978\)](#) show that the state price can be obtained from a cross-section of call options with different strike prices and the same time to maturity. That is, for future price S_j on date T ,

$$\text{date-}t \text{ state price of } S_j = R_{f,t} \left. \frac{\partial^2 \text{Call}_T}{\partial X^2} \right|_{X=S_j} \quad (9)$$

where Call_T is the price of a call option with expiration date T , and X is the strike price.

In equation (9), the second derivative must be computed using option prices for a finite number of strike prices. I employ the semiparametric estimator suggested by [Ait-Sahalia and Lo \(2000\)](#). [Appendix C](#) provides the details of the estimation. The resulting state price when the current stock price is S_i and the future stock price is S_j is denoted by $F_{i,j}$.

Next, I decompose the state prices into the SDF and physical probabilities. To do so, I use [Ross \(2015\)](#)'s recovery theorem.⁸ Consider a projection of the SDF onto the space spanned by the price of stock k . Let M_T^k denote the projected SDF.⁹ A positive constant U_i , which will be recovered later, is assigned to state S_i of the stock price. With these state-specific constants, we specify the projected SDF:

$$M_T^k = \beta \frac{U_j}{U_i} \quad (10)$$

when the current price is S_i and the future price is S_j ¹⁰. Here, β can be interpreted as representing

⁷The actual stock price dynamics are nonstationary. To reflect the nonstationarity, I define a new support of future prices each month. Specifically, I first calculate the average of the option-implied volatility observed in a month. Then, I set the lowest/highest prices in the support such that the lowest/highest annualized log returns are (average return) $\pm (2) \times (\text{implied volatility})$. The number of grids in the support is 30.

⁸I recognize the recent critique on the recovery theorem by [Borovicka et al. \(2016\)](#). The critique notes that the assumed structure in equation (10) may not allow the separation of physical probability from a martingale component associated with long-term risk adjustment. To complement this theoretical drawback, I perform the empirical test in the following section to check whether the recovery-based measure correctly captures the conditional equity premium.

⁹The description of the SDF using different stocks results in different estimates of M_T^k . However, this does not mean the existence of multiple SDFs. Instead, the different estimates are different projections of the unique SDF.

¹⁰The marginal utility U_i and probability $F_{i,j}$ are specific for each firm. The superscript k is dropped for a simple presentation.

the time preference, and U_i represents the marginal utility at state S_i . Then, the state price can be expressed as

$$F_{i,j} = \beta \frac{U_j}{U_i} \mathbb{P}_{i,j} \quad (11)$$

where $\mathbb{P}_{i,j}$ is the physical transition probability from state S_i to S_j . In a matrix form, the relation becomes

$$F = \beta U^{-1} \mathbb{P} U \quad (12)$$

where F is a matrix with $F_{i,j}$ in row i and column j , \mathbb{P} is a matrix with $\mathbb{P}_{i,j}$ in row i and column j , and U is a diagonal matrix with U_i in row i and column i .

Let $\mathbb{1}$ denote a column vector of ones with N elements. Because \mathbb{P} is the matrix of transition probabilities, $\mathbb{P}\mathbb{1} = \mathbb{1}$. Then, it follows that

$$\mathbb{P}\mathbb{1} = \beta^{-1} U F U^{-1} \mathbb{1} = \mathbb{1} \quad (13)$$

and

$$F U^{-1} \mathbb{1} = \beta U^{-1} \mathbb{1}. \quad (14)$$

Denoting $U^{-1}\mathbb{1}$ by vector \mathbf{z} , the result indicates that solving for U^{-1} becomes the problem of finding an eigenvector \mathbf{z} of F such that $F\mathbf{z} = \beta\mathbf{z}$. Importantly, [Ross \(2015\)](#) proves that a unique eigenvector exists for the problem, if there is no arbitrage and matrix F is irreducible. Let \widehat{U} and $\widehat{\beta}$ denote the obtained solutions from the eigenvector. Plugging these into equation (13), I also obtain $\widehat{\mathbb{P}}$.

Now, I can pin down the expected return in excess of the risk-free return using the recovered probability and stochastic discount factor. The date- t expected return on stock k is

$$\text{ER}_{k,t}^{\text{recovery}} = -R_{f,t} \text{Cov}_t \left(\begin{matrix} \widehat{\beta}^k \frac{\widehat{U}_T^k}{U_t^k}, \frac{S_T^k}{S_t^k} \end{matrix} \right) \quad (15)$$

where the superscript k is added to highlight the fact that all of the quantities in the right-hand side are recovered from stock k 's option prices.

If one follows the above procedure using the option prices observed in a certain month, the expected return from the perspective of the corresponding month is obtained. Thus, conducting this estimation month by month will generate a time series of the expected return on the stock.

2.2.2. The Lower-Bound Measure of Cost of Equity

The second option-implied measure considered in this study is the lower bound for the expected return. [Martin and Wagner \(2019\)](#) introduce this measure that is theoretically designed as the lower bound but empirically forecasts subsequent stock returns well – that is, the bound is tight. In this section, I highlight their main findings.

To characterize the expected return on stock k , this approach utilizes the risk-neutral variance of the return $\text{Var}_t^*(R_{k,T}) = \mathbb{E}_t^* [R_{k,T}^2] - (\mathbb{E}_t^* [R_{k,T}])^2$, where $\mathbb{E}_t^*[\]$ denotes the expectation under the risk-neutral measure. This risk-neutral variance can be computed from the stock's option price without any structural assumption about the economy:

$$\frac{1}{R_{f,t}} \text{Var}_t^*(R_{k,T}) = \frac{1}{S_{k,t}^2} \left[2 \int_0^\infty \text{Call}_T(K) dK - \frac{1}{R_{f,t}} (F_{k,t}^T)^2 \right] \quad (16)$$

where $\text{Call}_T(K)$ is the price of the call option with strike price K and expiration date T , and $F_{k,t}^T$ is the forward price of stock such that $F_{k,t}^T = \mathbb{E}_t^* [S_{k,T}]$.

[Martin and Wagner \(2019\)](#) find that a function of risk-neutral variances of the market portfolio and individual stocks defines a theoretical lower bound for the stock k 's expected return in excess of the risk-free rate. Under the regularity condition, the expected return satisfies

$$\text{ER}_{k,t}^{\text{lower bound}} \geq \frac{\text{Var}_t^*(R_{m,T})}{R_{f,t}} + \frac{1}{2} \left[\frac{\text{Var}_t^*(R_{k,T})}{R_{f,t}} - \sum_k w_{k,t} \frac{\text{Var}_t^*(R_{k,T})}{R_{f,t}} \right], \quad (17)$$

where $R_{m,t}$ is return on the market portfolio and $w_{k,t}$ is the weight of stock k in the stock market index.

For each stock, I conduct this measurement using option prices observed in each month. As a result, I obtain monthly estimates of the lower bound for every stock's expected return.

2.3. Estimating Future Productivity

The analysis of investment sensitivity described in equation (7) requires a measure of the future productivity $\mathbb{E}_t [H(A_t)]$ in addition to the cost of capital. I propose a forward-looking measure of future productivity that utilizes the forward prices of a stock. The measurement is based on the idea that these forward prices enable one to infer the stock's dividend growth in the future, as noted by [Golez \(2014\)](#). Then, a theoretical relation between the dividend growth and productivity growth

helps to determine the expected future productivity each date.

First, the implied dividend growth is identified as follows. For a tractable identification, I approximate a stock's dividends as continuous payments and express time- T forward price: $F_T = S_t e^{(r_{f,t} - \lambda_t)(T-t)}$, where λ_t is a continuously compounded dividend yield. The dividend yield leads to the growth in dollar amount of dividend

$$\frac{D_{t+1}}{D_t} = \frac{D_{t+1}}{S_t} \frac{S_t}{D_t} = (e^{\lambda_t} - 1) \frac{S_t}{D_t}, \quad (18)$$

where D_t is the amount of dividends during period t .

Next, the firm's problem described by the Bellman equation (3) provides a theoretical link from the dividend growth to productivity growth. To elucidate the connection, I let Δd_{t+1} denote the log of the dividend growth $\log(D_{t+1}/D_t)$ and Δa_{t+1} denote the log of the productivity growth $\log(A_{t+1}/A_t)$. The following lemma relates the long-run mean of these two growth rates.

Lemma 2.2. *The long-run mean of the dividend growth is given by*

$$\mathbb{E}_t \left[\lim_{T \rightarrow \infty} \frac{1}{T-t} \sum_{s=t+1}^T \Delta d_s \right] \approx \mathbb{E}_t \left[\lim_{T \rightarrow \infty} \frac{1}{T-t} \sum_{s=t+1}^T \Delta a_s \right] + \log \left(1 - \delta + \overline{I/K} \right) \quad (19)$$

where $\overline{I/K}$ is the unconditional mean of investment-capital ratio.

The derivation of the lemma is provided in [Appendix D](#). Intuitively, for the all-equity-financed firm, dividend and production output are tightly connected through the cash flow identity. Over the long run, if transitory fluctuations are averaged out, the dividend growth becomes identical to growth in production output. The lemma further shows that the growth in production output has two sources, productivity growth and capital-stock growth $(1 - \delta + \overline{I/K})$.

Plugging the dividend growth from the forward prices into equation (19), I obtain the time- t expectation of productivity growth $\mathbb{E}^F [\Delta a_{t+1}]$.¹¹ This productivity growth multiplied by the current productivity is the estimate of the expected future productivity.

¹¹Here I assume that the dividend growth in the next period is the same as the growth over the long run. The validity of this assumption is tested in section 3.4.3.

3. Empirical Results

3.1. Data

I conduct the analysis on US non-financial and non-utilities companies that were included in the S&P 500 at least once from 1996 to 2014. This focus on relatively large companies is to ensure a sufficient number of observations of each firm’s individual equity option prices.

Data on individual equity options are obtained from OptionMetrics for the period January 1996 through August 2014. The estimation of the expected return based on the two option-based approach requires the prices of European options, whereas most of the individual equity options traded in the market are American. To address this circumstance, I compute the price of the European equivalent to each American option following Carr and Wu (2009) and Martin and Wagner (2019); I use the volatility surface reported by OptionMetrics and enter the reported volatility into the Black-Scholes-Merton formula to find the European option price. Among calls with different strike prices, in-the-money calls tend to be illiquid, so the price of these options might deviate from the fair value. With these concerns in mind, for in-the-money calls, I instead use the price of the put option with the same strike price and maturity and compute the call price through the put-call parity. In addition, for a reliable estimation of a firm’s expected return in a certain month, I require the firm to have at least 30 option price observations in that month. Applying the filter results in 651 firms that have, on average, 110 monthly estimates of the recovery-based expected return and 776 firms that have, on average, 105 monthly estimates of the lower bound.

For these companies in the sample, I collect quarterly financial statements from Compustat. Firm-level variables are measured in standard ways in the literature. The goal is to measure quantities on a quarterly basis. Thus, for some variables that display a year-end value in the second/third/fourth quarter, I compute the value increase from the previous to the current quarter. A firm’s capital investment in quarter t is computed from capital expenditures (CAPXY). The investment-capital ratio in quarter t ($INVEST_{i,t}$) is defined as the quarter- t investment divided by property, plant and equipment (PPENTQ) in quarter $t - 1$. The book leverage ratio ($LEV_{i,t}$) is the sum of debt in current liabilities (DLCQ) and long-term debt (DLTTQ) divided by total assets (ATQ). The cash flow ($CF_{i,t}$) is operating profits (OIBDPQ). The productivity ($A_{i,t}$) is the quarter- t OIBDPQ divided by PPENTQ in quarter $t - 1$. Firm size ($SIZE_{i,t}$) is defined as the natural

log of ATQ. Cash holdings ($\text{Cash}_{i,t}$) are cash and short-term investments (CHEQ), and dividends ($\text{Dividends}_{i,t}$) are computed from cash dividends (DVY). Sales growth ($\text{Sales Growth}_{i,t}$) is the rate of growth in sales (SALEQ) from quarter $t - 1$ to t . Industry sales growth is the average growth rate for companies that belong to a three-digit SIC industry. For Tobin’s q ($Q_{i,t}$), I follow the measurement of [Erickson et al. \(2014\)](#). The numerator of Tobin’s q is DLTTQ plus DLCQ plus market equity minus current assets (ACTQ), where the market equity is the product of common shares outstanding (CSHOQ) and stock price (PRCCQ). The denominator of Tobin’s q is gross capital stock (PPEGTQ).

To determine firms’ overall cost of capital requires the cost of debt. I obtain yields on corporate bonds from Trade Reporting and Compliance Engine (TRACE). The observed yields in quarter t for firm i ’s bonds are aggregated to generate the weighted average of the yields ($\text{YIELD}_{i,t}$), where the weight is proportional to the par value of each bond. The risk-free returns are treasury constant maturity rates from Federal Reserve Economic Data.

For the purpose of comparison, I also measure the cost of equity under the conventional framework of the CAPM and [Fama and French \(2015\)](#)’s five-factor model. The factor data are from Kenneth French’s website. Each stock’s exposures to the risk factors are derived through the rolling-window regression of the past five-year monthly returns. In addition, I employ another return model suggested by [Lewellen \(2015\)](#). Without identifying firms’ risk exposures, this alternative model directly relates stock returns to firm characteristics – size, book-to-market ratio, past return, stock issuance, accruals, return on asset, asset growth. Fama-MacBeth cross-sectional regression of returns then provides monthly estimates of slopes on each characteristic. As suggested by the study, I use the prior 10-year rolling average of the slope estimates and a firm’s beginning-of-month characteristics to determine the firm’s expected return $\text{ER}_{i,t}^{\text{character}}$. Similarly, I obtain the risk-premium estimates for CAPM and Fama-French factor models; the risk premium for a certain month is the 10-year rolling average of monthly estimates from Fama-MacBeth regression. Based on the risk premium along with the risk-exposure estimates, I compute $\text{ER}_{i,t}^{\text{CAPM}}$ and $\text{ER}_{i,t}^{\text{FF}}$.

For every measure of the cost of equity, from either the option-based approaches or conventional approaches, I compute the weighted average costs of capital (WACC):

$$\text{WACC}_{i,t} = (1 - \text{LEV}_{i,t}) \cdot (\text{EP}_{i,t} + R_{f,t}) + \text{LEV}_{i,t} \cdot \text{YIELD}_{i,t}. \quad (20)$$

The investment analysis also needs a measure of financial constraints. I use three different measures suggested in the literature: the index of [Kaplan and Zingales \(1997\)](#) (KZ index), the index of [Whited and Wu \(2006\)](#) (WW index), and the text-based index recently proposed by [Hoberg and Maksimovic \(2014\)](#) (HM index). I compute the KZ index and the WW index by employing the estimation results reported in [Lamont et al. \(2001\)](#) and [Whited and Wu \(2006\)](#), respectively.¹² Distinct from the previous two indices, the HM index utilizes textual analysis to measure financial constraints. Simply put, the index quantifies managers' concerns about liquidity and the potential need to curtail investment that appear in each firm's 10-K. The HM index is obtained from Gerard Hoberg's website. Among four variables provided, I use the comprehensive index, *delaycon*.

Finally, I measure each firm's dependence on external financing apart from its financial constraints. As in [Rajan and Zingales \(1998\)](#), this dependence ($\text{External Finance}_{i,t}$) is calculated as capital expenditures minus cash flow from operations, divided by cash flow from operations. Cash flow from operations is the sum of funds from operations (FOPTY), decreases in inventories (INVCHY), decreases in receivables (RECCHY), and increases in payables (APALCHY). Similarly, the dependence on external equity, ($\text{Equity Finance}_{i,t}$), is measured by the ratio of the net amount of equity issues, which is sales of common and preferred stock (SSTKY) minus the purchase of common and preferred stock (PRSTKCY), to capital expenditures. Table 1 presents descriptive statistics of these variables.

3.2. A Look at the Estimates of the Option-Implied Expected Returns

Prior to the main analysis, I here describe the general features of the estimates of option-implied expected return. Figure 1 depicts the monthly time series of the expected return for selected firms:

¹²In the original studies, these two indexes are constructed using annual variables. To generate consistent indexes using quarterly variables in this study, I multiply the coefficients for the flow variables by four. The resulting KZ index is

$$\begin{aligned} \text{KZ index} = & -(4)(1.002) \times \frac{\text{CF}}{\text{Net Fixed Assets}} + 0.283 \times Q + 3.139 \times \text{LEV} - (4)(39.368) \times \frac{\text{Dividends}}{\text{Net Fixed Assets}} \\ & - 1.315 \times \frac{\text{Cash}}{\text{Net Fixed Assets}}. \end{aligned}$$

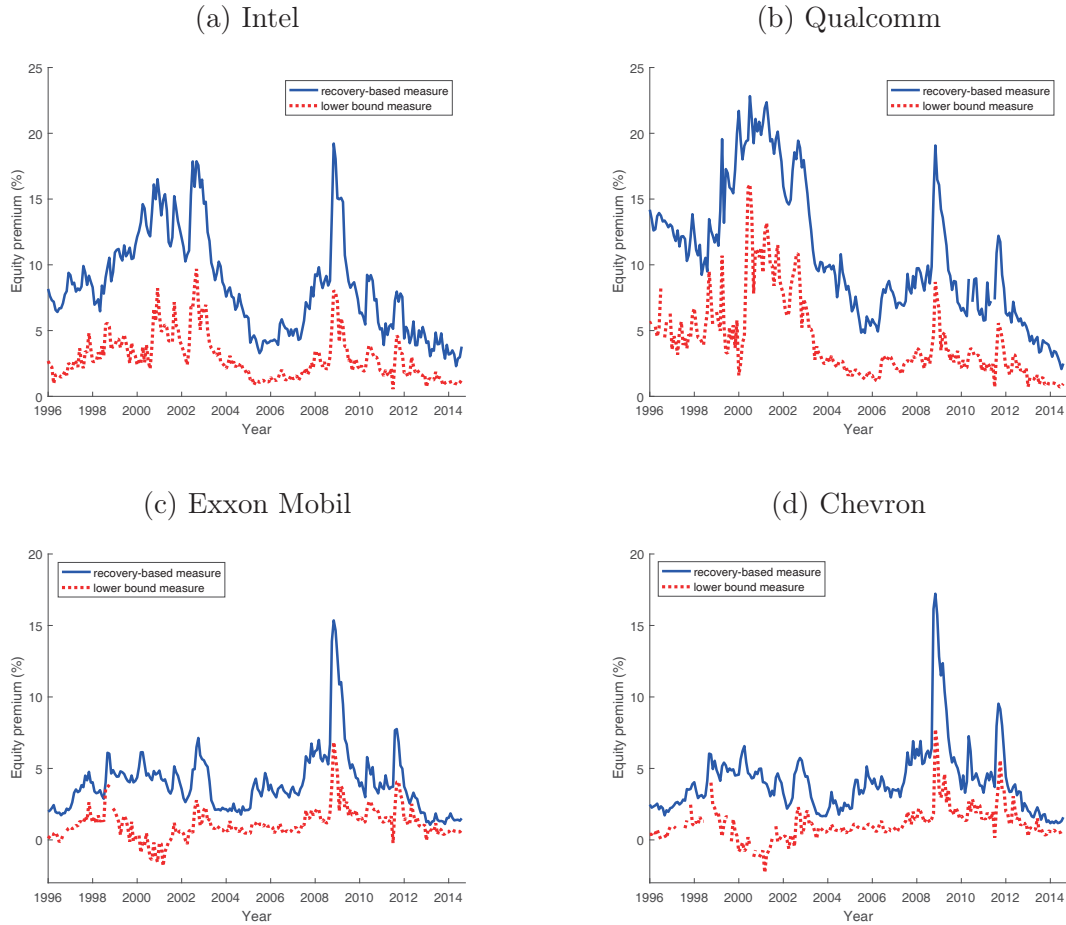
The WW index is

$$\begin{aligned} \text{WW index} = & -(4)(0.091) \times \frac{\text{CF}}{\text{Total Assets}} - 0.062 \times \mathbb{1}_{\text{Dividends} > 0} + 0.021 \times \text{LEV} - 0.044 \times \text{SIZE} \\ & + (4)(0.102) \times \text{Industry Sales Growth} - (4)(0.035) \times \text{Sales growth} \end{aligned}$$

where $\mathbb{1}_{\text{Dividends} > 0}$ is an indicator variable that has the value of one if dividends are positive and zero otherwise.

Figure 1: Examples of the Option-Implied Expected Returns

This figure presents the monthly estimates of the option-implied expected returns for selected firms. Selected are Intel and Qualcomm in the industry of semiconductors and related devices and Exxon Mobil and Chevron in the industry of petroleum products. The expected return is for a six-month horizon and estimated through the recovery-based approach and the lower-bound approach.



Intel and Qualcomm in the semiconductors and related devices industry and Exxon Mobil and Chevron in the petroleum product industry. Plotted are the expected returns for a six-month horizon.

All four of the companies display substantial time variations in the cost of equity. The estimates sometimes change rapidly, particularly during the 2007-08 financial crisis. Note that these time variations are obtained entirely from the data and not from a model assumption. These variations through time provide empirical support for the time-varying risk premiums on which many

theoretical asset pricing models are based.

In addition, the cost-of-equity estimates have a cross-sectional property that one would expect for the cost of capital. The cost of equity for companies from the same industry are more highly correlated than those for companies from different industries. As examples of intra-industry pairs, the correlation coefficient between the recovery-based measures is 0.95 for Exxon Mobil and Chevron and 0.72 for Intel and Qualcomm. In contrast, inter-industry pairs have lower correlation coefficients: 0.35 for Intel and Exxon Mobil and -0.15 for Qualcomm and Chevron.

Between the two option-implied measures, it appears that the lower-bound measure is consistently lower than the recovery-based measure. This is not surprising, however, because the lower-bound measure is theoretically designed as the lower bound for the expected return, whereas the recovery-based measure is to be an unbiased estimator of the return. Despite the difference in the level, the two measures fluctuate in a highly synchronized manner; for example, the correlation coefficient between the two measures for Intel is 0.89. The compatibility between these two measures will be discussed more in the next section.

3.3. Predictive Ability of the Option-Implied Expected Returns

The option-implied measures of expected returns are constructed theoretically. In this section, I test the empirical validity of these new measures. If the measures indeed detect the conditional expected return, they should be able to forecast realized returns going forward. After I confirm their empirical validity, I will employ the option-implied measures in the main analysis.

To examine the predictive ability of the estimate of expected return, I conduct a pooled regression,

$$R_{i,T}^e = \alpha + \beta \times ER_{i,t} + \epsilon_{i,T} \quad (21)$$

where $R_{i,T}^e$ is the realized stock returns from date t to T in excess of the risk-free rate and $ER_{i,t} \in \left(ER_{i,t}^{\text{recovery}}, ER_{i,t}^{\text{lower bound}} \right)$ is the date- t option-implied expected return for the period from t to T . If these measures are perfect empirically, I expect to find that $\alpha = 0$ and $\beta = 1$.

Panel A of Table 2 reports the regression results for the return horizons of six months and one year. The main finding is that both the recovery-based measure and the lower-bound measure of the expected return are a strong predictor of subsequent returns with a 1% level of significance. The t-statistics of the recovery-based measure are 6.39 for the six-month horizon in specification (1)

and 8.39 for the one-year horizon in specification (6). Similarly, the t-statistics of the lower-bound measure are 9.07 for the six-month horizon in specification (2) and 9.66 for the one-year horizon in specification (7). More importantly, the coefficients on the option-implied measures are close to one, consistent with the theory. The coefficients on the recovery-based measure (lower-bound measure) are 0.459 (0.898) for the six-month horizon and 1.052 (0.968) for the one-year horizon.

To test the null hypothesis that $\alpha = 0$ and $\beta = 1$, I conduct the Wald test. The null hypothesis is rejected for all specifications. However, the weaker hypothesis that $\beta = 1$ is accepted in most specifications where the predictions are based on the option-implied measures; p -values range from 0.30 and 0.75 except for the recovery-based measure of the six-month expected return. These results reveal that the option-implied measures correctly detect the magnitude of changes in the expected return as well as direction. This property is crucial for firms' capital budgeting decision because it requires a precise estimate of cost of capital beyond a directional guidance on increasing or decreasing the discount rate.

Nevertheless, the regressions show that the intercept is not zero, so a bias exists in predicting the level of the expected return. The presence of the bias is not surprising for the lower bound measure; theoretically, the measure is designed to capture the lower bound on the expected return. For the recovery-based measure, the bias may arise due to the possible misspecification noted by [Borovicka et al. \(2016\)](#). The authors show that the assumed structure in the recovery theorem might not be able to separate the physical probability from a martingale component associated with a long-term risk adjustment. Despite the theoretical drawbacks, however, the above empirical analysis establishes that both of these option-implied measures are a useful predictor of the cost of equity with regression coefficients close to one. Moreover, I focus on within-firm swings in the cost of capital and its impact on corporate policy, so the bias is not much problematic.

The predictive power of the option-implied measures may come from both cross-sectional variation and time-series variation in the stock returns that the measures explain. To focus on the measures' ability to predict the time-series variations, I perform the following panel regression

$$R_{i,T}^e = \alpha_i + \beta \times ER_{i,t} + \epsilon_{i,T} \quad (22)$$

where firm dummy α_i is included. The regression results are reported in panel B of [Table 2](#). The

option-implied expected returns continue to predict subsequent returns with even larger t-statistics after the firm fixed effect is accounted for. This result reassures that the option-implied measures significantly predict within-firm changes in the expected return through time.

In comparison, I also test the predictive ability of the conventional measures of the expected returns derived from the factor-based or characteristics-based models in specifications (3) through (5) and (8) through (10). The finding is that the associations between the conventional measures and subsequent returns are statistically significant, but the slope estimates are far below unity. For instance, in the panel regressions of six-month returns, the slope on the CAPM and the Fama French measure is 0.182 and 0.193, respectively, implying that these factor-based measures tend to substantially overshoot variations in the expected return. The characteristic-based expected return performs relatively better with the slope of 0.589, which is still noticeably lower than one.

Next, I examine the option-implied measures' performance in out-of-sample forecast. This out-of-sample performance will be of particular interest to firm managers who must update the cost of equity regularly using up-to-date data. Following [Welch and Goyal \(2008\)](#), I measure the performance by computing the out-of-sample R^2 :

$$R_{\text{OOS}}^2 = 1 - \frac{\text{SSE}_{\text{model}}}{\text{SSE}_{\text{avg}}} \quad (23)$$

where $\text{SSE}_{\text{model}}$ is the sum of squared errors in predicting subsequent returns with a forecast model, and SSE_{avg} is the sum of squared errors in predicting the returns with the five-year historical mean. To compute the forecast error of the option-implied equity premium, I preset the coefficients in equation (21) such that $\alpha = 0$ and $\beta = 1$, as the theory states.

Table 3 presents the out-of-sample results. I find that the option-implied measures far outperform the conventional benchmarks in predicting both six-month and one-year returns. For instance, the recovery-based measure (lower-bound measure) explains 17.52% (17.45%) of the total variance of one-year return. This performance is strikingly superior to the CAPM measure's -167% and the Fama-French measure's -175%; these negative values of R_{OOS}^2 indicate that the factor-based measures underperform even the historical-mean model. In fact, the factor models' poor performance is consistent with the finding of Table 2 that they noticeably overestimate changes in the expected return. Meanwhile, the characteristic-based model performs relatively better with the R_{OOS}^2 of 9.16%, confirming [Lewellen \(2015\)](#)'s finding.

This horse-race result highlights the advantage of the option-implied measures. These measurements rely only on the recent prices of financial assets. As a result, they are capable of quickly detecting changes in return-relevant information. On the contrary, in the conventional return models and the accompanying backward-looking estimation, recent information tends to be diluted by historical data, so the resulting return-estimate is less responsive to new information.

Finally, I investigate the compatibility between the two option-implied expected returns. Despite the difference in the theoretical designs of these two measures, the two should result in similar estimates if they contain the same return-relevant information. To check the compatibility, I run the following contemporaneous regression:

$$\text{ER}_{i,t}^{\text{recovery}} = \alpha + \beta \times \text{ER}_{i,t}^{\text{lower bound}} + \epsilon_{i,T} \quad (24)$$

and also run the panel regression that includes the firm fixed effect.

The regression results are presented in Table 4. Across different specifications, the regression slopes are significantly positive and close to one. The slopes are 1.281 (1.120) for the six-month horizon and 0.684 (0.594) for the one-year horizon in the pooled regression (in the panel regression). This result shows that the expected returns measured by the two approaches fluctuate in a synchronized manner and they also change in similar magnitudes. In addition, the R^2 values are impressively large. For instance, the statistic for the six-month horizon is 65.99% in the pooled regression and 72.63% in the panel regression, showing that the two measures are highly correlated. Based on the above properties, I argue that these two-option implied measures are almost equivalent empirically in detecting dynamics in the expected return.

To summarize, I have established that both the recovery-based and lower bound estimates of the expected returns are an empirically valid measure of the cost of equity. From now on, I employ the two measures to investigate the link from cost of capital to corporate investment.

3.4. The Cost of Capital and Capital Investment

3.4.1. The Option-Implied Cost of Equity

I now examine whether firms take the fluctuating cost of capital into account in their decisions on capital investment. As described in section 2.1, theoretically, a rise in cost of capital lowers the expected present value of marginal profits to investment (marginal q). Thus, if firms correctly

respond to changes in the cost of capital, I expect to find a negative response of investment. Among the components constituting cost of capital, I first focus on the cost of equity and its impact on investment.

To test the hypothesis, I conduct the following panel regression:

$$\text{INVEST}_{i,t} = \beta \times \text{ER}_{i,t-1} + \gamma \times X_{i,t-1} + \alpha_i + \epsilon_{i,t+1} \quad (25)$$

where $\text{INVEST}_{i,t}$ is firm i 's investment-capital ratio in quarter t , $\text{ER}_{i,t-1}$ is the quarter $t-1$ estimate of the cost of equity for the six-month horizon¹³ – either $\text{ER}_{i,t-1}^{\text{recovery}}$ or $\text{ER}_{i,t-1}^{\text{lower bound}}$ – and $X_{i,t-1}$ contains the control variables. The controls include Tobin's q ($Q_{i,t-1}$), firm size ($\text{SIZE}_{i,t-1}$), the leverage ratio ($\text{LEV}_{i,t-1}$), the risk-free return (r_{t-1}^f), the bond yield ($\text{YIELD}_{i,t-1}$), and cash flow-to-asset ratio ($\text{CF}_{i,t-1}$).

Notice that the above specification does not separate the cost-of-capital component and the productivity component, as would be ideal. Instead, Tobin's q , which is the composite of the two components, is included along with the cost of equity. I start with this specification to see whether the cost of equity can forecast the investment in the presence of other usual determinants noted in the literature.¹⁴ Section 3.4.3 will describe the investment analysis when the two components are properly separated.

Table 5 presents the regression results. The main finding is the option-implied cost of equity, whether it is measured by the recovery-based approach in specification (1) or the lower-bound approach in specification (2), negatively predicts capital investments. This negative association is statistically significant at the 1% level, with t-statistics of -4.06 for the recovery-based measure and -4.84 for the lower-bound measure. This finding uncovers that firms reduce (increase) their capital investment when the cost of equity increases (decreases), as the theory predicts. Furthermore, the association is economically significant. The coefficient estimate suggests that a within-firm increase in the lower-bound cost of equity by one standard deviation decreases investment by 4.5% (e.g., the annual investment-capital ratio changes from 0.211 to 0.201). Similarly, a one-standard-deviation

¹³I calculate the average of the monthly estimates to obtain the quarterly equity premium.

¹⁴Although Tobin's q is the composite measure, its cost-of-capital component is likely to be subsumed into the cost of equity and the cost of debt, both of which also enter the regression specification. As a result, I expect the coefficient on Tobin's q to mainly reflect the effect of the productivity component.

increase in the recovery-based cost of equity decreases investment by 4.2%. These impacts are quantitatively similar to the impact of Tobin's q ; a within-firm increase in Tobin's q by one standard deviation increases investment by 6.6%

Next, temporarily stepping back from the option-implied measures, I investigate how investment reacts to the conventional measures of cost of equity, the estimates from the CAPM, Fama-French five-factor model, and characteristics-based model. The idea behind this analysis is that if firm managers relied entirely on the CAPM or the multi-factor models in determining the cost of capital, these conventional measures would prevail as a determinant of capital investment and the option-implied measures might lose their predictive power.

Specifications (3) and (4) report that both measures of cost of equity from the CAPM and the five-factor model also negatively predict investment. This finding supports the survey result from [Graham and Harvey \(2001\)](#) that firm managers refer to these asset-pricing models to identify the cost of capital. At the same time, however, this negative association is contrary to the finding of [Frank and Shen \(2016\)](#) that these measures of the discount rate are puzzlingly positively associated with investment. I conjecture that this discrepancy results from different implementations of the factor models. Specifically, the prior study estimates the factor risk premium from each firm's time-series of returns, whereas I estimate it from the Fama-MacBeth cross-sectional analysis. On the whole, this fluid performance of the factor-based estimates manifests their disadvantage as a cost-of-capital measure. In the meantime, the characteristic-based measure in specification (5) is insignificant for predicting investment.

In specifications (6) and (7), I take a "kitchen sink" approach and include multiple measures of the cost of equity. It turns out the option-implied measures continue to negatively and strongly predict investment at the 1% level, even after the conventional measures are taken into account. This finding suggests that in capital budgeting practice, firm managers do not depend entirely on the CAPM or the multi-factor model. Instead, they appear to make substantial adjustments to the cost of capital beyond what these conventional frameworks suggest, and these adjustments help them properly update the discount rate for investment decisions.

3.4.2. *Weighted Average Costs of Capital*

The cost of equity depends on a firm’s financing choice according to the Modigliani-Miller theorem. In this section, I employ the “unlevered” version of the cost of capital, the WACC, as a determinant of capital investment. This exercise is to separate the financing impact and to focus on the role of the cost of capital associated with fundamental business risk. Letting the WACC be the composite measure, I conduct the regression

$$\text{INVEST}_{i,t} = \beta \times \text{WACC}_{i,t-1} + \gamma \times X_{i,t-1} + \alpha_i + \epsilon_{i,t+1}. \quad (26)$$

Note that the cost of debt and the risk-free rate are now merged into the WACC. The controls are otherwise the same as specification (25).

Table 6 presents the regression results. In specifications (1) and (2) where the option-implied costs of equity are entered into the unlevered cost of capital, the WACC estimates are a strong and negative predictor of investment at a 1% level of significance. The finding suggests that changes in the overall cost of capital, beyond the cost of equity, are taken into account in the investment practice, consistent with the theoretical guidance.

In comparison, I test the predictive ability of the WACC from the conventional measures of the cost of equity in specifications (3) through (5). It appears that the WACC derived from the CAPM or five-factor estimates also negatively predicts investment, exhibiting a similar statistical significance to the option-implied estimates. In other words, when it comes to the qualitative association with investment, the conventional measures seem to perform as well as the option-implied measures. In the following section, I take a further step toward contrasting these measures focusing on a quantitative aspect of the association.

3.4.3. *Decomposition of Marginal Q and Investment Sensitivity*

The analyses up to this point have established the qualitative relationship that the option-implied cost of capital negatively forecasts firm investment. The theoretical model further disciplines the relationship and makes the quantitative prediction – investment should be equally responsive to the cost-of-capital component and the productivity component described in equation (7) of lemma 2.1. Motivated by this prediction, in this section, I examine the empirical sensitivity of investment to the two component of the marginal q .

The decomposition of the marginal q requires the estimate of the cost of capital and the estimate of future productivity. Among the required inputs, the option-implied cost of equity has been proven empirically valid in section 3.3. Then, as the lemma guides, the resulting estimate of the weighted average costs of capital determines the cost-of-capital component $L(A)_{i,t}$. To apply a similar standard to the productivity estimate described in section 2.3, I first test its empirical ability to predict the subsequently realized productivity. After its empirical validity is ensured, I proceed toward identifying the productivity component of the marginal q .

I conduct the pooled regression of the realized productivity growth $\Delta a_{i,t+1}$ with the specification

$$\Delta a_{i,t+1} = \alpha_i + \beta \times \mathbb{E}_t^F [\Delta a_{i,t+1}] + \epsilon_{i,t+1} \quad (27)$$

where $\mathbb{E}_t^F [a_{i,t+1}]$ is the date- t expectation of future productivity derived from the forward prices. In the result reported in table 7 specification (1), the estimate of the expected growth in productivity indeed forecasts the realized growth with the t-statistic of 7.54. Furthermore, in specification (2), all of the inputs constituting the expected growth in productivity – the expected growth in dividend $\mathbb{E}_t^F [\Delta d_{i,t+1}]$ and the log of the price-dividend ratio pd_t – are strong predictors as the model states.

At the same time, I find that this productivity estimate does not provide a quantitatively perfect prediction. The regression slope is 0.04, indicating that the estimate tends to overshoot changes in actual productivity growth. Possibly, this result comes from the imperfect design of the estimate that I let the long-run average of productivity growth approximate the growth in the next quarter. Acknowledging this limitation, I employ the fitted value from the above regression, instead of $\mathbb{E}_t^F [a_{i,t+1}]$ itself, as a more sensible estimate of the expected growth. Multiplying the growth estimate by the current productivity, I identify the date- t expectation of future productivity, which enters the productivity component $L(A)_{i,t}$.

Having identified the cost-of-capital component and the productivity component, I investigate the investment's response to them under the specification

$$\log(\text{INVEST}_{i,t}) = \alpha_i + \beta_1 \times L(R)_{i,t-1} + \beta_2 \times L(A)_{i,t-1} + \epsilon_{i,t}. \quad (28)$$

According to lemma 2.1, I expect to find that $\beta_1 = -\beta_2$ if firms are equally sensitive to the two

components of the marginal q .

Table 8 presents the regression results. In specifications (1) and (2) where $L(R)_{i,t-1}^{\text{recovery}}$ and $L(R)_{i,t-1}^{\text{lower bound}}$ are derived from the option-implied measures of cost of equity, I find that both the cost-of-capital component and productivity component are strongly significant mostly at the 1% level, consistent with the theory. Notably, this empirical result contrasts with [Abel and Blanchard \(1986\)](#)'s finding that the cost-of-capital component of marginal q is insignificant in explaining investment. I conjecture that this discrepancy mainly results from different measurements of cost of capital. The prior study uses the ex-post returns as a proxy for the cost of equity, but this measure is subject to concerns that it cannot separate the subsequent return variation that investment itself causes conversely. Instead, I employ the ex-ante estimate of the required return, which is free of the reverse impact of investment by its design.

More importantly, the regression coefficients uncover that the cost-of-capital component is quantitatively as crucial as productivity component in determining investment. The coefficient on $L(R)_{i,t-1}^{\text{recovery}}$, which utilizes the recovery-based measure of expected return, is -0.898 and the coefficient on $L(R)_{i,t-1}^{\text{lower bound}}$, which utilizes the lower bound measure, is -1.030. These slopes are of similar magnitude in the absolute value to the coefficients on the productivity component, 1.373 in specification (1) and 1.403 in (2). Furthermore, the equal sensitivity of investment is confirmed by formal tests of the hypothesis $\beta_1 = -\beta_2$. In both specifications, this null hypothesis is accepted with the p-value for Wald' test 0.34 or 0.56. All of these findings suggest that a unit decrease in the cost-of-capital component raises firms' investment as much as a unit increase in productivity component does. This discovery is intriguing considering that in capital budgeting practice, managers customarily tend to focus more on predicting future cash flows and devote less attention to determining discount rates.

The above findings seem obvious from theoretical perspectives. I however find that the empirical results vary substantially across different measurements of cost of capital. In specifications (3) through (5), the investment is regressed on the alternative measures of the cost-of-capital components, which are derived from the conventional estimates of the cost of equity. Interestingly, the conventional measures end up with reporting a far lower economic significance of the cost-of-capital component. The coefficients on $L(R)_{i,t-1}$ range from -0.126 to -0.037, of which absolute values are conspicuously below the coefficients on $L(A)_{i,t-1}$ ranging from 1.421 to 1.451. As a result, the null

hypothesis that $\beta_1 = -\beta_2$ is rejected in all of the specifications (3) through (5).

In fact, these remarkably different results between the option-implied measures and the conventional measures of the cost of capital are not surprising considering their empirical validity reported in section 3.3. The option-implied expected returns have been found to predict the subsequent returns, in both magnitude and direction. As valid measures of cost of equity, they are capable of correctly detecting the quantitative aspect of investment response. On the contrary, the conventional measures of expected returns lack the ability to capture the magnitude of return changes in subsequent periods. Hence, the analyses based on these conventional measures as cost of capital lead to incorrect descriptions of the investment.

To summarize, my analysis establishes the empirical link that corporate investment responds negatively to fluctuations in both cost of equity and the overall cost of capital. Furthermore, the empirical investment is equally sensitive to the cost-of-capital component and productivity component, as the theory predicts.

3.5. Robustness Test: Alternative Forces for Capital Investment

It may seem surprising that firms empirically respond to the option-implied measures of cost of capital. Despite the measures' empirical validity, it is not likely that the required estimation has actually been conducted in corporate practice. Accordingly, one could suspect that the main finding might be spuriously driven by alternative forces other than cost of capital. This concern is legitimate. Due to its counter-cyclical nature as shown in Figure 1, the cost of capital tends to drive capital investment pro-cyclical. However, the literature has noted other forces beyond the cost of capital – financial constraints or irrational sentiment – as causing a similarly cyclical pattern in investment. In this section, I include these two forces and examine whether the cost of capital continues to explain investment.

3.5.1. Financial Constraints

Financial constraints affect capital investment by limiting firms' ability to spend. This effect is distinct from the channel of the cost of capital; a shift in cost of capital changes the fair value of investment projects, whereas financial constraints restrict firms' spending irrespective of the project values. However, these two forces are likely to generate similar investment patterns over economic cycles. As [Gertler and Gilchrist \(1994\)](#) and [Kashyap et al. \(1994\)](#) document, firms tend to be more

financially constrained during recessions, thus investing pro-cyclically. Therefore, it is a possible scenario that firms actually respond to financial constraints not to fundamental business risks and related discount rate.

Having this alternative explanation in mind, I include various measures of financial constraints in addition to the WACC in regression (26). Table 9 reports the regression results in specifications (1) through (6). I find that the impact of financial constraints varies across specifications. The WW-index in specifications (2) and (5) negatively predicts investment as expected, but the coefficients on the KZ-index in (1) and (4) and the HM-index in (3) and (6) are insignificant. This insignificance, in particular of the HM-index, appears to contrast with the result of [Hoberg and Maksimovic \(2014\)](#). I conjecture that this discrepancy may arise from the sampling difference; my sample consists of relatively large companies that they find less constrained according to the HM index.

Importantly, the WACC maintains the ability to predict investment at the 1% level of significance for all specifications. Moreover, the inclusion of financial constraints only slightly changes the point estimates on the cost of capital. This result confirms that the negative link between the capital investment and cost of capital is not spuriously driven by firms' response to financial constraints.

3.5.2. Irrational Sentiment

Another force causing capital investment to swing is investors' sentiment. According to [Baker and Wurgler \(2006\)](#), irrational sentiment affects investors' valuation of stocks. Consistently, [Arif and Lee \(2014\)](#) find that corporate investment at the aggregate level reacts to wave of investors' sentiment.¹⁵ Thus, one could suspect that the main findings in section 3.4 are in fact driven by irrational sentiment, which possibly influences both option prices and capital investment decisions.

To address this alternative explanation, I additionally control for investors' sentiment in the investment regression. As a measure of investors' sentiment, I use either of two indicators, one at the economy level and the other at the firm level. The economy-wide indicator is [Baker and Wurgler \(2006\)](#)'s sentiment index. The firm-level indicator is constructed from the option-implied expected returns and the realized returns. Specifically, to measure the sentiment in the date- t valuation of

¹⁵[Arif and Lee \(2014\)](#) provide two hypotheses on why investors' sentiment affects corporate investments. First, corporate managers may be subject to the same sentiment in valuations as investors in the market. Second, they may value rationally but exploit misvaluations in the market, such as investing more when investors are optimistic and require a lower cost of capital than they should rationally. Both of these hypotheses predict the same positive association between sentiment and corporate investment.

firm i 's stock, I compute an ex-post forecast error, $\text{Error}_{i,t \rightarrow T}$, in predicting stock returns over the period from t to T as follows:

$$\text{Error}_{i,t \rightarrow T} = R_{i,t \rightarrow T}^e - \text{ER}_{i,t}. \quad (29)$$

The idea behind this measure is that date t 's irrational optimism (pessimism) about stock i and its over-valuation (under-valuation) will be accompanied by a negative (positive) forecast error, as the irrational component in the valuation will be corrected later. In other words, a subsequent negative forecast error reveals optimism at date t in valuing stock i . Employing the expected return from either the recovery-based approach or the lower-bound approach, I have two measures of the forecast error that indicates firm-level sentiment.

Table 9 presents the regression results in specifications (7) and (10). The economy-wide indicator of sentiment in (7) and (9) has a strong significance and positively predicts investment, as the sentiment hypothesis suggests. Consistently, the ex-post forecast error in specifications (8) and (10) is negatively associated with investment; a firm tends to invest more when investors are optimistic about the firm, as reflected by lower forecast errors. These results confirm the finding of the prior studies that irrational sentiment influences firm investment.

Turning to the main interest of this study, I find that the WACC continues to negatively predict investment at the 1% level of significance. Interestingly, this result contrasts with the finding of [Arif and Lee \(2014\)](#). Using proxies of the cost of equity, such as credit spreads on corporate bonds and the difference between short-term and long-term treasury rates, they find that the cost of equity becomes an insignificant determinant of investment once the sentiment measures are taken into account. In contrast, my analysis based on the direct measures of the cost of equity reveals that the predictive ability of the cost of capital is robust to the inclusion of the sentiment indicators. This evidence helps to assure that the empirical link between investment and the cost of capital is distinct from the sentiment impact.

3.5.3. *Uncertainty*

Recent studies by [Bloom \(2009\)](#), [Kim and Kung \(2017\)](#), and [Alfaro et al. \(2017\)](#) have documented that firms tend to reduce capital investment when facing a higher level of business uncertainty. For this empirical finding, two possible explanation exist. One possibility is the real-option consideration

as in [Dixit and Pindyck \(1993\)](#). Specifically, the “option-like” value of deferring investment is higher in times of higher uncertainty, so firms invest less. The other possible channel is the uncertainty’s influence on the cost of capital. Because the uncertainty level tends to correlate positively with the quantity of risk, elevated uncertainty is likely to increase the cost of capital.

This multilateral effect of uncertainty may lead to concern that the main finding, the link from the cost of capital to investment, might misleadingly capture firms’ reaction to the real-option value distinct from the discount-rate effect. To address this concern, I isolate the cost-of-capital component that is not driven by the uncertainty level. Focusing on this particular component that is orthogonal to the real-option consideration, I aim to establish the investment’ response to the cost of capital apart from the alternative force.

The starting point of this analysis is the recognition that a firm’s cost of capital depends on both the quantity of risk facing the firm and the market price of risk, or the amount of compensation that investors require for taking a unit of risk, as stated in prior studies including [Cogley and Sargent \(2008\)](#) and [Ludvigson \(2013\)](#). Having this in mind, I identify changes in the cost of capital driven by the price of risk, which could be considered plausibly independent of the uncertainty. Using this instrumented cost, I run the investment regression.

Implementing this strategy requires a measure of the market price of risk. Under the assumption that the S&P 500 Index fairly represents the market portfolio, I use the option-implied expected return on the index and the standard deviation of its returns to measure the price of risk.¹⁶ Then, the price of risk is defined as

$$\text{Price of Risk}_t = \frac{\text{ER}_{t,\text{market}}}{\sigma(R)_{t,\text{market}}} \quad (30)$$

where $\sigma(R)_{t,\text{market}}$ is the annualized standard deviation of realized daily returns on the index for the past year. Having two estimates of the market’s expected return from the recovery-based approach and lower-bound approach, I also have two different measures of the price of risk.

I conduct the two-stage least squares regression to examine the connection between capital investment and the instrumented cost of capital. [Table 10](#) panel (a) reports the first stage results. There, the relevance of the instruments is confirmed. The price-of-risk estimates from the two

¹⁶Note that the estimation techniques, the recovery-based approach and the lower-bound approach, can also be applied to option prices of the S&P 500 Index. This leads to the date- t expected return on the market portfolio.

option-implied approaches positively predict the firm-level cost of capital at the 1% level. The F-statistics are 53.19 (33.20) for the price of risk estimated by the recovery-based approach (lower-bound approach).

Next, panel (b) reports the second-stage results. I find the negative response of investment to the uncertainty, measured from either realized returns in specifications (1) and (3) or the implied volatility from options in specifications (2) and (4). The result is consistent with the prior studies' finding that firms invest less in times of higher uncertainty. More importantly, the coefficients on the instrumented cost of capital are negative at the 1% level of significance in all specifications. This finding shows that an increase in the cost of capital stemming from shifts in the price of risk causes firms to reduce their investments. Besides, the correlation coefficients between the second-stage regression residuals and the instruments support the exclusion requirement for the instruments. The correlation is low, ranging from 0.001 to 0.002.

3.6. Which Firms Respond to Time-Varying Cost of Capital?

Do all firms respond unanimously to the dynamic cost of capital? Although the previous sections document that firms, by and large, adjust their discount rate correctly to the time-varying cost, it is possible that firms update the discount rate with different levels of accuracy. This is because considering the failure of the CAPM to update properly, firm managers must rely on their own subjective adjustment to do so; the subjective adjustment actually takes effect in capital budgeting practice according to [Graham et al. \(2015\)](#) and [Jagannathan et al. \(2016\)](#). If firms are indeed heterogeneous in responding to the cost of capital as hypothesized, it is worth exploring which firms respond particularly well and which firm characteristics are associated with those. This analysis can elucidate the underlying mechanism for how firms recognize the cost fluctuations.

Here, I conduct a two-step analysis. First, to check for heterogeneity across firms in adjusting their investment, I test the responsiveness of each individual firm to the cost of equity. Specifically, I run a time-series regression of investment for each firm, changing the firm dummy variable to a constant in equation (25), and examine the significance of the cost of equity. This firm-specific regression may raise concerns about a limited number of observations, which may mechanically lead to a low statistical significance. To address these concerns, I include only firms with at least 30 quarterly observations of all relevant variables. As a result, 131 firms are in the final sample.

Figure 2: T-statistics for Cost of Equity in Firm-Specific Time-Series Regressions

This figure presents the histogram of the t-statistics for the recovery-based cost of equity in the firm-specific time-series regressions. Included are 131 firms with at least 30 quarterly observations of all relevant variables. For each firm, a time-series regression is performed to predict its capital investment with regression equation (25). In the figure, the x-axis is the estimated t-statistics for the cost of equity, and the y-axis is the ratio of the number of firms that belong to each bin to the total number of firms.

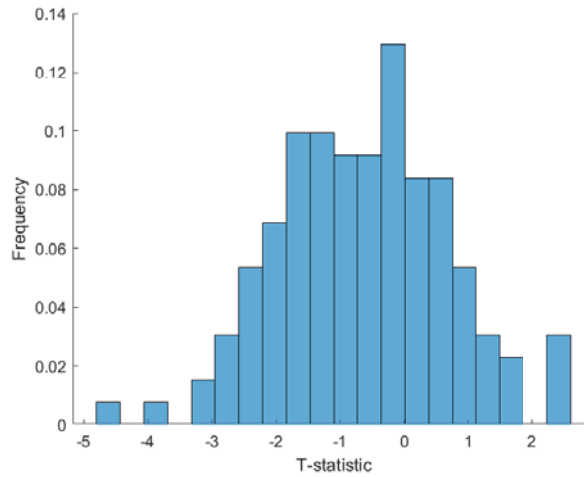


Figure 2 illustrates the results of the first step. The histogram shows the distribution of the 131 firm-specific estimates of t-statistics for the recovery-based cost of equity. While a majority of firms, 91 out of 131, show a negative relation between investment and the equity premium, a cross-sectional dispersion exists in the statistical significance of the relation. The 25th percentile, the median, and the 75th percentile of the t-statistic are -1.68, -0.47, and 0.27, respectively.

Then, the 131 firms are classified into two groups: “responders”, who negatively respond to the equity premium with a 10% or lower level of significance (t-statistic < -1.65), and “non-responders”, whose response is not significant or positive. Thirty-four firms are classified as the responders. Having these two group of firms, the second step is to run a logistic regression that enables me to identify the firm characteristics connected to the responders. In the logistic regression, the dependent variable is one if a firm is a responder and zero otherwise.

As for firm characteristics, I consider the following variables: age, size, indicators of financial constraints, and dependence on external financing/equity financing. For each firm, I compute the time-series average of the above variables. The rationale for using these characteristics is as follows.

In the absence of a formal framework to properly update the cost of capital, I conjecture that a learning-by-doing mechanism may be at work in this practice. In a different context of corporate practice, learning based on experience is found to boost firms' productivity, according to prior studies by [Benkard \(2000\)](#) and [Thornton and Thompson \(2001\)](#). More related to this study, [Berk et al. \(2004\)](#) propose the model where firms' investment make them better informed about the value of R&D projects. Extending these to the context of capital budgeting, I expect old firms to respond better (age). If experience matters, firms with more direct exposure to external financing might become more disciplined and respond better (dependence on external financing/equity financing). If determining cost of capital requires scrutiny, larger firms might respond better as they have more human resources (size). Alternatively, the responsiveness to the cost of capital might be pronounced for financially constrained firms because the indicators of financial constraints in the regression might not capture the impact to the full extent (indicators of financial constraints).

Table 11 reports the logistic regression results. Among the firm characteristics considered, the dependence on equity financing is the most strongly connected to the responders; in all specifications (4) through (6), the coefficient on equity financing is significantly positive at a 5% level. The next significant characteristics are the dependence on external financing and the firm's age. These characteristics are also associated with responders with a 5% level of significance in all but one specification.

These findings suggest that experience matters in firms' recognition of the dynamic cost of capital. In particular, firms' direct experience in raising equity capital, measured by the dependence on equity finance, helps them better identify the fluctuations in cost of equity. Interestingly, this result aligns with [Thornton and Thompson \(2001\)](#)'s finding that cross-plant learning is less effective for productivity growth than a plant's own experience. In the context of capital budgeting, firms could have learned from their peers to update discount rates, considering the high correlation of cost of capital among industry peers. Nonetheless, firms' own experience of issuing equity still appears to matter in learning. In addition, older firms with longer experience in capital budgeting tend to respond better. Meanwhile, none of the indicators for financial constraints distinguishes between the responders and non-responders. This result again confirms that firms' response to cost of equity is not spuriously driven by the force of financial constraints.

This logit analysis is a rather exploratory test, and I do not expect the considered variables to

capture the comprehensive aspects of how firms become aware of the dynamics in the cost of capital. For example, the awareness can be related to firm managers' personal characteristics such as their educational background or professional experience in the industry. I leave an exploration of this question to future research.

4. Conclusion

This study empirically investigates the temporal relationship between the cost of capital and capital investment. Facing fluctuation in the cost of capital, whether or not a firm reacts to it matters for firm value. Ignoring the time variation in the cost would result in sub-optimal investment that can cost as much as 11% of the total firm value (Kim and Routledge (2019)).

For the analysis, I first construct the conditional cost of equity using individual equity option prices. The new measures of cost of equity far outperform the conventional measures in identifying the cost variations over time. These option-implied measures are a strong predictor of subsequent stock returns in both in-sample and out-of-sample tests.

The main findings are as follows. First, the cost of capital, whether it is decomposed into the cost of equity and the cost of debt or aggregated as the WACC, negatively predicts capital investment. Second, once the marginal value of capital is decomposed, the empirical investment is equally responsive to the cost-of-capital component and the productivity component, as the theory predicts. All of these findings suggest that firm managers correctly update their discount rates despite the conventional frameworks such as the CAPM or multi-factor models being of little help in this regard.

Nevertheless, this study is far from providing a complete picture of how managers recognize the fluctuation in the cost. My exploratory test suggests that firms' experience in external financing may help tune their investment decisions. If such a learning mechanism indeed exists, an interesting interaction could emerge between firms' optimal investment and financing choices beyond what the literature has noted so far. I leave an exploration of this question to future research.

Table 1: Summary Statistics

This table presents the descriptive statistics of the annualized variables.

Variable	Mean	Std.Dev.	25%	50%	75%
<i>Firm-level variables</i>					
$ER_{i,t}^{\text{recovery}}$ (%)	8.928	5.329	5.161	7.721	11.451
$ER_{i,t}^{\text{lower bound}}$ (%)	5.042	5.262	2.102	3.611	5.975
$ER_{i,t}^{\text{CAPM}}$ (%)	6.011	10.732	-0.255	2.709	8.798
$ER_{i,t}^{\text{FF}}$ (%)	7.845	12.517	0.204	6.817	14.797
$INVEST_{i,t}$	0.211	0.145	0.119	0.179	0.261
$SIZE_{i,t}$	4.226	1.120	3.443	4.157	4.967
$LEV_{i,t}$	0.268	0.146	0.164	0.244	0.342
$Q_{i,t}$	1.980	1.012	1.336	1.710	2.314
$YIELD_{i,t}$ (%)	4.897	3.053	3.034	4.490	5.850
$KZ \text{ index}_{i,t}$	0.246	1.684	-0.218	0.372	0.928
$WW \text{ index}_{i,t}$	-0.231	0.094	-0.281	-0.238	-0.185
$HM \text{ index}_{i,t}$	-0.013	0.083	-0.073	-0.007	0.049
External Finance _i	-2.545	2.317	-3.655	-1.899	-0.850
Equity Finance _i	-0.567	0.795	-0.900	-0.360	-0.018
$L(R)_{i,t}^{\text{recovery}}$	-0.015	0.040	-0.015	-0.002	0.002
$L(A)_{i,t}^{\text{recovery}}$	-0.002	0.061	-0.015	-0.001	0.012
$L(R)_{i,t}^{\text{lower bound}}$	-0.006	0.017	-0.012	-0.002	0.006
$L(A)_{i,t}^{\text{lower bound}}$	-0.002	0.063	-0.015	-0.001	0.012
$\sigma(R)_{i,t}$ (%)	31.432	18.669	19.526	26.542	37.258
<i>Market-wide variables</i>					
$ER_{\text{market},t}^{\text{recovery}}$ (%)	6.189	3.091	3.865	5.460	7.786
$ER_{\text{market},t}^{\text{lower bound}}$ (%)	3.908	2.190	2.241	3.469	4.767
Price of Risk _t ^{recovery}	0.322	0.128	0.238	0.295	0.384
Price of Risk _t ^{lower bound}	0.200	0.083	0.146	0.189	0.241

Table 2: The Option-Implied Expected Returns, the Benchmark Measures and Subsequent Stock Returns

This table presents regressions of realized stock returns on the estimates of expected returns. The dependent variable is the realized excess return over six months or one year from time t . The explanatory variable in each regression is one of the followings: the recovery-based measure ($ER_{i,t}^{\text{recovery}}$), the lower-bound measure ($ER_{i,t}^{\text{lower bound}}$), CAPM measure ($ER_{i,t}^{\text{CAPM}}$), Fama-French measure ($ER_{i,t}^{\text{FF}}$) or characteristic-based measure ($ER_{i,t}^{\text{character}}$) for the corresponding horizon. Panel A reports the result of the pooled regression for the specification

$$R_{i,T}^e = \alpha + \beta \times EP_{i,t} + \epsilon_{i,T}.$$

Panel B reports the result of the panel regression including firm fixed effects as follows:

$$R_{i,T}^e = \alpha_i + \beta \times EP_{i,t} + \epsilon_{i,T}.$$

The t-statistics are presented in parentheses below the parameter estimates. *, **, *** denotes significance at 10%, 5%, 1%, respectively. The panel A's second and third last rows report p-values for Wald's test of hypothesis.

Panel A. Pooled Regression

Dependent variable:	$R_{i,T}^e$									
Return horizon:	6 months	6 months	6 months	6 months	6 months	12 months	12 months	12 months	12 months	12 months
Specifications:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$ER_{i,t}^{\text{recovery}}$	0.459*** (6.39)					1.052*** (8.39)				
$ER_{i,t}^{\text{lower bound}}$		0.898*** (9.07)					0.968*** (9.66)			
$ER_{i,t}^{\text{CAPM}}$			0.185*** (34.13)					0.100*** (29.95)		
$ER_{i,t}^{\text{FF}}$				0.196*** (36.06)					0.105*** (31.40)	
$ER_{i,t}^{\text{character}}$					0.246** (2.70)					0.200* (1.94)
const.	0.0217*** (4.51)	0.0328*** (11.86)	0.0508*** (21.74)	0.0506*** (21.61)	0.0615*** (24.84)	0.0170 (1.65)	0.0693*** (12.20)	0.114*** (21.78)	0.114*** (21.74)	0.126*** (23.18)
N	48,203	56,150	56,150	56,150	56,150	38,826	53,044	53,044	53,044	53,044
$H_0 : \alpha = 0 \beta = 1$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$H_0 : \beta = 1$	0.00	0.30	0.00	0.00	0.00	0.68	0.75	0.00	0.00	0.00
adj. R^2	0.006	0.012	0.051	0.056	0.001	0.014	0.018	0.022	0.023	0.001

Panel B. Panel Regression

Dependent variable:	$R_{i,T}^e$									
Return horizon:	6 months	6 months	6 months	6 months	6 months	1 year	1 year	1 year	1 year	1 year
Specifications:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$ER_{i,t}^{\text{recovery}}$	0.721*** (8.75)					1.333*** (9.30)				
$ER_{i,t}^{\text{lower bound}}$		1.251*** (12.71)					1.291*** (13.22)			
$ER_{i,t}^{\text{CAPM}}$			0.182*** (33.63)					0.0984*** (29.39)		
$ER_{i,t}^{\text{FF}}$				0.193*** (35.47)					0.103*** (30.79)	
$ER_{i,t}^{\text{character}}$					0.589*** (5.10)					0.537*** (4.07)
N	48,203	56,150	56,150	56,150	56,150	38,826	53,044	53,044	53,044	53,044
adj. R^2	0.043	0.048	0.081	0.086	0.034	0.079	0.080	0.077	0.078	0.060

Table 3: Out-of-Sample Forecast of the Option-Implied Expected Returns and the Benchmark Measures

This table presents out-of-sample forecast performances of the option-implied expected returns and the benchmark measures. The option-implied expected returns are the recovery-based measure ($ER_{i,t}^{\text{recovery}}$) and the lower-bound measure ($ER_{i,t}^{\text{lower bound}}$). The benchmark measures are the estimates of the expected return derived from the CAPM ($ER_{i,t}^{\text{CAPM}}$), the Fama-French five factor model ($ER_{i,t}^{\text{FF}}$), and the characteristic-based model ($ER_{i,t}^{\text{character}}$). Each measure of the expected return serves as the predictor in the return-forecast model with the specification

$$R_{i,T}^e = ER_{i,t} + \epsilon_{i,T}.$$

The performance of each measure is measured by out-of-sample R^2 , which is defined as

$$R_{\text{OOS}}^2 = 1 - \frac{\text{SSE}_{\text{model}}}{\text{SSE}_{\text{avg}}}$$

where $\text{SSE}_{\text{model}}$ is the sum of squared errors in predicting subsequent excess returns with a forecast model, and SSE_{avg} is the sum of squared errors in predicting with the five-year historical mean.

Return Horizon:	R_{OOS}^2 (%)	
	6 months	12 months
$ER_{i,t}^{\text{recovery}}$	2.71	17.52
$ER_{i,t}^{\text{lower bound}}$	5.80	17.45
$ER_{i,t}^{\text{CAPM}}$	-95.12	-175.01
$ER_{i,t}^{\text{FF}}$	-90.11	-67.31
$ER_{i,t}^{\text{character}}$	0.73	9.16

Table 4: Compatibility of the Two Option-Implied Expected Returns

This table presents the contemporaneous regressions using two option-implied measures of the expected return. In the pooled regression, the specification is

$$ER_{i,t}^{\text{recovery}} = \alpha + \beta \times ER_{i,t}^{\text{lower bound}} + \epsilon_{i,T}$$

where the estimates of the expected return are for the six month or one year horizon. In the panel regression, the firm dummy α_i is also included. The t-statistics are presented in parentheses below the parameter estimates. *, **, *** denotes significance at 10%, 5%, 1%, respectively.

Dependent variable: Return Horizon: Specifications:	ER _{i,t} ^{recovery}			
	6 months (1)	6 months (2)	12 months (3)	12 months (4)
const	0.043*** (47.63)		0.062*** (43.58)	
ER _{i,t} ^{lower bound}	1.281*** (61.25)	1.120*** (51.22)	0.684*** (37.21)	0.594*** (34.99)
fixed effect	No	Yes	No	Yes
adj-R ² (%)	65.99	72.63	54.15	63.16
observations	66,080	66,080	55,911	55,911

Table 5: The Cost of Equity and Capital Investment

This table presents panel regressions of capital investment on its determinants. The dependent variable is $INVEST_{i,t}$, firm i 's investment-capital ratio in quarter t . The regression specification is

$$INVEST_{i,t} = \alpha_i + \beta \times ER_{i,t-1} + \gamma \times X_{i,t-1} + \epsilon_{i,t}$$

where $ER_{i,t-1}$ is the cost of equity and $X_{i,t-1}$ denotes the control variables. The cost of equity is measured by the option-implied expected returns, $ER_{i,t-1}^{\text{recovery}}$ or $EP_{i,t-1}^{\text{lower bound}}$, or the conventional measures, $ER_{i,t-1}^{\text{CAPM}}$, $ER_{i,t-1}^{\text{FF}}$ or $ER_{i,t-1}^{\text{character}}$. The controls include Tobin's q ($Q_{i,t-1}$), the log of book value of total assets ($SIZE_{i,t-1}$), the book value of leverage ratio ($LEV_{i,t-1}$), the value-weighted yields on corporate bonds ($YIELD_{i,t-1}$), 10-year treasury constant maturity (r_{t-1}^f), and cash flow-to-asset ratio ($CF_{i,t-1}$). The t-statistics are presented in parentheses below the parameter estimates. *, **, *** denotes significance at 10%, 5%, 1%, respectively.

Dependent variable: Specification:	(1)	(2)	(3)	INVEST _{<i>i,t</i>}			
				(4)	(5)	(6)	(7)
$ER_{i,t-1}^{\text{recovery}}$	-0.0545*** (-4.06)					-0.0609*** (-4.54)	
$ER_{i,t-1}^{\text{lower bound}}$		-0.1170*** (-4.84)					-0.123*** (-5.01)
$ER_{i,t-1}^{\text{CAPM}}$			-0.0027*** (-3.42)				
$ER_{i,t-1}^{\text{FF}}$				-0.0026*** (-3.23)		-0.0021** (-2.62)	-0.00168** (-2.05)
$ER_{i,t-1}^{\text{character}}$					-0.0194 (-0.72)	-0.0554** (-1.98)	-0.0512* (-1.85)
$Q_{i,t-1}$	0.0089*** (5.47)	0.0086*** (5.34)	0.0094*** (5.66)	0.0094*** (5.66)	0.0095*** (5.82)	0.0084*** (5.28)	0.0082*** (5.19)
$SIZE_{i,t-1}$	-0.0013 (-0.41)	-0.0015 (-0.46)	-0.0002 (-0.07)	-0.0002 (-0.07)	-0.0001 (-0.04)	-0.0016 (-0.50)	-0.0017 (-0.52)
$LEV_{i,t-1}$	-0.0208 (-1.62)	-0.0206 (-1.62)	-0.0211* (-1.67)	-0.0212* (-1.67)	-0.0212* (-1.67)	-0.0191 (-1.49)	-0.0191 (-1.51)
$r_{f,t-1}$	0.0578 (0.91)	0.0405 (0.65)	0.0974 (1.58)	0.0973 (1.57)	0.0903 (1.47)	0.0614 (0.95)	0.0443 (0.70)
$YIELD_{i,t-1}$	-0.0690* (-1.75)	-0.0601 (-1.58)	-0.0957** (-2.50)	-0.0958** (-2.50)	-0.0965* (-2.50)	-0.0720* (-1.83)	-0.0639* (-1.68)
$CF_{i,t-1}$	0.0184 (0.67)	0.0130 (0.47)	0.0268 (0.99)	0.0269 (0.99)	0.0269 (0.98)	0.0153 (0.57)	0.0105 (0.39)
N	7,136	7,136	7,136	7,136	7,136	7,136	7,136
adj. R^2	0.553	0.554	0.551	0.551	0.550	0.554	0.555

Table 6: The Weighted Average Costs of Capital and Capital Investment

This table presents panel regressions of capital investment on its determinants. The dependent variable is $INVEST_{i,t}$, firm i 's investment-capital ratio in quarter t . The regression specification is

$$INVEST_{i,t} = \alpha_i + \beta \times WACC_{i,t-1} + \gamma \times X_{i,t-1} + \epsilon_{i,T}$$

where $WACC_{i,t}$ is the weighted average costs of capital derived from the cost of equity and corporate bond yield and $X_{i,t-1}$ denotes the control variables. The cost of equity is measured by the option-implied expected returns, $ER_{i,t-1}^{\text{recovery}}$ or $EP_{i,t-1}^{\text{lower bound}}$, or the conventional measures, $ER_{i,t-1}^{\text{CAPM}}$, $ER_{i,t-1}^{\text{FF}}$ or $ER_{i,t-1}^{\text{character}}$. The controls include Tobin's q ($Q_{i,t-1}$), the log of book value of total assets ($SIZE_{i,t-1}$), the leverage ratio ($LEV_{i,t}$), and cash flow-to-asset ratio ($CF_{i,t-1}$). The t-statistics are presented in parentheses below the parameter estimates. *, **, *** denotes significance at 10%, 5%, 1%, respectively.

Dependent variable: Specification:	INVEST _{<i>i,t</i>}				
	(1)	(2)	(3)	(4)	(5)
WACC _{<i>i,t-1</i>} ^{recovery}	-0.0365*** (-3.68)				
WACC _{<i>i,t-1</i>} ^{lower bound}		-0.0646*** (-3.62)			
WACC _{<i>i,t-1</i>} ^{CAPM}			-0.0014*** (-2.88)		
WACC _{<i>i,t-1</i>} ^{FF}				-0.0013** (-2.62)	
WACC _{<i>i,t-1</i>} ^{character}					0.0052 (0.33)
Q _{<i>i,t-1</i>}	0.0090*** (5.45)	0.0091*** (5.46)	0.0095*** (5.67)	0.0095*** (5.66)	0.0098*** (5.86)
SIZE _{<i>i,t-1</i>}	-0.0020 (-0.60)	-0.0022 (-0.64)	-0.0009 (-0.27)	-0.0009 (-0.27)	-0.0006 (-0.17)
LEV _{<i>i,t-1</i>}	-0.0273** (-2.06)	-0.0254* (-1.93)	-0.0228* (-1.72)	-0.0228* (-1.72)	-0.0228* (-1.71)
CF _{<i>i,t-1</i>}	0.0214 (0.74)	0.0181 (0.63)	0.0325 (1.16)	0.0327 (1.17)	0.0337 (1.19)
<i>N</i>	7,136	7,136	7,136	7,136	7,136
adj. <i>R</i> ²	0.549	0.550	0.547	0.546	0.546

Table 7: The Expected Future Productivity from Forward Prices and Realized Productivity

This table presents panel regressions of realized productivity on the expected future productivity. The dependent variable is the log of realized productivity growth from quarter t to $t + 1$, $\Delta a_{i,t+1}$. The expected future productivity is estimated from forward prices as in equation 19. In specification (1), the explanatory variable is the quarter- t estimate of the expected future productivity $\mathbb{E}_t^F [\Delta a_{i,t+1}]$. The regression specification is

$$\Delta a_{i,t} = \alpha_i + \beta \times \mathbb{E}_t^F [\Delta a_{i,t+1}] + \epsilon_{i,t}.$$

In specification (2), the inputs constituting the productivity estimate – the expected growth in dividend $\mathbb{E}_t^F [\Delta g_{i,t+1}]$ and the log of the price-dividend ratio pd_t – are entered separately. The t-statistics are presented in parentheses below the parameter estimates. *, **, *** denotes significance at 10%, 5%, 1%, respectively.

Dependent variable:	$\Delta a_{i,t+1}$	
Specification:	(1)	(2)
$\mathbb{E}_t^F [\Delta a_{i,t+1}]$	0.0403*** (7.54)	
$\mathbb{E}_t^F [\Delta d_{i,t+1}]$		0.0104** (2.37)
pd_t		0.0619*** (8.03)
N	15,289	15,289
adj. R^2	0.117	0.123

Table 8: The Decomposed Marginal Q and Capital Investment

This table presents panel regressions of capital investment on the decomposed marginal q . The dependent variable is $\log(\text{INVEST}_{i,t})$, the log of firm i 's investment-capital ratio in quarter t . The regression specification is

$$\log(\text{INVEST}_{i,t}) = \alpha_i + \beta_1 \times L(R)_{i,t-1} + \beta_2 \times L(A)_{i,t-1} + \alpha_i + \epsilon_{i,t}$$

where $L(R)_{i,t-1}$ is the cost-of-capital component and $L(A)_{i,t-1}$ is the productivity component. $L(R)_{i,t-1}$ is a known function of the weighted average costs of capital. $L(R)_{i,t-1}^{\text{recovery}}$ and $L(R)_{i,t-1}^{\text{lower bound}}$ are obtained from the option-implied measures of cost of equity, and $L(R)_{i,t-1}^{\text{CAPM}}$, $L(R)_{i,t-1}^{\text{FF}}$, and $L(R)_{i,t-1}^{\text{character}}$ are obtained from the conventional measures of cost of equity. $L(A)_{i,t-1}$ is a known function of the date- $t-1$ expectation of future productivity, which is obtained from the forward prices of stock. The t-statistics are presented in parentheses below the parameter estimates. *, **, *** denotes significance at 10%, 5%, 1%, respectively. The second last row reports the p-value for Wald's test of the hypothesis that coefficients β_1 and β_2 are exactly the opposite.

Dependent variable: Specification:	$\log(\text{INVEST}_{i,t})$				
	(1)	(2)	(3)	(4)	(5)
$L(R)_{i,t}^{\text{recovery}}$	-0.898*** (-2.97)				
$L(R)_{i,t}^{\text{lower bound}}$		-1.030** (-2.08)			
$L(R)_{i,t}^{\text{CAPM}}$			-0.0373** (-2.48)		
$L(R)_{i,t}^{\text{FF}}$				-0.126*** (-5.09)	
$L(R)_{i,t}^{\text{character}}$					-0.0936*** (-4.00)
$L(A)_{i,t}$	1.373*** (3.29)	1.403*** (3.35)	1.441*** (3.43)	1.451*** (3.49)	1.421*** (3.43)
N	3,912	3,921	3,921	3,921	3,921
$H_0 : \beta_1 = -\beta_2$	0.34	0.56	0.00	0.00	0.00
adj. R^2	0.509	0.505	0.504	0.507	0.506

Table 9: The Cost of Capital and Alternative Forces

This table presents the regressions of capital investment on its determinants. The dependent variable is $INVEST_{i,t}$, firm i 's investment-capital ratio in quarter t . In specifications (1) through (6), the regression equation is

$$INVEST_{i,t} = \alpha_i + \beta_1 \times WACC_{i,t-1} + \beta_2 \times Financial\ Constraints_{i,t-1} + \gamma \times X_{i,t-1} + \epsilon_t.$$

$WACC_{i,t-1} \in (WACC_{i,t-1}^{recovery}, WACC_{i,t-1}^{lower\ bound})$ is the WACC derived from the option-implied measures of cost of capital. $Financial\ Constraints_{i,t-1}$ is measured by the KZ index, WW index, or HM index. X_{t-1} denotes the control variables that include Tobin's q ($Q_{i,t-1}$), the log of book value of total assets ($SIZE_{i,t-1}$), the book value of leverage ratio ($LEV_{i,t-1}$), and cash flow-to-asset ratio ($CF_{i,t-1}$). In specifications (7) and (10), the regression equation is

$$INVEST_{i,t} = \alpha_i + \beta_1 \times WACC_{i,t-1} + \beta_2 \times Sentiment_{i,t-1} + \beta_3 \times WW\ index_{i,t-1} + \gamma \times X_{i,t-1} + \epsilon_t,$$

where $Sentiment_{i,t-1}$ is an indicator of irrational sentiment. It is measured by either economy-wide index ($SentIndex_t$) by Baker and Wurgler (2006) or forecast errors in predicting the subsequent excess returns. The forecast error is

$$Error_{i,t \rightarrow T} = R_{i,t \rightarrow T}^e - ER_{i,t}.$$

The t-statistics are presented in parentheses below the parameter estimates. *, **, *** denotes significance at 10%, 5%, 1%, respectively.

Dependent variable: Specification:	INVEST _{i,t}									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$WACC_{i,t-1}^{recovery}$	-0.0381*** (-3.90)	-0.0387*** (-3.97)	-0.0412** (-3.28)				-0.0239** (-2.56)	-0.0394*** (-3.33)		
$WACC_{i,t-1}^{lower\ bound}$				-0.0601*** (-3.62)	-0.0601*** (-3.67)	-0.0701*** (-3.39)			-0.0514*** (-3.28)	-0.0561*** (-2.85)
KZ index _{i,t-1}	-0.0003 (-0.65)			-0.0003 (-0.61)						
WW index _{i,t-1}		-0.0211*** (-2.97)			-0.0202*** (-2.76)		-0.0203*** (-3.02)	-0.0209*** (-3.32)	-0.0196*** (-2.88)	-0.0211*** (-3.44)
HM index _{i,t-1}			0.0020 (0.17)			0.0046 (0.40)				
SentIndex _{t-1}							0.0135*** (9.18)		0.0144*** (9.57)	
$Error_{i,t-1 \rightarrow T-1}^{recovery}$								-0.0049*** (-2.67)		
$Error_{i,t-1 \rightarrow T-1}^{lower\ bound}$										-0.0044** (-2.37)
$Q_{i,t-1}$	0.0086*** (5.26)	0.0086*** (5.21)	0.0093*** (4.26)	0.0088*** (5.36)	0.0088*** (5.31)	0.0094*** (4.34)	0.0073*** (4.63)	0.0075*** (4.10)	0.0071*** (4.56)	0.0078*** (4.45)
$SIZE_{i,t-1}$	-0.0021 (-0.70)	-0.0032 (-1.04)	-0.0038 (-1.13)	-0.0021 (-0.68)	-0.0031 (-0.99)	-0.0039 (-1.13)	-0.0039 (-1.34)	-0.0050 (-1.28)	-0.0043 (-1.43)	-0.0051 (-1.35)
$LEV_{i,t-1}$	-0.0331*** (-3.05)	-0.0340*** (-3.11)	-0.0316** (-2.64)	-0.0310*** (-2.83)	-0.0318*** (-2.87)	-0.0303** (-2.44)	-0.0258** (-2.45)	-0.0343*** (-2.80)	-0.0246** (-2.31)	-0.0314** (-2.55)
$CF_{i,t-1}$	0.0207 (0.77)	0.0171 (0.62)	0.0206 (0.58)	0.0175 (0.64)	0.0144 (0.52)	0.0151 (0.42)	0.0178 (0.66)	0.0157 (0.52)	0.0117 (0.43)	0.0151 (0.48)
N	8,006	8,005	5,522	7,894	7,893	5,438	8,005	7,045	7,893	7,061
adj. R^2	0.547	0.548	0.570	0.546	0.547	0.571	0.559	0.555	0.561	0.551

Table 10: The Instrumented Cost of Capital and Uncertainty and Capital Investment

This table presents 2SLS regressions of the capital investment on its determinants. The dependent variable is $INVEST_{i,t}$, firm i 's investment-capital ratio in quarter t . The regression specification is

$$INVEST_{i,t} = \alpha_i + \beta_1 \times WACC_{i,t-1} + \beta_2 \times Uncertainty_{i,t-1} + \gamma \times X_{t-1} + \epsilon_t.$$

The WACC derived from the option-implied cost of equity, $WACC_{i,t}$, are instrumented by the date- t market price of risk. The market price of risk is measured by the ratio of the option-implied expected return on S&P Index to the annualized standard deviation of daily returns on the index for the past year. The uncertainty is measured either from the realized daily stock returns ($\sigma_{i,t-1}^{realized}$) or by the implied volatility from option prices ($\sigma_{i,t-1}^{implied}$). X_{t-1} denotes the control variables. The controls include Tobin's q ($Q_{i,t-1}$), the log of book value of total assets ($SIZE_{i,t-1}$), the book value of leverage ratio ($LEV_{i,t-1}$), the cash flow-to-asset ratio ($CF_{i,t-1}$) and the WW index. Panel (a) presents the first stage results, and panel (b) presents the second stage results. The t-statistics are presented in parentheses below the parameter estimates. *, **, *** denotes significance at 10%, 5%, 1%, respectively.

(a) First Stage Results

Dependent variable: Specification:	$WACC_{i,t}^{recovery}$ (1)	$WACC_{i,t}^{lower\ bound}$ (2)
Price of Risk $_{i,t-1}^{recovery}$	0.0746*** (7.29)	
Price of Risk $_{i,t-1}^{lower\ bound}$		0.0764*** (5.76)
N	7,772	7,663
F -Statistic	53.19	33.20
adj. R^2	0.467	0.364

(b) Second Stage Results

Dependent variable: Specification:	$INVEST_{i,t}$			
	(1)	(2)	(3)	(4)
$WACC_{i,t-1}^{recovery}$	-0.3895*** (-4.63)	-0.4023*** (-5.02)		
$WACC_{i,t-1}^{lower\ bound}$			-0.9642*** (-4.78)	-0.9696*** (-4.91)
$\sigma(R)_{i,t-1}^{realized}$	-0.2184*** (-3.14)		-0.2023** (-2.22)	
$\sigma(R)_{i,t-1}^{implied}$		-0.0165** (-2.43)		-0.0155* (-1.80)
$Q_{i,t-1}$	0.0074*** (3.30)	0.0074*** (3.18)	0.0073** (2.50)	0.0072** (2.50)
$SIZE_{i,t-1}$	-0.0036 (-0.80)	-0.0034 (-0.76)	-0.0035 (-0.60)	-0.0034 (-0.59)
$LEV_{i,t-1}$	-0.0278* (-1.82)	-0.0270* (-1.73)	-0.0272 (-1.35)	-0.0264 (-1.34)
$CF_{i,t-1}$	0.0196 (0.50)	0.0213 (0.51)	0.0169 (0.33)	0.0183 (0.35)
WW index $_{i,t-1}$	-0.0204** (-2.49)	-0.0198** (-2.29)	-0.0201* (-1.91)	-0.0195* (-1.81)
N	7,542	7,475	7,437	7,392
adj. R^2	0.556	0.556	0.556	0.554

Table 11: Logistic Analysis on Responders to Time-Varying Cost of Equity

This table presents the logistic regressions that investigate the association between firm characteristics and individual firm's response to time-varying cost of equity. Included are firms with 30 or more observations of all relevant variables. The dependent variable is equal to one if a firm's t-statistic for the equity premium is -1.65 or below in the firm-specific time-series regression of capital investment specified by equation (25). Otherwise, the dependent variable is zero. In the logistic regressions, the explanatory variables are the log of book value of total assets ($SIZE_i$), the log of firm' age (AGE_i), dependence on external finance ($External\ Finance_i$), dependence on equity finance ($Equity\ Finance_i$), and indicators of financial constraints ($KZ\ index_i$, $WW\ index_i$, $HM\ index_i$). The p-values are presented in bracket below the parameter estimates. *, **, *** denotes significance at 10%, 5%, 1%, respectively. The likelihood ratio statistic is asymptotically distributed chi-square with degrees of freedom equal to the number of explanatory variables, under the null hypothesis that all slope coefficients are zero.

Dependent variable:	Binary number with 1 for responders and 0 otherwise					
Specifications:	(1)	(2)	(3)	(4)	(5)	(6)
$SIZE_i$	-0.094 [0.71]	0.282 [0.63]	-0.131 [0.63]	0.007 [0.33]	0.558 [0.45]	-0.979 [0.72]
AGE_i	0.997** [0.05]	1.07** [0.04]	1.212** [0.03]	0.878* [0.07]	1.010** [0.04]	1.154** [0.03]
$External\ Finance_i$	0.310** [0.04]	0.252* [0.06]	0.299** [0.04]			
$Equity\ Finance_i$				0.472** [0.04]	0.446** [0.04]	0.584** [0.02]
$KZ\ index_i$	-0.249 [0.32]			0.006 [0.99]		
$WW\ index_i$		7.068 [0.51]			11.31 [0.28]	
$HM\ index_i$			-0.411 [0.91]			0.373 [0.92]
likelihood ratio statistic	9.94	9.77	11.02	8.60	9.76	10.91
p-value	0.04	0.04	0.03	0.07	0.04	0.03
observations	129	129	117	131	131	119

Appendix A. Identity Between Investment Return and Stock Return

In equation (3), the ex-dividend price of stock on date t is $\mathbb{E}_t [M_{t+1}V(\omega_{t+1}, K_{t+1})]$. Then, the gross return from date t to $t + 1$ is

$$R_{E,t+1} = \frac{V_{t+1}}{\mathbb{E}_t [M_{t+1}V_{t+1}]} \quad (\text{A.1})$$

The numerator of equation (A.1) can be written as $K_{t+1}\partial V_{t+1}/\partial K_{t+1}$ due to the linear homogeneity of the firm value V_{t+1} . Similarly, the denominator becomes

$$\mathbb{E}_t [M_{t+1}V_{t+1}] = \mathbb{E}_t \left[M_{t+1}K_{t+1} \frac{\partial V_{t+1}}{\partial K_{t+1}} \right] = K_{t+1}\phi' \left(\frac{I_t^*}{K_t} \right), \quad (\text{A.2})$$

where the first-order condition of optimal investment is used in the last equality. Then, the return on stock can be written as

$$\frac{V_{t+1}}{\mathbb{E}_t [M_{t+1}V_{t+1}]} = \frac{K_{t+1} \frac{\partial V_{t+1}}{\partial K_{t+1}}}{K_{t+1}\phi' \left(\frac{I_t^*}{K_t} \right)} = R_{I,t+1}, \quad (\text{A.3})$$

confirming that the return on investment equals the stock return.

Appendix B. Decomposition of Marginal q

Taking logarithm of equation (6) leads to the following expression of the optimal investment

$$(\eta - 1) \log \left(\frac{I_t^*}{K_t} \right) = \log \left(\mathbb{E}_t [H(A_{t+1})] \right) - \log \left(\text{WACC}_t \right). \quad (\text{B.1})$$

The first term in the right-hand side can be approximated through the Taylor series expansion around the unconditional mean of productivity \bar{A} . Then, the first term becomes

$$\begin{aligned} \log \left(\mathbb{E}_t [H(A_{t+1})] \right) &\approx \log \left(\mathbb{E}_t \left[H(\bar{A}) + \frac{\partial H}{\partial A} (A_{t+1} - \bar{A}) \right] \right) \\ &= \log \left(H(\bar{A}) \mathbb{E}_t \left[1 + \frac{1}{H(\bar{A})} \frac{\partial H}{\partial A} (A_{t+1} - \bar{A}) \right] \right) \\ &= \log \left(H(\bar{A}) \right) + \log \left(1 + \frac{1}{H(\bar{A})} (\mathbb{E}_t [A_{t+1}] - \bar{A}) \right), \end{aligned} \quad (\text{B.2})$$

where I use the fact $\partial H/\partial A = 1$ as explained below. To determine the derivative $\partial H/\partial A$, we first apply the envelope theorem to the Bellman equation (3) and find $\partial V_t/\partial A_t = K_t$. Given that $V_t = H(A_t)K_t$, the derivative is

$$\frac{\partial H(A_t)}{\partial A_t} = \frac{1}{K_t} \frac{\partial V_t}{\partial A_t} = 1. \quad (\text{B.3})$$

Substituting equation (B.2) for the productivity component in equation (B.1), I obtain

$$\log\left(\frac{I_t^*}{K_t}\right) \approx \underbrace{\frac{\log\left(H(\bar{A})\right)}{\eta-1}}_{\equiv \kappa_0} + \frac{1}{\eta-1} \underbrace{\log\left(1 + \frac{1}{H(\bar{A})} (\mathbb{E}_t(A_{t+1}) - \bar{A})\right)}_{\equiv L(A)_t} - \frac{1}{\eta-1} \underbrace{\log\left(\text{WACC}_t\right)}_{\equiv L(R)_t}. \quad (\text{B.4})$$

Appendix C. Estimating the State Prices

In the semiparametric approach of [Ait-Sahalia and Lo \(2000\)](#), the call option price is given by

$$\text{Call}_{BSM}(S^F, X, \tau, R_{f,t}, \sigma(X/F, \tau)) \quad (\text{C.1})$$

where S^F is the forward price of the stock, σ is the implied volatility, $\tau (= T - t)$ is time to maturity, and Call_{BSM} is the Black-Scholes-Merton formula. The function of implied volatility is estimated for each month. To do so, I perform the kernel regression using option prices observed in that month as follows:

$$\hat{\sigma}(X/S^F, \tau) = \frac{\sum_{i=1}^n k\left(\frac{X/S^F - X_i/S_i^F}{h_{X/S^F}}\right) k\left(\frac{\tau - \tau_i}{h_\tau}\right) \sigma_i}{\sum_{i=1}^n k\left(\frac{X/S^F - X_i/S_i^F}{h_{X/S^F}}\right) k\left(\frac{\tau - \tau_i}{h_\tau}\right)} \quad (\text{C.2})$$

where i denotes an observation in the month, σ_i is the implied volatility of observation i , $k(z)$ is the Gaussian kernel function such that $k(z) = 1/\sqrt{2\pi} \exp(-z^2/2)$, and $h_{X/F}$ and h_τ are bandwidth parameters. The bandwidth parameters are chosen to minimize the sum of squared errors of observations as suggested in [Härdle \(1994\)](#). [Ait-Sahalia and Lo \(2000\)](#) show that this semiparametric estimator captures the salient features in the option market, the volatility smile or smirk, which are likely to carry risk-relevant information. As a result, the equity premium that will be recovered from the estimated state price is expected to reflect these option market features.

The above method is designed for the setting where the stock price is a continuous state variable. I modify the method to apply to the discrete states of stock prices, (S_1, \dots, S_N) as follows. The

state price of S_j on date T when the current price is S_i is

$$F_{i,j} = R_{f,t} \frac{\partial^2 \text{Call}_{BSM}(S_i^F, X, \tau, R_{f,t}, \hat{\sigma}(X/S_i^F, \tau))}{\partial X^2} \Big|_{X=S_j} \left(\frac{S_{j+1} - S_{j-1}}{2} \right). \quad (\text{C.3})$$

where S_i^F is the date- T forward price of current stock price S_i . Note that the increment of stock price, $(S_{j+1} - S_{j-1})/2$, is multiplied to obtain the state prices over the discrete states.

Appendix D. Connection between Dividend and Productivity Growths

Consider the all-equity-financed firm described in equation (3). According to the cash flow identity, date- t dividend (cashflow to stock holders) is equal to date- t cash flow from assets, which is production output minus investment expenditure for the firm. Then, the dividend growth is

$$\frac{D_{t+1}}{D_t} = \frac{A_{t+1}K_{t+1} - \phi\left(\frac{I_{t+1}}{K_{t+1}}\right)K_{t+1}}{A_tK_t - \phi\left(\frac{I_t}{K_t}\right)K_t} = \frac{1 - \phi\left(\frac{I_{t+1}}{K_{t+1}}\right)/A_{t+1}}{1 - \phi\left(\frac{I_t}{K_t}\right)/A_t} \frac{K_{t+1}}{K_t} \frac{A_{t+1}}{A_t}. \quad (\text{D.1})$$

Taking the log of both sides of the above equation, I express the log of the dividend growth

$$\underbrace{\log D_{t+1} - \log D_t}_{\equiv \Delta d_{t+1}} = \log\left(1 - \frac{\phi_{t+1}}{A_{t+1}}\right) - \log\left(1 - \frac{\phi_t}{A_t}\right) + \log(1 - \delta + e^{\xi_t}) + \underbrace{\log A_{t+1} - \log A_t}_{\equiv a_{t+1}}, \quad (\text{D.2})$$

where ϕ_t denotes $\phi(I_t/K_t)$ and ξ_t is the log of investment-capital ratio $\log(I_t/K_t)$. Next, I approximate the dividend growth using the Taylor's expansion with respect to ξ_t . Then,

$$\Delta d_{t+1} \approx \log\left(1 - \frac{\phi_{t+1}}{A_{t+1}}\right) - \log\left(1 - \frac{\phi_t}{A_t}\right) + \underbrace{\log(1 - \delta + e^{\bar{\xi}})}_{\equiv \kappa_1} + \underbrace{\frac{e^{\bar{\xi}}}{1 - \delta + e^{\bar{\xi}}}}_{\equiv \rho} (\xi_t - \bar{\xi}) + \Delta a_{t+1}. \quad (\text{D.3})$$

where $\bar{\xi}$ is the unconditional mean of the investment-capital ratio and κ_1 is log of the unconditional mean of capital growth rate $1 - \delta + \overline{I/K}$.

Adding the dividend growths from $t+1$ to T leads to

$$\sum_{s=t+1}^T \Delta d_s \approx \log\left(1 - \frac{\phi_T}{A_T}\right) - \log\left(1 - \frac{\phi_t}{A_t}\right) + (T-t)\kappa_1 + \rho \sum_{s=t+1}^T (\xi_s - \bar{\xi}) + \sum_{s=t+1}^T \Delta a_{s+1}. \quad (\text{D.4})$$

As a result, the long-run average of the dividend growth can be written as

$$\begin{aligned}
\lim_{T \rightarrow \infty} \frac{1}{T-t} \sum_{s=t+1}^T \Delta d_s &\approx \lim_{T \rightarrow \infty} \frac{1}{T-t} \log \left(1 - \frac{\phi_T}{A_T} \right) - \lim_{T \rightarrow \infty} \frac{1}{T-t} \log \left(1 - \frac{\phi_t}{A_t} \right) + \kappa_1 \\
&+ \lim_{T \rightarrow \infty} \frac{1}{T-t} \rho \sum_{s=t+1}^T (\xi_s - \bar{\xi}) + \lim_{T \rightarrow \infty} \frac{1}{T-t} \sum_{s=t+1}^T \Delta a_{t+1} \\
&\approx \kappa_1 + \lim_{T \rightarrow \infty} \frac{1}{T-t} \sum_{s=t+1}^T \Delta a_{t+1},
\end{aligned} \tag{D.5}$$

where we use the fact $\lim_{T \rightarrow \infty} \sum_{s=t+1}^T (\xi_s - \bar{\xi})$ converges to zero.

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