Does your hedge fund manager smooth returns intentionally or inadvertently?

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Abstract

We propose an econometrically logical approach that distinguishes intentional from inadvertent smoothing of hedge fund return. Other than the hedge fund return (Y) we introduce an explanatory variable: a market portfolio of hedge fund returns (X). By connecting X and Y, some critical parameters are found to be uniquely related to the two types of return smoothing. Using those parameters, we develop distinct desmoothing algorithms against intentional and inadvertent smoothing. Our empirical results show that although intentional smoothing is partly attributable to hedge fund smoothing, return smoothing is mainly caused by the nature of underlying assets; moreover, intentional smoothing is done more consistently than inadvertent smoothing.

Keywords: intentional smoothing, inadvertent illiquidity smoothing, desmoothing algorithm, latent factor model, single equation error correction model

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1. Introduction

Many hedge funds hold illiquid securities or difficult-to-price, over-the-counter securities of which publicly available traded prices often do not exist. Unlike mutual funds, hedge funds are not regulated entities, and hence, some fund managers do not disclose their strategies and positions and are known to report their returns at their discretion. The lack of securities' prices and transparency may induce hedge fund managers to manage, either intentionally or inadvertently, their positions for reporting returns. Although most hedge fund managers are good and honorable people, they have a strong incentive to show returns that are consistent and uncorrelated with traditional markets, and some managers likely engage in the unsavory practice of intentional return smoothing (Getmansky et al. 2004). Liang (2003) shows that the returns of hedge funds audited are "more precise and consistent" across databases. Thus, it is generally suspected that data generated by hedge funds are contaminated by inadvertent return smoothing due to pricing problems or intentional return smoothing due to the manager's agenda. These behaviors often cast reasonable doubts on benefits from the portfolio diversification of hedge funds.

Bollen and Pool (2009) infer hedge fund managers' return smoothing through a discontinuity at zero in the hedge fund net return distribution; that is, the number of small gains far exceeds the number of small losses. They argue that their finding is caused by fund managers' manipulating their returns to avoid showing small losses. However, Jorion and Schwarz (2014) show that the incentive fees for hedge funds as well as asset illiquidity and the bounding of yields at zero for fixed-income securities can generate such distribution discontinuities. Therefore, they conclude that the observed hedge fund return discontinuities are not direct evidence of intentional smoothing.

Cassa and Gerakos (2011) study the two types of smoothing by incorporating hedge fund due diligence reports into hedge fund return data and notice that funds using less verifiable pricing sources and funds that provide managers with greater discretion in pricing investment positions are more likely to have consistent intentional smoothing. More recently, Cao et al. (2017) examine the extent to which hedge fund return smoothing is due to intentional smoothing using a new hedge fund data set from a separate account platform that trades pari passu with matching main hedge funds and that features third-party reporting and permissive share restrictions. They find that 33% of reported smoothing is classified as intentional smoothing and 67% of reported return smoothing as inadvertent return smoothing due to the properties of the underlying assets and other factors common to main funds and separate accounts. These studies are done based on data that are not easily available, such as due diligence reports or separate accounts, to distinguish between the two types of smoothing.

In the literature, there are also econometric modeling approaches for return smoothing. In other words, autoregressive model-based solutions to adjust biases caused by return smoothing are suggested by Brook and Kat (2002) and Getmansky et al. (2004). These bias adjustments, however, fail to disentangle the effects of underlying assets' illiquidity smoothing from intentional smoothing by hedge fund managers because of insufficient endogenous or explanatory information (refer to Section 2 below). The above discussions show that the desmoothing problem likely suffers from a shortage of data as well as a lack of proper explanatory variables.

In this paper, we develop an econometric approach that distinguishes intentional smoothing from inadvertent illiquidity smoothing. For this, we consider not only the hedge fund return Y itself but also a market portfolio of hedge fund returns X as an explanatory variable. By connecting X and Y via single equation error correction model (SEECM), some critical parameters are found to be uniquely related to intentional smoothing and inadvertent illiquidity smoothing. *Based on these critical parameters and AR(1) type model by Brook and Kat (2002), desmoothing algorithms against intentional smoothing and inadvertent illiquidity smoothing are developed.*

As empirical applications of our algorithms, we test smoothing behavior for individual hedge funds in the TASS database. Our empirical findings are intuitive as well as consistent with previous results (Getmansky et al., 2004; Cassa and Gerakos, 2011; Cao et al., 2017). Funds containing illiquid securities for which managers and brokers have any discretion in marking their position (e.g., fixed income arbitrage funds and convertible arbitrage funds) appear to be prone to involve return smoothing. In contrast, return smoothing is found to be less involved by funds that contain liquid assets easily marked to market (e.g., long/short equity funds and managed futures funds). We also find that although intentional smoothing is partly attributable to hedge fund smoothing, return smoothing is mainly caused by the nature of the funds' underlying assets (inadvertent smoothing) and that intentional smoothing is done more consistently than inadvertent illiquidity smoothing. These consistent and intuitive findings verify that our methodology resolving the shortage of data and explanatory variables is a logically reasonable tool for detecting intentional smoothing.

We organize this paper into the following sections. Section 2 discusses our methodology of desmoothing and testing return smoothing; two distinct desmoothing algorithms are described in this section. Section 3 describes our data and discusses the main results of empirical tests. Section 4 offers concluding remarks.

2. Desmoothing and Testing Methodology

To desmooth the smoothed hedge fund returns, Brook and Kat (2002) consider a model

$$Y_t^* = \tau Y_{t-1}^* + (1-\tau)Y_t \tag{1}$$

where $|\tau| < 1, Y_t^*$ is a smoothed return, and Y_t is an original (true) return without smoothing at time t (presumably unavailable unless $\tau = 0$). Note that (1) leads to

$$Y_t^* = (1-\tau)\sum_{j=0}^{t-1} \tau^j \, Y_{t-j} \tag{2}$$

which shows that the smoothed hedge fund return at time t (Y_t^*) is a weighted average of its true returns over the past periods. It is noted from equation (1) that a sufficiently small $|\tau|$ implies an insignificant amount of return smoothing because a small $|\tau|$ makes $Y_t^* \cong Y_t$. A value of $|\tau|$ close to 1 implies substantial return smoothing because fund managers report Y_t^* with a relatively higher weight of Y_{t-1}^* (smoothed return at a previous time) than that of Y_t (true return). Thus, τ is an important smoothing profile parameter, and the choice of τ is critical for a successful desmoother recovering true return Y_t. From equation (1), desmoother $Y_t(\hat{\tau})$ with estimate $\hat{\tau}$ is given by

$$Y_t(\hat{\tau}) = \frac{Y_t^* - \hat{\tau} Y_{t-1}^*}{(1-\hat{\tau})}. \tag{3}$$

Brook and Kat (2002) sets a smoothing profile estimate $\hat{\tau}$ equal to the smoothed returns' autocorrelation coefficient at lag 1, which forces the first order autocorrelation of desmoothed data $Y_t(\hat{\tau})$ to be zero. Thus, their desmoothing implicitly assumes that the original (and unavailable) hedge fund returns are independent, and return smoothing only causes serial autocorrelation in hedge fund returns. This naive assumption could be subject to serious bias because the autocorrelation might result from other factors besides return smoothing. Getmansky et al.'s (2004) methods set the observed (smoothed) hedge fund return at time t (Y_t^*) as a weighted average of its true return Y_t over the most recent k+1 periods, including the current period

$$Y_t^* = \theta_0 Y_t + \theta_1 Y_{t-1} + \cdots \theta_k Y_{t-k} \tag{4}$$

where $0 \leq \theta_i \leq 1, i = 0,1,2,\cdots k$, and $\theta_0 + \theta_1 + \cdots + \theta_k = 1$. The performance of Getmansky et al. (2004) critically depends on the choice of k and θ_is, which requires a knowledge of the autocorrelation of Y_t. For this, Getmansky et al. (2004) introduces a linear single-factor model for Y_t. By doing so, it separates the effects of illiquidity from the intentional return smoothing. As discussed there, the most difficult challenge in implementing their method is to correctly identify the single common factor with proper additional information. Overall, the difficulty with correctly identifying intentional smoothing mainly comes from the unavailability of Y_t. Also refer to Asness et al. (2001), Bollen and Pool (2008), Cassar and Gerakos (2011), and Cao et al. (2017) for related references.

In order to resolve the unavailability of Y_t more efficiently, we consider model (1) and introduce the market portfolio of hedge fund returns X as an additional explanatory variable. We assume that X and an individual hedge fund return Y are modeled as

$$X_t = \theta_x W_t + \delta_x u_{x,t} \qquad Y_t = \theta_y W_t + \delta_y u_{y,t} \tag{5}$$

where W_t represents a common latent factor with the non-zero loadings θ_x and θ_y while $u_{x,t}$ and $u_{y,t}$ are idiosyncratic factors unique to X_t and Y_t, respectively, with the loadings δ_x and δ_y. It is assumed that W_t, $u_{x,t}$ and $u_{y,t}$ are stochastic processes with zero mean and unit variance; that is,

$$W_t \sim (0,1),\ u_{x,t} \sim (0,1)\ \ and\ u_{y,t} \sim (0,1).$$

To complete the specification of the common factor model, all factors are assumed to be independent:

$$\mathrm{E}(u_{x,t}u_{y,t}) = 0,\quad \mathrm{E}(u_{x,t}W_t) = 0,\quad \mathrm{E}(u_{y,t}W_t) = 0.$$

In addition, we employ an AR(1) model for idiosyncratic shocks that occur in fund Y as

$$u_{y,t} = \rho u_{y,t-1} + a_{u,t} \tag{6}$$

where $\mathrm{E}(a_{u,t}W_t) = 0, \mathrm{E}(a_{u,t}u_{y,t}) = 0,\ 0 < \rho < 1,$ and $a_{u,t} \sim iid(0,1)$. The AR(1) model imposed by (6) is appropriate because the idiosyncratic factors for a hedge fund's returns certainly progress dynamically over time. Note that $\mathrm{Var}(u_{y,t}) = \frac{1}{1-\rho^2}$, that is, the volatility of the idiosyncratic factor of Y_t is determined by ρ. *As* Y_t *in (5) contains* W_t *as systematic illiquidity factors and* $u_{y,t}$ *as an idiosyncratic factor via (6), the two smoothing behaviors to the hedge fund could be tested and analyzed via a set of parameters in (5) and (6), respectively.*

It is econometrically reasonable to assume that the dynamic system behind the returns of an individual hedge fund Y_t and a market portfolio of hedge funds X_t keeps a long-term equilibrium. From this point of view, one may employ a single equation error correction model (SEECM) and link it to the latent factor model (5) with (6). The SEECM is useful for estimating both short-term and long-term effects of one time series on another. Using Y_t and X_t, we employ the SEECM as follows:

$$\Delta Y_t = \alpha + \beta_0 \Delta X_t + \beta_1 Y_{t-1} + \beta_2 X_{t-1} + \varepsilon_t$$

$$= \alpha + \beta_0 \Delta X_t + \beta_1 (Y_{t-1} - \gamma X_{t-1}) + \varepsilon_t \qquad (7)$$

Where $\gamma = -\frac{\beta_2}{\beta_1}, \Delta Y_t \equiv Y_t - Y_{t-1}, \Delta X_t \equiv X_t - X_{t-1}$, and ε_t is the independent and identically distributed (iid) error. It is assumed here that Y_t and X_t are stationary (De Boef and Keele, 2008). The part of the equation in parentheses in SEECM (7) is the error correction mechanism, where $(Y_{t-1} - \gamma X_{t-1}) = 0$ when X and Y are in equilibrium. The coefficient β_0 specifies the short-term effects of an increase in X on an increase in Y, while β_1 specifies the speed at which X and Y return to equilibrium from a state of disequilibrium. The coefficient γ specifies the long-term effects of a one-unit increase in X on Y. Note that when $\beta_1 < 0$ ($\beta_1 > 0$), the system converges to equilibrium (diverges from equilibrium). Taking into account the latent factor model (5) with (6), we have a SEECM as follows:

$$\Delta Y_t = \frac{\theta_y}{\theta_x} \Delta X_t - (1-\rho)\left(Y_{t-1} - \frac{\theta_y}{\theta_x(1-\rho)} X_{t-1}\right) + \varepsilon_t \qquad (8)$$

where $\varepsilon_t = -\frac{\theta_y}{\theta_x} \delta_x u_{x,t} + \delta_y a_{u,t} - \rho \theta_y W_{t-1}$(Kim and Lee, 2017). By comparing SEECM (8) with SEECM (7), it is clear that $\beta_0 = \beta_2 = \frac{\theta_y}{\theta_x}$ and $\beta_1 = \rho - 1$. Then, it is easy to see that $\beta_1 = \rho - 1$ is related to $u_{y,t}$, the idiosyncratic factor of Y_t in (5) with (6), while $\beta_0 = \beta_2 = \frac{\theta_y}{\theta_x}$ is related to W_t, the common latent shock of Y_t in (5). *Thus, it is reasonable to assume that ρ is mainly related to intentional smoothing, and β_0 (or β_2) is mainly related to inadvertent illiquidity smoothing*. Below we develop two desmoothing algorithms: one is against intentional smoothing based on the parameter ρ and the other is against inadvertent illiquidity smoothing based on the parameter β_0 (or β_2).

2.1 Desmoother against Intentional Smoothing and Testing

Using (3) and the fact that ρ is a critical parameter for the intentional smoothing of Y_t, a desirable desmoother against intentional smoothing is expected to recover a sequence of data

$$\{Y_{tdI}(\hat{\tau}_{dI}) = \frac{Y_t^* - \hat{\tau}_{dI}\, Y_{t-1}^*}{(1-\hat{\tau}_{dI})} : t = 1 \cdots, n\} \tag{9}$$

where $\hat{\tau}_{dI}$ is determined so that estimate $\hat{\rho}_{dI}$ from $\{Y_{tdI}(\hat{\tau}_{dI}); t = 1 \cdots, n\}$ would be close to the estimate $\tilde{\rho}$ for ρ when (8) is applied to unreported non-intentionally smoothed data $\{Y_t: t = 1, \cdots, n\}$. This can be implemented into the following algorithm:

(R1) generates $\{Y_{tdI}(\hat{\tau}_{dI}) = \frac{Y_t^* - \hat{\tau}_{dI}\, Y_{t-1}^*}{(1-\hat{\tau}_{dI})} : t = 1 \cdots, n\}$ with an optimal smoothing profile estimate $\hat{\tau}_{dI}$, which minimizes $|\hat{\rho}_{dI} - \tilde{\rho}|$.

To run (R1), it is essential to find $\tilde{\rho}$, the estimate of ρ of (8) with non-intentionally smoothed data $\{Y_t: t = 1 \cdots, n\}$. Since $\{Y_t: t = 1 \cdots, n\}$ is not available, instead we obtain an estimate of $\tilde{\rho}$ from

$$\Delta Y_t' = \frac{\theta_y}{\theta_x}\Delta X_t - (1-\rho)\left(Y_{t-1}' - \frac{\theta_y}{\theta_x(1-\rho)}X_{t-1}\right) + \varepsilon_t' \tag{10}$$

where Y_t' is the return of the hedge fund style index as a surrogate predicted variable in place of Y_t in (8), and X_t is the return of the hedge fund market portfolio as a predictor. The estimation of $\tilde{\rho}$ by the fund style from (10) is reasonable because the return of the hedge fund style index Y_t' is expected to diversify away individual hedge funds' idiosyncratic return smoothing (or intentional smoothing) and hence provides a reasonable estimate $\tilde{\rho}$.

Using $\{Y_{tdI}(\hat{\tau}_{dI}): t = 1 \cdots, n\}$, we can test whether a given hedge fund is subject to significant intentional smoothing. Observe that we are equipped with two data sets: $\{Y_{tdI}(\hat{\tau}_{dI}): t = 1 \cdots, n\}$ (desmoothed against an intentionally smoothed one) and $\{Y_t^*: t = 1 \cdots, n\}$ (observed but presumably an intentionally smoothed one). If SEECM (8) is applied to each data set, we have

$$\Delta Y_t^* = \frac{\theta_y^*}{\theta_x^*} \Delta X_t - (1 - \rho^*) \left(Y_{t-1}^* - \frac{\theta_y^*}{\theta_x^*(1-\rho^*)} X_{t-1} \right) + \varepsilon_t^* \tag{11}$$

for observed returns Y_t^* and

$$\Delta Y_{tdI} = \frac{\theta_{ydI}}{\theta_{xdI}} \Delta X_t - (1 - \rho_{dI}) \left(Y_{t-1,dI} - \frac{\theta_{ydI}}{\theta_{xdI}(1-\rho_{dI})} X_{t-1} \right) + \varepsilon_{tdI} \tag{12}$$

for desmoothed against intentionally smoothed returns, Y_{tdI}. Then, the hypotheses of concern are as follows:

H_{0I}: There is no intentional smoothing (or $\rho^* = \rho_{dI}$).

H_{1I}: There is intentional smoothing (or $\rho^* \neq \rho_{dI}$).

The above null hypothesis implies that the observed returns maintain the volatility of the idiosyncratic factor of a non-intentionally smoothed hedge fund return ($\tilde{\rho}$)[1], and hence, the fund manager is not involved in intentional smoothing. If we reject the null hypothesis, the fund manager is significantly involved in intentional smoothing, which causes a significant change in the volatility of the idiosyncratic factor of the observed hedge fund returns. In order to implement the test for a given hedge fund, we apply a standard t-test in model (7) with

[1] Note that $\hat{\rho}_{dI} \approx \tilde{\rho}$ from (R1).

$\{Y_t^*: t = 1 \cdots, n\}$, i.e., $H'_{0I}: \beta_1^*(:= \rho^* - 1) = \hat{\beta}_{1dI}(:= \hat{\rho}_{dI} - 1)$ where $\hat{\beta}_{1dI}$ is estimated from $\{Y_{tdI}(\hat{\tau}_{dI}): t = 1 \cdots, n\}$. In this case, the t statistic has Student's t distribution with (*n*-3) degree of freedom.

2.2 Desmoother against Inadvertent Illiquidity Smoothing and Testing

Hedge funds' return smoothing due to illiquidity of underlying assets is mainly related to the parameter $\frac{\theta_y}{\theta_x}$ in (8) that depends on the systematic factor W_t in (5)[2]. Using (3) and the fact that $\frac{\theta_y}{\theta_x}$ is a critical parameter for the inadvertent illiquidity smoothing of Y_t, a desirable desmoother against inadvertent illiquidity smoothing is expected to recover a sequence of data

$$\{Y_{tdL}(\hat{\tau}_{dL}) = \frac{Y_t^* - \hat{\tau}_{dL}\, Y_{t-1}^*}{(1 - \hat{\tau}_{dL})} : t = 1 \cdots, n\}$$

where $\hat{\tau}_{dL}$ is determined so that estimate $\frac{\hat{\theta}_{ydL}}{\hat{\theta}_{xdL}}$ from $\{Y_{tdL}(\hat{\tau}_{dL}); t = 1 \cdots, n\}$ would be close to the estimate $\frac{\tilde{\theta}_y}{\tilde{\theta}_x}$ for $\frac{\theta_y}{\theta_x}$ when (8) is applied to non-inadvertently smoothed data $\{Y_t: t = 1 \cdots, n\}$. This can be implemented in the following algorithm:

[2] In fact, the systematic factor W contains not only illiquidity exposure but also other factors common to X and Y. Therefore, in the strict sense, our desmoother against inadvertent illiquidity smoothing in this paper implies a desmoother against all inadvertent smoothings, not limited to against illiquidity smoothing. However, the illiquidity of underlying assets is a main factor that causes hedge fund return smoothing, and hence we call our desmoothing algorithm a desmoother against inadvertent illiquidity smoothing.

(R2) generates $\{Y_{tdL}(\hat{\tau}_{dL}) = \frac{Y_t^* - \hat{\tau}_{dL}\, Y_{t-1}^*}{(1-\hat{\tau}_{dL})} : t = 1 \cdots, n\}$ with an optimal smoothing profile estimate $\hat{\tau}_{dL}$, which minimizes $|\frac{\widehat{\theta}_{\mathrm{ydL}}}{\widehat{\theta}_{\mathrm{xdL}}} - \frac{\widetilde{\theta}_{\mathrm{y}}}{\widetilde{\theta}_{\mathrm{x}}}|$.

To run (R2), it is essential to find $\frac{\widetilde{\theta}_{\mathrm{y}}}{\widetilde{\theta}_{\mathrm{x}}}$, the estimate of $\frac{\theta_{\mathrm{y}}}{\theta_{\mathrm{x}}}$ of (8) with non-inadvertently smoothed data $\{Y_t : t = 1 \cdots, n\}$. Since $\{Y_t : t = 1 \cdots, n\}$ is not available, instead we obtain an estimate of $\frac{\widetilde{\theta}_{\mathrm{y}}}{\widetilde{\theta}_{\mathrm{x}}}$ from

$$\Delta Y_t'' = \frac{\theta_y}{\theta_{\mathrm{x}}} \Delta X_t - (1-\rho)\left(Y_{t-1}'' - \frac{\theta_{\mathrm{y}}}{\theta_{\mathrm{x}}(1-\rho)} X_{t-1}\right) + \varepsilon_{\mathrm{t}}'' \tag{13}$$

where Y_t'' is the return of a portfolio of hedge funds that contains mostly liquid securities as a surrogate predicted variable in place of Y_t in (8), and X_t is the return of a hedge fund market portfolio as a predictor. This is reasonable because hedge funds that contain mostly liquid securities Y_t'' are not expected to involve inadvertent illiquidity smoothing and hence provide a reasonable estimate $\frac{\widetilde{\theta}_{\mathrm{y}}}{\widetilde{\theta}_{\mathrm{x}}}$.

Using $\{Y_{tdL}(\hat{\tau}_{dL}) : t = 1 \cdots, n\}$, we can test whether a given hedge fund is subject to significant inadvertent illiquidity smoothing. Observe that we are equipped with two data sets: $\{Y_{tdL}(\hat{\tau}_{dL}) : t = 1 \cdots, n\}$ (desmoothed against an inadvertently smoothed one) and $\{\mathrm{Y}_{\mathrm{t}}^* : t = 1 \cdots, n\}$ (observed but presumably an inadvertently smoothed one). If SEECM (8) is applied to the data set $\{Y_{tdL}(\hat{\tau}_{dL}) : t = 1 \cdots, n\}$, we have

$$\Delta Y_{tdL} = \frac{\theta_{ydL}}{\theta_{xdL}} \Delta X_t - (1-\rho_{dL})\left(Y_{t-1,dL} - \frac{\theta_{ydL}}{\theta_{xdL}(1-\rho_{dL})} X_{t-1}\right) + \varepsilon_{tdL}. \tag{14}$$

Then, the hypotheses of concern are established from equations (11) and (14) as follows:

H_{0L}: There is no inadvertent illiquidity smoothing (or $\frac{\theta_y^*}{\theta_x^*} = \frac{\theta_{ydL}}{\theta_{xdL}}$).

H_{1L}: There is inadvertent illiquidity smoothing (or $\frac{\theta_y^*}{\theta_x^*} \neq \frac{\theta_{ydL}}{\theta_{xdL}}$).

The above null hypothesis implies that the observed returns maintain the effects of a systematic factor ($\frac{\tilde{\theta}_y}{\tilde{\theta}_x}$) in which the non-inadvertently smoothed hedge fund returns are involved[3], and hence the fund is not involved in inadvertent illiquidity smoothing. If we reject the null hypothesis, the fund is significantly involved in inadvertent illiquidity smoothing, which causes a significant change in the effect of the systematic factor of the observed hedge fund returns. In order to implement the test for a given hedge fund, we apply a standard t-test in model (7) with $\{Y_t^*: t = 1 \cdots, n\}$, i.e., H_{0L}': $\beta_0^* (\coloneqq \frac{\theta_y^*}{\theta_x^*}) = \hat{\beta}_{0dL} (\coloneqq \frac{\hat{\theta}_{ydL}}{\hat{\theta}_{xdL}})$ where $\hat{\beta}_{0dL}$ is estimated from $\{Y_{tdL}(\hat{\tau}_{dL}): t = 1 \cdots, n\}$. In this case, the t statistic has Student's t distribution with (n-3) degree of freedom.

3.Empirical Analysis

3.1. Data Description

For an empirical analysis, we use the TASS database for individual hedge funds (Y) and the

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[3] Note that $\frac{\hat{\theta}_{ydL}}{\hat{\theta}_{xdL}} \approx \frac{\tilde{\theta}_y}{\tilde{\theta}_x}$ from (R2).

Credit Suisse hedge fund database for hedge fund indices (Y'). The TASS database consists of monthly returns and accompanying information from January 1994 to September 2013. As of September 2013, the combined database, including both live and dead funds, contains 19,187 funds with at least one monthly return observation. The funds are classified according to one of 12 different investment styles: convertible arbitrage (CA), emerging markets (EM), event driven (ED), fixed income arbitrage (FIA), equity market neutral (EMN), long/short equity (LSE), managed futures (MF), multi-strategy (MS), dedicated short bias (DSB), global macro (GM), options strategy (OS), and fund of funds (FOF). The Credit Suisse hedge fund indices (Y') are based on the TASS database and provide 10 style indices that correspond to the investment styles in the TASS database except for OS and FOF. Therefore, we exclude funds classified as OS and FOF in the TASS database and match 10 fund styles in the TASS and the Credit Suisse hedge fund databases.

Most funds in the TASS report returns net of various fees on a monthly basis. We eliminate funds that report only gross returns and/or quarterly returns. Since our desmoothing algorithm is based on an equilibrium between an individual hedge fund (Y) and a market portfolio of hedge funds (X), we impose an additional filter of including only those funds with at least a 10-year lifetime, leaving a total of 1,495 funds, including live and dead funds. This obviously produces additional survivorship bias in our sample, but this filter may not be problematic because our main objective is not to make inferences about a hedge fund's overall performance. The representation of 10 investment styles is not evenly distributed but is concentrated among five categories: EM (125), ED (158), LSE (589), MF (193), and MS (181). These five categories account for 83% of funds in the combined database. To apply our desmoothing algorithms and test the intentional and inadvertent illiquidity smoothing of a given hedge fund, we use 237 monthly returns for individual hedge funds (Y) and Credit Suisse indices (Y′) from January 1994 to September 2013. Table 1 reports the monthly means and standard deviations of basic summary statistics for the 1,495 funds in our combined TASS database in Panel A and basic summary statistics for the monthly Credit Suisse hedge fund style indices in Panel B.

Panel A of Table 1 shows a great deal of variation in monthly mean returns and volatilities both across and within styles. For example, the 125 EM hedge funds in our sample exhibit the highest monthly mean return of 1.14% with a standard deviation of 0.66% and a monthly mean volatility of 6.44% with a standard deviation of 4.35%. Alternatively, DSB funds in our sample show the lowest monthly mean return of 0.23% with a standard deviation of 0.45% and a monthly mean volatility of 5.97% with a standard deviation of 2.50%. Average first autocorrelation coefficients also vary considerably across fund styles. CA (0.41) and FIA (0.35), which include some of the most illiquid securities traded and hence appear to provide more opportunities to smooth returns, are found to have the highest averages of serial correlation. In contrast, fund styles composed mostly of exchange-traded securities such as MF (0.05) show the lowest averages of serial correlation.

The Credit Suisse hedge fund database tracks approximately 9,000 funds that (i) are valued at US $50 million (minimum), (ii) possess a 12-month track record, and (iii) have audited financial statements. Credit Suisse calculates and rebalances the index monthly and reflects the performance net of all fees and expenses. Due to this survivorship bias in our sample, the results in Panel B of Table 1 are rather different from those in Panel A. The GM fund style index exhibits the highest monthly mean return of 0.94% with a standard deviation of 2.72%, whereas the DSB fund style index shows the lowest monthly mean return of -0.35% with the highest standard deviation of 4.81%. Similar to the results of the first autocorrelation in Panel A, the CA (0.56) and FIA (0.53) style indices are found to be mostly serially correlated.

Table 1
Mean and standard deviation of basic summary statistics for hedge funds in the TASS hedge fund database and Credit Suisse hedge fund style indices

Panel A Individual hedge funds in the TASS database (Y)

This table reports monthly means and standard deviations of basic summary statistics for 1,495 hedge funds in the combined TASS database. The 1,495 funds have at least a 10-year return history during the period from January 1994 to September 2013.

Style	Number of Funds	Lifetime (months)		Monthly Mean (%)		Monthly SD (%)		Skewness		Kurtosis		First Autocorrelation	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Convertible Arbitrage	55	156.85	28.41	0.69	0.31	2.54	1.81	-1.48	1.98	13.61	15.69	0.41	0.20
Dedicated Short Bias	13	156.85	25.49	0.23	0.45	5.97	2.50	0.03	0.76	3.49	3.53	0.11	0.07
Emerging Markets	125	159.50	31.45	1.14	0.66	6.44	4.35	-0.38	1.56	8.09	10.11	0.22	0.11
Emerging Market Neutral	60	159.93	29.80	0.75	0.46	3.64	3.87	-0.65	3.57	16.96	39.40	0.14	0.23
Event Driven	158	167.05	37.25	0.85	0.34	2.57	1.69	-0.65	1.14	6.39	6.14	0.27	0.15
Fixed Income Arbitrage	58	156.45	26.76	0.76	0.48	1.99	1.69	-1.37	2.76	15.72	26.68	0.35	0.30
Global Macro	63	157.02	26.97	0.91	0.53	4.26	2.37	0.47	1.45	5.62	8.18	0.07	0.14
Long/Short Equity	589	159.30	32.47	0.92	0.46	4.58	2.53	0.26	1.33	5.49	10.53	0.14	0.13
Managed Futures	193	174.12	36.06	0.82	0.52	5.36	3.23	0.52	1.76	6.50	19.13	0.05	0.15
Multi Strategy	181	156.43	30.53	0.76	0.54	2.77	2.20	-0.51	2.07	10.09	16.93	0.26	0.21
Total	1495	161.44	33.01	0.87	0.50	4.19	3.00	-0.11	1.82	7.63	15.83	0.18	0.19

Panel B Credit Suisse hedge fund style indices (Y′)

This table reports basic summary statistics for the monthly aggregate hedge fund index and 10 style indices in the Credit Suisse hedge fund database. The number of monthly return observations for each index is 237 from January 1994 to September 2013.

Style	Monthly Mean (%)	Monthly SD (%)	Skewnes	Kurtosis	First Autocorrelation
Aggregate Hedge Fund Index	0.71	2.12	-0.18	2.71	0.21
Convertible Arbitrage	0.62	1.94	-2.74	17.14	0.56
Dedicated Short Bias	-0.35	4.81	0.72	1.54	0.09
Emerging Markets	0.69	4.16	-0.78	5.53	0.30
Emerging Market Neutral	0.45	2.89	-12.10	170.77	0.07
Event Driven	0.77	1.78	-2.26	11.04	0.36
Fixed Income Arbitrage	0.46	1.60	-4.55	32.93	0.53
Global Macro	0.94	2.72	0.05	4.18	0.09
Long/Short Equity	0.79	2.79	-0.03	3.44	0.20
Managed Futures	0.45	3.35	0.03	-0.04	0.03
Multi Strategy	0.66	1.51	-1.73	6.27	0.32

3.2 Intentional Smoothing

Using the algorithm (R1) in Section 2.1, we estimate the smoothing profile parameter τ by $\hat{\tau}_{dI}$ of (9) and test intentional smoothing for each fund. To implement our methodology from Section 2.1, we set $Y_t{}'$ in (10) to be the returns of the Credit Suisse hedge fund style index that includes 237 monthly observations during the sample period and Y_t^* to be the corresponding 237 monthly observed returns of individual hedge funds. As a predictor X_t, the same set of 237 monthly returns of the Credit Suisse "aggregate" hedge fund index is incorporated. For (11), a sequence $\{Y_t^*: t = 1 \cdots ,237\}$ is analyzed for obtaining $\hat{\rho}^*$. For (12), a desmoothed sequence $\{Y_{tdI}(\hat{\tau}_{dI}): t = 1 \cdots ,237\}$ is generated by the algorithm (R1) from which $\hat{\rho}_{dI}$ is obtained. Then, we perform the t-test for testing intentional smoothing or the null hypothesis H_{0I}: $\rho^* = \rho_{dI}$. Table 2 reports the means and standard deviations of $\hat{\tau}_{dI}$ by fund style for 1,495 hedge funds in the TASS combined database. Table 2 also reports rejection rates in the t-test for each fund style to determine which style of hedge funds involves intentional smoothing more in the last column.

Table 2
Means and standard deviations of intentional smoothing profile estimate $\hat{\tau}_{dI}$ and intentional smoothing test results. This table reports the means and standard deviations of estimated $\hat{\tau}_{dI}$ for 1,495 hedge funds in the TASS database with at least ten years of return history during the period from January 1994 to September 2013. The rejection rate for testing the intentional smoothing of each fund style is reported in the last column. The test results are based on the 5% significance level.

Style	Number of Funds	$\hat{\tau}_{dI}$		Rejection Rate
		Mean	SD	
CA	55	0.09	0.28	0.33
DSB	13	0.02	0.13	0.15
EM	125	0.02	0.12	0.14
EMN	60	0.02	0.31	0.19
ED	158	0.10	0.14	0.35
FIA	58	0.17	0.33	0.71
GM	63	0.14	0.16	0.51
LSE	589	0.05	0.14	0.23
MF	193	0.04	0.15	0.18
MS	181	0.13	0.22	0.46
Total	1495			

The mean value of $\hat{\tau}_{dI}$ in Table 2 is the average across all funds in each style and indicates the average level of the intentional smoothing of each style. The higher the mean value of $\hat{\tau}_{dI}$ implies more intentional return smoothing. Table 2 shows that five styles seem to exhibit a higher average value of $\hat{\tau}_{dI}$ than the other styles: FIA (0.17), GM (0.14), MS (0.13), ED (0.1), and CA (0.09). These styles also show higher rejection rates for testing intentional smoothing than those of the other styles. These styles, except for GM, also show high serial correlation in Panel A of Table 1, implying that the serial correlation is related to intentional smoothing, but not always. In contrast, DSB, EM, EMN, LSE, and MF funds that exhibit lower average values of $\hat{\tau}_{dI}$ and rejection rates than those of other styles seem to involve less intentional smoothing. Overall, the results in Table 2 appear to be consistent with general intuition about the nature of the styles and securities involved in each fund style. Funds investing less verifiable pricing sources are more likely to involve intentional return smoothing (Cassa and Gerakos, 2011; Cao et al., 2017).

To demonstrate how our methodology applies to individual hedge funds, we report the estimates of $\hat{\tau}_{dI}$ and p-values for testing intentional smoothing for 20 randomly selected individual funds from the sample in Table 3. We select one intentionally smoothed and one non-intentionally smoothed fund from each style in our sample. Panels A and B of Table 3 contain 10 intentionally and non-intentionally smoothed hedge funds, respectively. The $\hat{\tau}_{dI}$s among these 20 funds range from 0.0078 to 0.9249. As expected, funds that are found to involve intentional smoothing in Panel A report a higher value of $\hat{\tau}_{dI}$ than those that are not found to involve intentional smoothing in Panel B. Consider, for example, Fund 10 in Table 3, which has a surprisingly high value of 0.9249 of $\hat{\tau}_{dI}$. By (1), this implies that only 7.51% (or $(1-\hat{\tau}_{dI})$) of that fund's current monthly non-intentionally smoothed return (Y_t) would be reported. This large value of $\hat{\tau}_{dI}$ suggests a significant amount of intentional smoothing by Fund 10. In contrast, Fund 13 in Panel B is not found to involve intentional smoothing, given its quite low value of $\hat{\tau}_{dI}$ of 0.0078, implying 99.22% of that fund's current monthly non-intentionally smoothed return (Y_t) would be reported.

Table 3
Intentional smoothing profile estimates of 20 randomly selected funds from the sample. This table reports $\hat{\tau}_{dI}$ for 20 randomly selected funds from the 1,495 hedge funds in the TASS database that have at least ten years of return history during the period from January 1994 to September 2013. One intentionally smoothed and one non-intentionally smoothed fund is randomly selected from each style.

Panel A Ten intentionally smoothed hedge funds from the sample

Hedge Fund	Style	Period	Number of months	$\hat{\tau}_{dI}$	p-value
Fund 1	CA	Oct 2002 – Sep 2013	132	0.3171	0.0152
Fund 2	DSB	Jul 1999 – Sep 2013	171	0.2938	0.0443
Fund 3	ED	Jan 1994 – Apr 2009	184	0.2768	0.0015
Fund 4	EM	Jul 1995 – Jun 2009	168	0.2126	0.0088
Fund 5	EMN	Jul 1997 – Dec 2008	138	0.8536	<0.0001
Fund 6	FIA	Oct 1999 – Jan 2012	148	0.6924	<0.0001
Fund 7	GM	Apr 2003 – Sep 2013	126	0.3913	<0.0001
Fund 8	LSE	Oct 1996 – Dec 2006	123	0.1980	0.0189
Fund 9	MF	Jan 1997 – Sep 2013	201	0.5809	<0.0001
Fund 10	MS	Jan 1994 – Mar 2009	183	0.9249	<0.0001

Panel B Ten non-intentionally smoothed hedge funds from the sample

Hedge Fund	Style	Period	Number of months	$\hat{\tau}_{dI}$	p-value
Fund 11	CA	Apr 1998 – Sep 2013	186	0.0910	0.4233
Fund 12	DSB	Mar 1997 – Oct 2007	128	0.0385	0.6782
Fund 13	ED	Jan 1994 – Dec 2009	192	0.0078	0.9004
Fund 14	EM	Sep 1996 – Aug 2007	132	0.0440	0.7005
Fund 15	EMN	Jul 1995 – Jan 2009	163	0.0109	0.9231
Fund 16	FIA	Mar 2001 – Sep 2013	149	0.0114	0.8953
Fund 17	GM	Nov 1998 – Sep 2013	179	0.0671	0.2718
Fund 18	LSE	Mar 1999 – Jun 2010	136	0.0890	0.1318
Fund 19	MF	Jan 1994 – Nov 2007	167	0.0924	0.2575
Fund 20	MS	Jul 1998 – Nov 2011	161	0.0316	0.5676

3.3 Inadvertent Illiquidity Smoothing

Using the algorithm (R2) in Section 2.1, we estimate the smoothing profile parameter τ by $\hat{\tau}_{dL}$ and test inadvertent illiquidity smoothing for each fund. To implement our methodology from Section 2.1, we set Y_t'' in (13) to be the returns of the MF hedge fund style index that includes 237 monthly observations during the sample period and Y_t^* to be the corresponding 237 monthly observed returns of individual hedge funds. As a predictor X_t, the same set of 237 monthly returns of the Credit Suisse aggregate hedge fund index is incorporated again. Notice that the MF funds contain mostly liquid securities and hence do not seem to involve significant inadvertent illiquidity smoothing. This is supported by Panel B of Table 1 showing that MF funds report the lowest serial correlation. For (11), a sequence $\{Y_I^*: t = 1 \cdots, 237\}$ is analyzed for estimating $\frac{\theta_y^*}{\theta_x^*}$. For (14), a desmoothed sequence $\{Y_{tdL}(\hat{\tau}_{dL}): t = 1 \cdots, 237\}$ is generated by algorithm (R2) from which $\frac{\theta_{\mathrm{ydL}}}{\theta_{\mathrm{xdL}}}$ is estimated. Then, we perform the t-test for testing inadvertent illiquidity smoothing or the null hypothesis H_{0L}: $\frac{\theta_y^*}{\theta_x^*} = \frac{\theta_{\mathrm{ydL}}}{\theta_{\mathrm{xdL}}}$. Table 4 reports the means and standard deviations of $\hat{\tau}_{dL}$ by 1,495 hedge funds in the TASS combined database. Table 4 also reports rejection rates in the t-test for each fund style to determine which style of hedge funds involves more inadvertent illiquidity smoothing in the last column.

The mean value of $\hat{\tau}_{dL}$ in Table 4 is the average across all funds in each style and indicates the average level of inadvertent illiquidity smoothing. The higher the value of $\hat{\tau}_{dL}$ implies more inadvertent illiquidity smoothing. Table 4 shows that four styles exhibit a higher average value of $\hat{\tau}_{dL}$ than the other styles: CA (0.88), FIA (0.85), MS (0.80), and EMN

(0.77). Note that CA and FIA funds have more illiquidity exposure than the other styles. These styles also show higher rejection rates for testing inadvertent illiquidity smoothing than those of the other styles. In contrast, DSB, LSE, and MF funds that exhibit lower average values of $\hat{\tau}_{dL}$ and rejection rates than other styles seem to involve less inadvertent illiquidity smoothing due to less illiquidity exposure. Overall, the results in Table 4 are also consistent with accepted intuition about the underlying assets' illiquidity exposure of the styles. Taken together, the results in Tables 2 and 4, where $\hat{\tau}_{dL}$ is higher than $\hat{\tau}_{dI}$ for all styles, imply that the level of inadvertent illiquidity smoothing is more significant than that of intentional smoothing. Moreover, the standard deviations of $\hat{\tau}_{dL}$ are higher than those of $\hat{\tau}_{dI}$. These results suggests that (i) return smoothing is mainly caused by the nature of the funds' underlying assets, although managerial discretion (intentional smoothing) is partly attributable to hedge fund smoothing (Getmansky et al., 2004; Cassa and Gerakos, 2011; Cao et al., 2017), and (ii) intentional smoothing is done more consistently than inadvertent illiquidity smoothing.

Table 4
Means and standard deviations of inadvertent illiquidity smoothing profile estimates and inadvertent illiquidity smoothing test results. This table reports the means and standard deviations of estimated $\hat{\tau}_{dL}$ for 1,495 hedge funds in the TASS database with at least ten years of return history during the period from January 1994 to September 2013. The rejection rate for testing the inadvertent illiquidity smoothing of each fund style is reported in the last column. The test results are based on the 5% significance level.

Style	Number of Funds	$\hat{\tau}_{dL}$		Rejection Rate
		Mean	SD	
CA	55	0.88	0.57	0.14
DSB	13	0.31	0.24	0.00
EM	125	0.60	0.90	0.09
EMN	60	0.77	0.89	0.12
ED	158	0.55	0.74	0.05
FIA	58	0.85	0.85	0.22
GM	63	0.57	0.89	0.06
LSE	589	0.41	0.80	0.03
MF	193	0.43	0.97	0.02
MS	181	0.80	1.09	0.20
Total	1495			

Table 5 reports $\hat{\tau}_{dL}$ and p-values for testing inadvertent illiquidity smoothing for 20 randomly selected individual funds from the sample. We randomly selected one inadvertently smoothed and one non-inadvertently smoothed fund from each style in our sample. Panels A and B of Table 5 contain 10 inadvertently and non-inadvertently smoothed hedge funds, respectively. In contrast to the average of Table 4, the $\hat{\tau}_{dL}$s among these 20 funds ranges from 0.0260 to 0.9850. As expected, funds that are found to involve inadvertent illiquidity smoothing in Panel A report a higher value of $\hat{\tau}_{dL}$ than those that are not found to involve inadvertent illiquidity smoothing in Panel B. Take, for example, Fund 1 in Table 5, which has 0.9850 of $\hat{\tau}_{dL}$. This implies that only 1.50% $(1-\hat{\tau}_{dL})$ of that fund's current monthly non-inadvertently smoothed return (Y_t) would be reported. The surprisingly large value of $\hat{\tau}_{dL}$ suggests a significant amount of inadvertent illiquidity smoothing by Fund 1. In contrast, Fund 17 in Panel B is not found to involve inadvertent illiquidity smoothing with $\hat{\tau}_{dL}$ of 0.0260, implying 97.40% $(1-\hat{\tau}_{dL})$ of that fund's current monthly non-inadvertently smoothed return (Y_t) would be reported. The small value of $\hat{\tau}_{dL}$ suggests that Fund 17 does not involve inadvertent illiquidity smoothing.

Table 5
Inadvertent smoothing profile estimates of 20 randomly selected funds from the sample. This table reports $\hat{\tau}_{dL}$ for 20 randomly selected funds from the 1,495 hedge funds in the TASS database that have at least ten years of return history during the period from January 1994 to September 2013. One inadvertently smoothed and one non-inadvertently smoothed fund is randomly selected from each style.

Panel A Ten inadvertently smoothed hedge funds from the sample

Hedge Fund	Style	Period	Number of months	$\hat{\tau}_{dL}$	p-value
Fund 1	CA	Oct 1996 – Dec 2008	147	0.9850	0.0226
Fund 2	DSB*	Not Available			
Fund 3	ED	Jul 2003 – Sep 2013	123	0.8820	0.0247
Fund 4	EM	Aug 1999 – Sep 2013	170	0.7310	0.0207
Fund 5	EMN	May1994 – Dec 2008	176	0.6510	0.0002
Fund 6	FIA	Jun 2001 – Sep 2013	148	0.9750	0.0262
Fund 7	GM	Feb 1997 – Nov 2009	154	0.4200	0.0233
Fund 8	LSE	Jan 1994 – Sep 2013	237	0.7590	0.0242
Fund 9	MF	Sep 1996 – Sep 2013	206	0.9640	0.0170
Fund 10	MS	Jan 1994 – Sep 2013	237	0.7840	0.0316

* No DSB fund is found to involve illiquidity smoothing.

Panel B Ten non-inadvertently smoothed hedge funds from the sample

Hedge Fund	Style	Period	Number of months	$\hat{\tau}_{dL}$	p-value
Fund 11	CA	Jan 2002 – Dec 2012	132	0.5110	0.3233
Fund 12	DSB	May 1996 – Sep 2013	209	0.5560	0.2743
Fund 13	ED	Jun 1994 – Oct 2005	137	0.0620	0.1354
Fund 14	EM	Jan 1994 – Jan 2007	157	0.4400	0.3133
Fund 15	EMN	Jan 1995 – Sep 2013	225	0.0290	0.1467
Fund 16	FIA	Mar 1996 – Sep 2013	211	0.6530	0.1615
Fund 17	GM	Jan 1994 – Mar 2006	149	0.0260	0.1962
Fund 18	LSE	Aug 1995 – Apr 2009	165	0.1490	0.4736
Fund 19	MF	Nov 1994 – Mar 2012	208	0.2330	0.4479
Fund 20	MS	Jan 2002 – Jun 2012	126	0.2130	0.4674

4. Conclusion

Though significant serial correlation in reported hedge fund returns suggests smoothing due to the lack of securities' prices or transparency, no econometric model has distinguished between intentional and inadvertent smoothing. The idea of separating out intentional from inadvertent smoothing is a very important practical problem. In this paper, we develop an econometric approach for finding smoothed returns for hedge funds and propose two distinct desmoothing algorithms against intentional smoothing and inadvertent illiquidity smoothing.

Our empirical results using the TASS database find that the nature of funds' underlying assets is the main source of return smoothing, and managerial discretion (intentional smoothing) is partly attributable to hedge fund smoothing; in addition, intentional smoothing is done more consistently than inadvertent illiquidity smoothing. Though it is not handled specifically in this paper, a case where further intentional smoothing is done over the returns that are already inadvertently smoothed can be handled straightforwardly by executing the two algorithms sequentially, i.e., implement the desmoother against intentional smoothing (R1) and then apply the desmoother against inadvertent illiquidity smoothing (R2) to the data that are already desmoothed against intentional smoothing.

Our methodology of desmoothing and testing an individual hedge fund's intentional or inadvertent illiquidity smoothing provides important references for hedge fund researchers and investment managers who use commercial databases and publicly available information on institutions' portfolio holdings. In principle, our desmoothing algorithm can be used for other smoothed variables such as real estate prices, not limited to hedge fund return smoothing.

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