Endogenous Market Structure: Over-the-Counter versus Exchange Trading *

Ji Hee Yoon†

Job Market Paper

October 25, 2017

Most Recent Version: Here

Abstract

For many assets, traders favor either over-the-counter (OTC) or centralized markets. This paper examines how traders' choice between these trading venues depends on asset and trader characteristics. Traders have private information with heterogeneous precision and their values depend on a common and idiosyncratic component. A trader's incentive to choose an OTC market depends on the benefit of learning the asset value and the potential cost due to price impact. Traders choose OTC markets over centralized exchanges when the idiosyncratic component dominates in asset values or their private information is sufficiently inaccurate, and thus, the benefit from learning is high. Market structures are endogenously determined by traders' individual market choices. This paper provides comparative statics of endogenous market structures. When traders are asymmetric, the OTC and centralized markets can coexist. Furthermore, the OTC market decreases information efficiency by being conducive to trade only between informed traders.

Keywords: Noncompetitive trading, Over-the-counter markets, Exchanges, Price impact, Liquidity, Efficiency

* I gratefully acknowledges the financial support from the Leon Mears Graduate Fellowships for this paper under the title "Incentives to Trade in Over-the-Counter Markets."

†Ph.D. Candidate, University of Wisconsin - Madison, Economics. Email: jyoon43@wisc.edu

1 Introduction

Over-the-counter markets have been an important alternative trading venue for many assets, goods, commodities, financial derivatives, and securities. In over-the-counter markets, buyers and sellers are paired and privately choose their own trading terms, while public exchanges use a centralized trading mechanism such as uniform-price auctions. Many commentators have raised concerns about the implications of the various market mechanisms for their role in facilitating information aggregation and efficiency. The goal of this paper is to examine these implications when traders individually choose a trading venue and the resulting market structure is endogenously formed by traders' choices.

Certain types of assets appear to be traded mostly in over-the-counter markets, whereas others have been traded in centralized exchanges. Corporate bonds, interest rate swaps, index derivatives, and many liquid financial products are mostly traded in over-the-counter markets, even with their standardized structures and high volumes of trade. For some assets, the over-the-counter and centralized markets coexist. For instance, foreign exchange is traded in both over-the-counter spot markets and centralized futures markets. A good portion of over-the-counter FOREX trading is in fast venues, such as Currenex, EBS, and Reuters, in which the foreign exchange spot price is generally the same as the price in centralized futures markets. This paper examines traders' incentives to choose over-the-counter markets and whether this type of trading venue can harm the efficiency of the economy.

This paper characterizes the endogenous market structure in which traders play a Bayesian Nash equilibrium in the over-the-counter and centralized trading venues. In the first period $t = 0$, each trader chooses to enter either a centralized market or an over-the-counter market that opens at $t = 1$. In the *centralized market*, all traders' demand and supply schedules determine the single market price. In the *over-the-counter market*, a pair of traders are matched to be pairwise stable; and then they trade bilaterally at a pair-specific price. Entering into the over-the-counter market, a trader observes types of others – the correlation between his and their asset valuations (buyers and sellers) and their information precision (informed and uninformed). In both the centralized market and over-the-counter bilateral trades, trade takes place in the uniform-price double auction, in which all traders simultaneously submit their (net) demand schedules $q_i(p) : p \mapsto \mathbb{R}$, and the trades clear at price p^* such that $\sum_i q_i(p^*) = 0$. Traders are uncertain about the value of a risky asset and receive a private signal about the value before any market opens. Signal precision can be heterogeneous among traders. Their asset valuations are interdependent and have two components: a common component, which is the same for all traders, and an idiosyncratic component, which can be correlated heterogeneously across traders. The common value component captures the asset return or a future price in the dynamic market. In turn, the idiosyncratic value component captures an individual portfolio return that is correlated to the asset traded. If traders' portfolios are correlated (e.g., they contain the same assets), then the trader's idiosyncratic values are also correlated.

This trading mechanism is the canonical model for non-competitive markets for divisible goods (e.g., Kyle (1989), Vives (2011), and Rostek and Weretka (2012)). The uniform-price double auction allows an explicit treatment of price impacts, which are the key equilibrium objects that determine traders' trading behaviors, as well as, the market choices. The literature based on the uniform-price mechanism so far has maintained a joint symmetry assumptions on traders risk aversions, the correlation in traders' asset values, and variance of values and uncertainty. Rostek and Yoon (2017) dispense with any symmetry restrictions to allow heterogeneity in all characteristics and primitives. One message in this paper is that heterogeneity in trader and asset characteristics matter for understanding traders' incentives to choose a market and endogenous market structure.

Heterogeneity in characteristics affects *liquidity* and *learning* for traders. Liquidity is measured by the price impact, which is endogenously defined as the change of price as a trader's demand increases by one unit in equilibrium.[1] Larger price impacts, i.e. lower liquidity, reduce traders' demands and lower their utilities. Allowing traders to condition their private information and prices, the demand schedule incorporates inference about values. Hence, price impact and inference are interdependent. Heterogeneous correlations and information precisions affect the liquidity and learning separately and determine the incentives for traders to choose an over-the-counter market.

The first result shows that when the idiosyncratic component dominates the common value, in the sense that if the dispersion of correlations is larger than the average level of correlations,[2] over-the-counter markets are more attractive to traders in terms of both learning and liquidity. Equilibrium price is a weighted average of traders' signals and thus aggregates *out* idiosyncratic components in centralized markets. When a trader's value relies more on the idiosyncratic value, he learns more about this component in an over-the-counter market. On the other hand, the liquidity incentive is independent of whether the common or idiosyncratic component dominates. An over-the-counter market allows a trader to choose a counterparty who would more likely have opposite trading needs (i.e., more negatively correlated asset values), so that the trader has a lower price impact, while the centralized price mitigates the difference between values of buyers and sellers. This effect is stronger when traders' asset values are interdependent through the idiosyncratic values rather than the common component.

The literature has studied various traders' incentives to trade in over-the-counter markets or alternative trading venues. Traders can benefit in over-the-counter trades by searching better

[1] Other frictions considered in the literature (e.g. search costs, a chance that bilateral trades fail, bid-ask spreads by dealers, etc.) can increase traders' incentives to choose centralized markets over over-the-counter markets, but the results in this paper still hold quantitatively.

[2] Suppose that $\rho_{i,j}$ denotes the correlation between individual asset values of two traders i, j. A trader i's value is correlated all other traders $\{\rho_{i,j}\}_{j\neq i}$. The variance of these correlation represents the idiosyncratic component in trader i's asset value, while the average of correlations represents the common component. See Section 2.

prices in an over-the-counter market (e.g., Vayanos and Wang (2007), Vayanos and Weill (2008), and Zhu (2013)) or by clearing their large trading needs that cannot be fully exhausted in the centralized market (e.g., Bessembinder and Venkataraman (2014), Ready (2014), and Degryse, Jon, and Kervel (2015)). In this paper, even when there is no difference in prices between an over-the-counter trade and centralized market, the over-the-counter market can open by traders' choices. This is because, for certain trader and asset characteristics, trading over-the-counter offers the benefit of improving learning and lowering price impact. Moreover, traders choose the over-the-counter market when the centralized exchange is competitive. These results show that the heterogeneous asset values of traders from large idiosyncratic value components are the new incentive of over-the-counter trading that this paper contributes to the literature.

Second, this paper shows that the incentives to choose over-the-counter versus centralized markets differ between informed and uninformed traders. Traders with low information precision (i.e., *uninformed traders*) benefit from an over-the-counter market because it helps them learn counterparties' information. On the other hand, the over-the-counter market discourages those whose asset values are less idiosyncratic or those with high information precision (i.e., *informed traders*) to participate, since it decreases the likelihood of meeting a counterparty to trade with and also may increase price impact. The trade-off between information and liquidity incentives in over-the-counter markets creates a cutoff level of information precision. If a trader's information precision is higher than the cutoff level, the liquidity incentive dominates and he chooses to trade in the centralized market. Likewise, with a precision lower than the cutoff, the learning incentive dominates and traders choose the over-the-counter market.

Taking into account that the market choice are individual, the over-the-counter market will attract uninformed traders or both informed and uninformed traders. In the over-the-counter matching, there are two structures of over-the-counter markets depending on a dominant incentive: When the learning incentive dominates for all traders, all prefer to trade with informed counterparties. An uninformed trader cannot be matched with his preferred counterparty, and thus, he chooses another uninformed counterparty instead. It creates a *same-type* (i.e., positive assortative) matching. On the other hand, if the dominant incentive differs between informed and uninformed, a *cross-type* (i.e., negative assortative) matching occurs. With an available centralized market, however, the cross-type matching does not occur in equilibrium. When an informed trader values low price impact more than improving learning, he is better off trading in the centralized market than in the over-the-counter trading with uninformed counterparties. With only uninformed traders entering in the over-the-counter market, information is not transmitted between the informed and uninformed traders and the over-the-counter market aggravates the information asymmetry in the economy.

Moreover, this paper shows that asymmetry in traders is necessary for the two trading venues to coexist in equilibrium. When traders asset values are interdependent with the same profile of correlations[3] and their information precisions are the same, the trading strategies and incentives in the market choices are symmetric for all traders. With these symmetric incentives, the endogenous distribution of traders in two trading venues has a corner solution, in the sense that either all traders choose the centralized market or all choose the over-the-counter market. With asymmetric traders, endogenizing market structure creates a fixed point problem between traders' incentives in market choices and the distribution of traders in two trading venues. Taking into account that the learning and liquidity incentives are functions of endogenized market structures, this paper identifies the types of traders trading in the over-the-counter market in equilibrium. The over-the-counter trading occurs between traders who have relatively smaller idiosyncratic value components or lower information precisions.

Lastly, this paper develops an algorithm to construct a pairwise stable over-the-counter matching when the asset characteristic is asymmetric across traders and shows that a stable matching exists with any arbitrary interdependence in traders' asset values. Although the existence of a stable equilibrium in one-sided matching is not trivial, traders' preference on counterparties following the ranking of negative correlations ensures that a pairwise stable over-the-counter matching exists. The stable matching may not be unique if some traders are indifferent between two or more counterparties, but it does not affect the qualitative results on the endogenous market structure and the identities of traders who are in each market.

The results in this paper help explain which traders choose the over-the-counter or centralized markets, which assets are traded in either type of trading venues, and when centralized and over-the-counter markets can coexist. Biais and Green (2007) show that transaction costs and liquidity are key determinants on why most trades for bonds are held in over-the-counter markets, while Attanasi, Centorrino, and Moscati (2016) explore the effects of lack of information in the over-the-counter market on efficiency. This is consistent with this paper's prediction. Many financial derivatives such as forwards contracts, interest rate swaps, or equity or credit linked securities are traded in over-the-counter markets, even though their trading volumes (liquidity) are large. When these products are held by traders until, or close to, the maturity, it suggests that the purpose of trading can be hedging of traders' outside portfolios so that they are idiosyncratically valued. On the other hand, centralized markets attract assets traded mostly by arbitrageurs or short-term investors, such as stocks or bonds with short maturity, which are valued by future prices that are common to all traders. High-yield bonds that have low credit ranking are often traded in the over-the-counter markets (e.g., Hendershott and Madhavan (2015)). Low past trading volume and volatile return prevent the traders' access to quality information, i.e., low information precision. Moreover, it is possible to increase the information asymmetry between insiders and other traders.

[3] Rostek and Weretka (2012) define a symmetric interdependence in traders' asset valuation by an *equicommonal model*, $\frac{1}{I-1}\sum_{j\neq i} Corr(\tilde{\theta}_i, \tilde{\theta}_j) = \bar{\rho}$ for all i. The symmetry condition in this paper is stronger than the equicommonal model. The profiles of correlations $\{Corr(\tilde{\theta}_i, \tilde{\theta}_j)\}_{j\neq i}$ are the same for all traders i. However, the symmetric model incorporates the heterogeneous correlations $Corr(\tilde{\theta}_i, \tilde{\theta}_j)$ across pairs of traders (i, j).

Related Literature: Literature has developed theoretical frameworks for over-the-counter markets for a fixed market structure. How liquidity affects traders' behavior and efficiency are studied in the literature (e.g., Duffie, Garleanu, and Pedersen (2005), Vayanos and Weill (2008), Weill (2008), Atkeson, Eisfeldt, and Weill (2014)). Other studies show how private information is aggregated in over-the-counter markets (e.g., Duffie, Malamud, and Manso (2014), Maurin (2015), Babus and Kondor (2016), Back, Liu, and Teguia (2016)). In addition to these papers focusing on over-the-counter markets, several papers compare these two trading venues in terms of welfare or individual profit. Observing that an over-the-counter markets can dominate centralized markets in welfare terms, several authors examine determinants that favor either market: such as default, search friction, price impacts, or information asymmetry between sellers and buyers (e.g., Acharya and Bisin (2010), Malamud and Rostek (2014), Duffie and Wang (2016), and Glode and Opp (2017)). Praz (2015) and Zhu (2014) studies how the presence of an alternating trading venue affect equilibrium in centralized markets.

Another strand of the literature endogenizes the over-the-counter structure itself by studying incentives to choose a counterparty in over-the-counter markets (e.g., Duffie, Carleanu, and Pedersen (2005), Golosov, Lorenzoni, and Tsyvinski (2014), Farboodi (2015)). Golosov, Lorenzoni, and Tsyvinski (2014) consider a dynamic incentive of uninformed traders to learn information at a cost of low liquidity in trading with an informed counterparty. In this paper, the joint effect (not necessarily a trade-off) of learning and price impact is the main determinant of counterparty choice. The learning and liquidity effects exist in the static trading due to the heterogeneity in precision (informed and uninformed traders) and correlation (asset characteristics). On the other hand, with heterogeneous preferences, traders search for a counterparty who can provide a better surplus (Chang and Zhang (2016)) or a better price (in Zhu (2012)). This paper shows that traders can strictly prefer one market to the other market or a counterparty to others even though equilibrium prices are the same across. Hence, the role of heterogeneity in determinants of counterparty choice is new in the endogenous market structure that this paper contributes to the literature.

The model in Babus and Parlatore (2017) is close to this paper. The authors examine over-the-counter dealer networks when trading is based on a uniform-price double auction as in this paper. In their paper, the endogenous choice of a segmented market (or the choice of a dealer) is determined by a trade-off between the price impact and the level of uncertainty in asset values. The contribution of this paper, compared to Babus and Parlatore (2017), is introducing a heterogeneous learning effect in endogenous over-the-counter matching and also in endogenous market choice.

The objective of this paper is to understand endogenous market structures when centralized and over-the-counter markets are both available. The choice between centralized and over-the-counter markets has been explored by several authors. Up to my knowledge, this paper is first to consider the market formation by all traders. Kirilenko (2000) and Viswanathan and

Wang (2002) consider a choice between trading venues for dealers by non-strategic agents (e.g. designers, authorities, consumers) maximizing profit or efficiency of the market. Bolton, Santos, and Scheinkman (2015) consider an entry problem of an informed seller to either market: a centralized (organized) market or an over-the-counter market with uninformed dealers. In this paper, all buyers and sellers, informed and uninformed, strategically choose a trading venue. Endogenizing the market choices of all traders lets the advantage of over-the-counter markets to be functions of endogenized participation and distribution of heterogeneous traders in two markets, rather than functions of fixed market structures. Furthermore, no trader has an incentive to change his market choice given chances in equilibrium with all traders' market choices, which provides a notion of the market stability.

2 Model

This paper considers a static economy where two trading venues open simultaneously: a centralized market where all traders' bids are executed at a single market price and an over-the-counter market where a pair of traders are matched and they trade bilaterally at a pair-specific price. Figure 2.1 summarizes the economy. Before the markets are open ($t = 0$), traders can choose which market they would trade in. If a trader chooses the over-the-counter market, then he also chooses a counterparty he would like to trade with. The market choice and bilateral matching occur at the end of period $t = 0$. Traders can trade only once, in one market and with one counterparty if they are in the over-the-counter market. At trading period $t = 1$, two assets – a risky asset (asset) and a riskfree asset (numeraire) – are traded in both markets. The assets are perfectly divisible. Traders submit their demands to the market they chose at the entering period, and each market cleared independently. I describe the details below, including (1) traders and payoffs, (2) information, (3) markets, (4) strategies, and (5) equilibrium.

Strategic Traders: There are $I < \infty$ strategic traders. Each of trader has a constant absolute risk-aversion (CARA) utility on quantity trading q_i net of payment $-pq_i$, where p is the price in the market he participates in. The ex-post utility is define as

$$u_i(q_i, p) = -\exp\Big(-\mu(\tilde{\theta}_i q_i - pq_i)\Big).$$

Here, $\mu > 0$ is the risk-aversion that is common for all traders, and $\tilde{\theta}_i$ is the individual value of the risky asset for trader i that is randomly drawn from $\tilde{\theta}_i \sim \mathcal{N}(E[\tilde{\theta}_i], \sigma_\theta^2)$. The conditional expected utility is equivalent to the mean-variance utility:

$$E[u_i(q_i, p)|\cdot] = -\exp\Big(-\mu\big(E[\tilde{\theta}_i|\cdot]q_i - pq_i - \frac{\mu}{2}Var(\tilde{\theta}_i|\cdot)q_i^2\big)\Big). \tag{1}$$

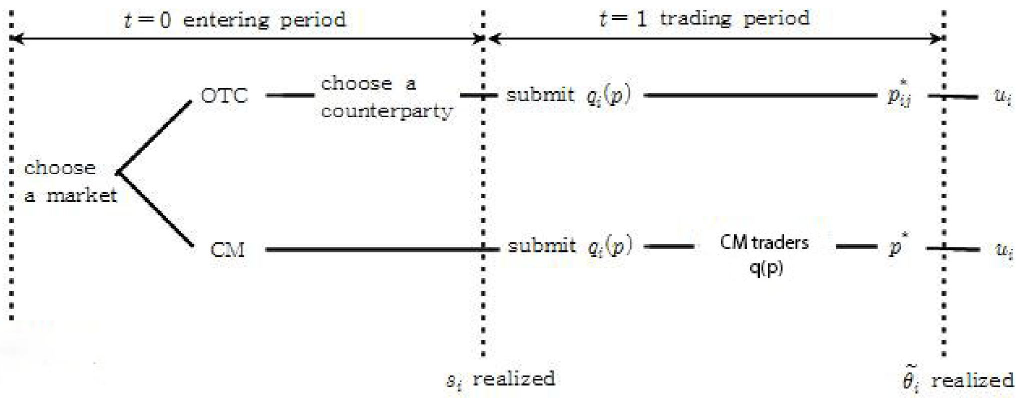

Figure 2.1: Timing of the economy. $q_i(p)$ is his quantity demand at a market price p. Trader i has a private information s_i before trading, and their individual asset value $\tilde{\theta}_i$ is realized at the end of economy and so does the utility u_i based on the realization of $\tilde{\theta}_i$ and equilibrium outcomes q_i, p^*.

In the CARA-Gaussian settings,[4] the conditional variance $V(\tilde{\theta}_i|\cdot)$ is a non-random constant independent of q_i, w_i or any realization in the markets, while the conditional expectation $E[\tilde{\theta}_i|\cdot]$ is a random variable that is determined by conditioning variables. Therefore, the expected utility (1) is equivalent to a *quadratic* utility with the coefficient on the first order term being random, which represented traders' expectation on asset value.

$$v_i(q_i, p) \equiv -\frac{1}{\mu}\log(-E[u_i(q_i)|\cdot]) = E[\tilde{\theta}_i|\cdot]q_i - pq_i - \frac{\mu}{2}Var(\tilde{\theta}_i|\cdot)q_i^2.$$

Traders' values $(\tilde{\theta}_i)_{i\in I}$ are interdependent. The correlation matrix for $(\tilde{\theta}_i)_i$ is denoted by $\widetilde{\Sigma} = (\tilde{\rho}_{ij})_{i,j\in I}$ with $\tilde{\rho}_{ij} \equiv Corr(\tilde{\theta}_i, \tilde{\theta}_j)$. The model allows any arbitrary Gaussian structure for traders' asset values. The interdependence of $(\theta_i)_i$ is interpreted as a combination of a *common value* component, which is attributed to the future asset return in the market, and an *idiosyncratic value* component, which comes from individual portfolio return consisting of other assets that are correlated to the trading asset in the market. The idiosyncratic value component is assumed to be independent to the common value component, but it can be correlated to other traders' idiosyncratic value components.[5] The decomposition of asset value $\tilde{\theta}_i$ is formalized as

[4]The CARA-normal settings is the standard of the non-competitive market literature. See Kyle (1989), Vives (2011), and Rostek and Weretka (2012). The non-competitive trading literature so far has focused on the cases when traders are symmetric in terms of asset valuations, interdependency, and/or information precision. One contribution of this paper is incorporating the heterogeneity in both interdependent asset valuation and information precision.

[5]This model with arbitrary interdependence of asset values can incorporates various interpretations and settings. For instance, as another interpretation of common and idiosyncratic value components: Each trader gets a random initial endowment before he enters the market, that can be correlated with other traders endowments. This private endowment forms his idiosyncratic value component. When the market has more trading rounds ($\tau > t = 1$) after the rounds we are considering in the model ($t = 1$), the asset value at t is determined by

follows:

$$\tilde{\theta}_i = \theta + \delta_i, \quad \forall\, i \in I,$$

where θ is the common value component and δ_i is the idiosyncratic value component of trader i. The common and idiosyncratic value components are independent and drawn from normal distribution, $\theta \sim \mathcal{N}(E[\theta], \sigma_{cv}^2)$ and $(\delta_i)_i \sim \mathcal{N}(0, \sigma_{iv}^2 \Sigma)$. The idiosyncratic values $(\delta_i)_i$ are interdependent by a correlation matrix,

$$\Sigma = \big(Corr(\delta_i, \delta_j)\big)_{i,j} = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1I} \\ \rho_{12} & 1 & \cdots & \rho_{2I} \\ \vdots & & \ddots & \vdots \\ \rho_{1I} & \rho_{2I} & \cdots & 1 \end{bmatrix}.$$

The correlations ρ_{ij} are *heterogeneous across pairs of traders* (i,j). I impose an assumption, without loss of generality, that the sum of correlations in a row $\sum_{j \neq i} \rho_{ij}$ is normalized to zero for all i. This is for keeping the heterogeneity from being additional source of common value, so that for making a clear separation of information aggregation.

Example 1 shows how the common value and idiosyncratic value components determine Σ in a simplest and intuitive setting.

Example 1 (Symmetric Interdependence in Asset Values) There are two groups of strategic traders – *buyers and sellers* – with equal group sizes.[6] Each trader has individual asset value that is decomposed into two independent random variables:

$$\tilde{\theta}_i = \theta + \delta_i = \begin{cases} \theta + \delta & \text{if } i \text{ is a buyer,} \\ \theta - \delta & \text{if } i \text{ is a seller.} \end{cases}$$

Suppose that $Var(\theta) = \sigma_{cv}^2$ and $Var(\delta) = \sigma_{iv}^2$. Then, the correlation matrix of idiosyncratic values $(\delta_i)_i$ is

$$\Sigma = \left[\begin{array}{c|c} \mathbf{1} & \mathbf{-1} \\ \hline \mathbf{-1} & \mathbf{1} \end{array}\right],$$

where each block represents $(\frac{I}{2} \times \frac{I}{2})$ matrix. From the distributions of common and idiosyncratic components, the correlation matrix of *total asset values* $(\tilde{\theta}_i)_i$ is

$$\widetilde{\Sigma} = \frac{1}{\sigma_{cv}^2 + \sigma_{iv}^2} \left[\begin{array}{c|c} \sigma_{cv}^2 + \sigma_{iv}^2 & \sigma_{cv}^2 - \sigma_{iv}^2 \\ \hline \sigma_{cv}^2 - \sigma_{iv}^2 & \sigma_{cv}^2 + \sigma_{iv}^2 \end{array}\right].$$

the marginal value function, which is a linear combination of individual asset return and future market prices. Hence, traders' valuation is interpreted as a combination of idiosyncratic value by holding the asset and common value by selling it at market price.

[6] Buyers and sellers are not explicitly determined in this model.

The interdependence of traders asset values is *symmetric* in this example, in the sense that the profile of correlation in each row is the same, $\{\rho_{ik}\}_{k\neq i} = \{\rho_{jk}\}_{k\neq j}$ for any $i \neq j$. However, the correlations $(\rho_{ij})_{i,j}$ are still heterogeneous across pairs of traders (i, j). I will consider this example for further analysis in Section 4, and will show the effects of the heterogeneous correlations across pairs and asymmetric interdependence across traders on endogenous market structures. □

Information: Each strategic trader gets a private information (signal) on his own valuation, $s_i = \tilde{\theta}_i + \varepsilon_i$ with an independent noise $\varepsilon_i \sim \mathcal{N}(0, \sigma^2_{i,\varepsilon})$. The information precision $\phi_i \equiv 1/\sigma^2_{i,\varepsilon}$ can differ across traders. The private signal s_i is realized at the beginning of trading period $t = 1$ and privately observed by trader i. The realizations of other traders' signals $(s_j)_{j\neq i}$ and prices in all markets are observed after all trades are done at the end of trading period $t = 1$.

Heterogeneity across traders, which is the key component in the model, is in both the correlation structure Σ and information precision $(\phi_i)_i$. Throughout this paper, I call this pair of asset and trader characteristics a *type*. Each trader's type represents his identity, such as buyers or sellers in Example 1. It is worth to remark that the type defined in this paper does not represent the realization of private signal s_i. This definition of type is different from the conventional definition in games with incomplete information. Traders' types and prior distribution of asset values and signals are common knowledge.

2.1 Centralized Market (CM) Mechanism

The centralized market is a large market where many buyers and sellers trade at a single price. I design the centralized market as a uniform-price double auction that is a canonical model in markets with divisible goods. A strategic trader i who enters the centralized exchange submits his net demand schedule $q_i(p) : \mathbb{R} \to \mathbb{R}$ (or a combination of limit and market orders) as a continuous function of price.

$$\max_{q_i(\cdot)} v_i(q_i, p) = \max_{q_i(\cdot)} \Big\{ E[\tilde{\theta}_i|s_i, p]q_i - pq_i - \frac{\mu}{2} Var(\tilde{\theta}_i|s_i, p)q_i^2 \Big\}, \quad \forall\, p \in \mathbb{R}. \tag{2}$$

Additional $L \geq 2$ traders, called *liquidity traders*, are introduced in the centralized market who are not given a choice to enter the over-the-counter market. Their presence ensures that the centralized market always function even when none of I strategic traders choose the centralized market.[7] The liquidity traders can be those who could not meet a requirement to enter the OTC, for example, not enough deposit or credibility. In a later section, I also show that liquidity

[7]Without this assumption on the presence of liquidity traders, there always exists a trivial equilibrium in which all strategic traders are in the over-the-counter market, independently of asset or trader characteristics. Two or more liquidity traders in the centralized market allows a strategic trader $i \in I$ who is currently in the over-the-counter market to consider an individual deviation to the centralized market.

traders can be those who do not observe other traders types so that they optimally choose to stay in the centralized exchange. Each liquidity trader submits their demand schedule $q_{lq}(p)$ based on his own private information. The precision of private information for liquidity traders are homogeneous and equal to the least informed traders: $\sigma^2_{lq,\epsilon} = \max_{i\in I} \sigma^2_{i,\varepsilon}$. Also, their asset values $\tilde{\theta}_{lq}$ are equal to the common value θ without any idiosyncratic value component. These assumptions on liquidity traders are to ensure that their presence does not affect the market choice of strategic traders $i \in I$.

After all demands are submitted, the centralized market is cleared at a price of which the total demand of I strategic traders and L liquidity traders is equal to zero; p^* such that $\sum_{i\in I} q_i(p^*) + \sum_{j\in L} q_{lq,j}(p^*) = 0$. The equilibrium allocation is determined by the demand schedule traders submitted, $q_i^* = q_i(p^*)$ for any $i \in (I \cup L)$.

2.2 Over-the-Counter Market (OTC) Mechanism

An over-the-counter market is an off-exchange trading venue in which bilateral trades occur between large institutions. Each trader who is in the over-the-counter market chooses a desired counterparty based on their types, information precision, and correlation. The choice should be mutual for two traders to be matched, in the sense that the over-the-counter matching is *pairwise stable*.[8] Each trader has an individual ranking on other traders, and based on the ranking, the matching will be determined by the algorithm of Irving (1985). If two traders are matched, they trade and leave the market. If the counterparty choice is not mutual, the matching fails and the trader searches for another counterparty until he succeeds at matching. I assume that the search cost is zero to focus on the difference of endogenous incentives in two trading venues. The over-the-counter market ends when all traders participate in exactly one bilateral trade or when only a single trader is left.

Once the matching occurs, each bilateral trade is operated by the same mechanism as in the centralized market: the uniform price double auction. Two traders simultaneously submit their demand schedules $q_i(p)$ as functions of price p by solving the optimization problem (2). The equilibrium price p_{ij} is determined by the market clearing condition: $q_i(p_{ij}) + q_j(p_{ij}) = 0$. The price p_{ij} is pair-specific in the over-the-counter market. If an equilibrium price does not exist, then there is no trade and the over-the-counter market ends without any further trade. The utility of traders in such case are set to be the autarky utility $v_i(q_i = 0) = 0$.

[8] The equivalence is due to the fact that traders' private signals are realized after their choice of the counterparty, and thus, the over-the-counter matching is determined by traders' ex-ante utility in the trade with each potential counterparty.

2.3 Market Choice and Trading

Based on the trading mechanisms in two markets described in Section 2.1 and 2.2, the strategies of traders and equilibrium are determined.

STRATEGIES: At $t = 0$, each strategic trader $i \in I$ chooses a market where he enters, $m_i \in \{OTC, CM\}$ and a type of counterparty τ_i upon his entering to the over-the-counter exchange $m_i = OTC$. When the market choice of a trader is $m_i = CM$, we will notate $\tau_i = \emptyset$ for the convenience. At $t = 1$, the trader chooses his demand function $q_i(\cdot : m_i, \tau_i)$ in market (m_i, τ_i). Therefore, the strategy profile of trader i is $\{(m_i, \tau_i), q_i(\cdot : m_i, \tau_i)\}$. A liquidity trader $j \in L$ in the centralized market has a strategy $\{(CM, \emptyset), q_j(\cdot; CM, \emptyset)\}$ since she cannot enter the over-the-counter market.

EQUILIBRIUM: Definition 1 provides three conditions for equilibrium: (i) Bayesian Nash equilibrium in the double auction in each market, (ii) no incentive to deviate from the over-the-counter market to the centralized market, and (iii) pairwise stable over-the-counter matching including a pairwise deviation from the centralized to over-the-counter market. For each trader $i \in I$, $E[u_i(m_i, \tau_i)]$ denotes the expected utility with (m_i, τ_i) for given equilibrium distribution of traders in both markets.

Definition 1 (Equilibrium) *An equilibrium is defined by* $\{(m_i, \tau_i), q_i(\cdot : m_i, \tau_i)\}_{i \in (I \cup L)}$ *such that*

(i) traders' optimal bid schedules $\{q^i(\cdot : m_i, \tau_i)\}_i$ *solving the optimization problem (2) characterize a Bayesian Nash equilibrium in each market;*

(ii) no trader in the over-the-counter market has a strictly positive incentive to deviate to the centralized market: i.e., if $(m_i^*, \tau_i^*) = (OTC, \tau_i^*)$ *for trader* i*, then*

$$E[u_i(m_i^*, \tau_i^*)] \geq E[u_i(CM, \emptyset)], \quad \forall i \in I; \quad \textit{and}$$

(iii) the over-the-counter matching is pairwise stable: i.e., there exists no pair of traders (i, j) *who are not matched such that both traders* i *and* j *strictly benefit from breaking their respective matchings and creating a new matching between them.*

$$E[u_i(m_i^*, \tau_i^*)] \geq E[u_i(OTC, j)] \quad \textit{or} \quad E[u_j(m_j^*, \tau_j^*)] \geq E[u_j(OTC, i)], \qquad \forall i \neq j \in I.$$

The inequalities in Definition 1 (iii) include the case where either trader i or j (or both) choose the centralized market, $m_i^* = CM$ or $m_j^* = CM$, in equilibrium. This equilibrium condition ensures that the over-the-counter matching is immune to an entry of a trader from the centralized market as well as within the over-the-counter counterparty choice. Furthermore, with this pairwise deviation from the over-the-counter to centralized market, a market structure where all traders choosing the centralized market (i.e. $m_i^* = CM$ for all i) is a trivial equilibrium, independently of asset or trader characteristics.

The following sections characterize equilibrium defined in Definition 1: Equilibrium bid strategies and outcomes in Bayesian Nash equilibrium for a given market - part (i) - is characterized in Section 3. The characterization allows us to develop comparative statics on traders' expected utilities over the market, asset, or traders characteristics. Section 4 and 5 show endogenous market structures that are formed by traders' market and counterparty choice - part (ii) and (iii) - and analyze influences of the characteristics in traders' market choices and thus in endogenous market structures.

3 Equilibrium in Double Auctions

This section shows traders' bidding strategies in a given market. Equilibrium characterization in a market provides traders' equilibrium utility and its dependence on market, asset, or trader characteristics.

Suppose that there are N traders in a market with a correlation structure of asset values $\widetilde{\Sigma}$ and information precision $\{\phi_i\}_i$. Each trader i maximizes his expected utility as in equation (1). The first order condition is characterized as follows:

$$E[\tilde{\theta}_i|s_i, p] - \mu Var(\tilde{\theta}_i|s_i, p)q_i - p - \lambda_i q_i = 0, \quad \forall\, p \in \mathbb{R}, \tag{3}$$

where $\lambda_i \equiv \partial p/\partial q^i$ is the *price impact* that represents the change of price when trader i increases his demand by one unit. A larger price impact implies that each unit of a trader's demand leads to a further increase in equilibrium price so that the trader's demand is reduced by higher price impact. This demand reduction due to the price impact represents *market illiquidity* that is endogenously determined by the traders' strategies. A competitive market with infinitely many traders is perfectly liquid and the price impact is zero. In terms of primitives, when there are fewer traders in the market or when traders are more sensitive to price changes due to inference or risk-aversion, the market becomes less liquid.

In a trader' first-order condition (3), he takes an expectation of asset value conditioning on the equilibrium price p, as well as, his own private information. A trader chooses his bid at each potential realization of price, and thus, his behavior incorporates the information revealed by the price as if he observed the price. Therefore, even in the static model, traders make inference about their asset values by the schedule bidding.

Proposition 1 states three equilibrium conditions for a given market with N traders whose asset values are correlated by Σ and whose information precisions are $\{\phi_i\}_i$: (i) a trader's strategy for a given price impact (illiquidity) and inference on asset values (learning); (ii) the consistency condition for equilibrium price impact; and (iii) the inference coefficients by equilibrium price distribution.

Proposition 1 (Equilibrium Representation in a Market) *In a market, a profile of demand schedules $\{q_i(\cdot)\}_i$ is a linear Bayesian Nash equilibrium (hereafter, equilibrium) if*

(i) a demand schedule $q_i(\cdot : \lambda_i)$ maximizing trader i's utility is

$$q_i = \frac{E[\tilde{\theta}_i|s_i,p]-p}{\mu Var(\tilde{\theta}_i|s_i,p)+\lambda_i} = \frac{c_{\theta,i}E[\theta_i]+c_{s,i}s_i-(1-c_{p,i})p}{\mu Var(\tilde{\theta}_i|s_i,p)+\lambda_i},$$

where $E[\tilde{\theta}_i|s_i,p] = c_{\theta,i}E[\theta_i]+c_{s,i}s_i+c_{p,i}p$,

(ii) price impacts satisfy the consistency condition

$$\lambda_i = -\Big(\sum_{j\neq i}\frac{\partial q_j(\cdot)}{\partial p}\Big)^{-1} = \Big(\sum_{j\neq i}\frac{1-c_{p,j}}{\mu_i Var(\tilde{\theta}_j|s_j,p)+\lambda_j}\Big)^{-1} \geq 0, \quad \forall i, \tag{4}$$

(iii) inference coefficients $\{c_{\theta,i}, c_{s,i}, c_{p,i}\}$ in $E[\tilde{\theta}_i|s_i,p]$ and conditional variance $Var(\tilde{\theta}_i|s_i,p)$ are determined by the Projection Theorem, with equilibrium price distribution following

$$p = (\sum_i \frac{1-c_{p,i}}{\mu Var(\tilde{\theta}_i|s_i,p)+\lambda_i})^{-1}\sum_i \frac{c_{\theta,i}E[\theta_i]+c_{s,i}s_i}{\mu Var(\tilde{\theta}_i|s_i,p)+\lambda_i}.$$

A trader' indirect utility in equilbrium can be written as a function of his price impact λ_i, expected asset value $E[\tilde{\theta}_i|s_i,p]$ conditioning on (s_i,p), and conditional variable $Var(\tilde{\theta}_i|s_i,p)$. Ex-ante utility of trader i in a give market is

$$E[u_i] = E\Big[-\exp\Big(-\mu(\frac{\mu Var(\tilde{\theta}|s_i,p)+2\lambda_i}{2(\mu Var(\tilde{\tilde{\theta}}_i|s_i,p)+\lambda_i)^2}(E[\tilde{\theta}_i|s_i,p]-p)^2)\Big)\Big].$$

The Gaussian structure of $\{\tilde{\theta}_i, s_i\}_i$ generates the difference in individual expected asset value from equilibrium price, $(E[\tilde{\theta}_i|s_i,p]-p)$, follows a normal distribution. Thus, the expectation on the right hand side of the above equation is in the form of the moment generating function for χ^2_k distribution. It provides an explicit formula for the ex-ante indirect utility:

$$E[u_i] = -\Big(1+\underbrace{\frac{1+2\widehat{\lambda}_i}{(1+\widehat{\lambda}_i)^2}}_{\text{liquidity effect}}\underbrace{\frac{Var(E[\tilde{\theta}_i|s_i,p]-p)}{Var(\tilde{\theta}_i|s_i,p)}}_{\text{learning effect}}\Big)^{-1/2}, \quad \forall i. \tag{5}$$

Here, $\widehat{\lambda}_i \equiv (\mu Var(\tilde{\theta}_i|s_i,p))^{-1}\lambda_i$ is a normalized price impact by the quadratic coefficient $\mu Var(\tilde{\theta}_i|s_i,p)$ of trader i's mean-variance utility. Trader i's ex-ante utility $E[u_i]$ can be decomposed into two parts: the value of liquidity and learning. The benefit of liquidity is captured by the term $(1+2\widehat{\lambda}_i)/(1+\widehat{\lambda}_i)^2 = 1-(\widehat{\lambda}_i/(1+\widehat{\lambda}_i))^2$. Recall that trader i's demand is reduced by a fraction $\widehat{\lambda}_i/(1+\widehat{\lambda}_i)$. In that,

$$q_i = \Big(1 - \frac{\lambda_i}{\mu Var(\theta_i|s_i,p)+\lambda_i}\Big)\frac{E[\tilde{\theta}_i|s_i,p]-p}{\mu Var(\tilde{\theta}_i|s_i,p)} = \Big(1-\frac{\widehat{\lambda}_i}{1+\widehat{\lambda}_i}\Big)q_i^{**}(p),$$

where $q_i^{**}(p)$ is the demand of trader i in a competitive market for a given price p. The demand reduction lowers utility with the same fraction. The liquidity benefit in utility terms, called *liquidity effect*, increases as the normalized price impact $\widehat{\lambda}_i$ increases. On the other hand, the utility (5) contains the term $Var(E[\tilde{\theta}_i|s_i,p]-p)/Var(\tilde{\theta}_i|s_i,p)$ that captures the benefit of learning from price and private information. Equilibrium price aggregates all market participants' private information on asset values. Traders learn the aggregated information from conditioning on price. It decreases the risk in uncertainty of the trader's own value θ_i and thus increases his expected utility through the term $Var(\tilde{\theta}_i|s_i,p)$. In addition, the price determines the net surplus $E[\tilde{\theta}_i|s_i,p]-p$ of buying a unit of asset. Such information on the future surplus influences the trader's utility through $Var(E[\tilde{\theta}_i|s_i,p]-p)$. The total benefit of learning the trader's own and others' valuation, called *learning effect*, is incorporated in a form of ratio in the expected utility (5).

3.1 Equilibrium Utilities: Learning and Price Impact

Now, this section characterizes the main objects of an equilibrium. Three characteristics can be considered in this model: market size (market characteristic), interdependence of traders' asset values (asset), and precision of private information (traders). Each characteristic affects learning and liquidity in traders' utilities. This section examines these effects of the three characteristics in a model with symmetric interdependent asset values and symmetric information precisions across traders.

Definition 2 (Symmetric Traders) *Traders are symmetric, if*

(i) the profile of correlations $\{\rho_{ij}\}_{j\neq i}$ is the same for all i; and

(ii) the information precision $\phi_i = \phi$ is the same for all i.

With the symmetric traders defined in Definition 2, traders submit symmetric strategies in each market, but the correlation of asset values is still heterogeneous across pairs, $Corr(\tilde{\theta}_i,\tilde{\theta}_j) = \rho_{ij}$ can differ for each pair (i,j), which is the key heterogeneity in traders' market choice. Asymmetric traders with asymmetric correlations and information precision will be considered in Section 5.[9] Example 2 and Proposition 2 show how the three key characteristics affect the ex-ante utility of each trader in symmetric markets.

Example 2 (Symmetric Interdependence and Precision) *Consider a market with N traders. All traders have a symmetric information precision $\phi_i = 1/\sigma^2_{i,\varepsilon} \equiv \phi$ and a symmetric average correlation to the residual market $\bar{\rho}_i = \frac{1}{I-1}\sum_{j\neq i}\rho_{ij} \equiv \bar{\rho}$ for all i. Each trader's optimal schedule and equilibrium price are*

$$\begin{aligned} q_i &= \frac{E[\tilde{\theta}_i|s_i,p]-p}{\mu Var(\tilde{\theta}_i|s_i,p)+\lambda} = \frac{c_\theta E[\theta]+c_s s_i-(1-c_p)p}{\mu Var(\tilde{\theta}_i|s_i,p)+\lambda}, \quad \forall\ i, \\ p &= \frac{1}{1-c_p}(c_\theta E[\theta]+c_s\frac{1}{I}\sum_i s_i) = \frac{1}{1-c_p}(c_\theta E[\theta]+c_s\bar{s}). \end{aligned}$$

Here, the liquidity and learning effects in trader i's ex-ante utility are characterized by the price impact and conditional variances:

$$\widehat{\lambda}_i = \frac{\lambda_i}{\mu Var(\tilde{\theta}_i|s_i,p)} = \frac{(1+(I-1)\bar{\rho})(1+\sigma^2-\bar{\rho})}{(I-2)(1+\sigma^2)+((I-1)^2+1-2(I-1)(1+\sigma^2))\bar{\rho}-(I-1)(I-2)\bar{\rho}^2},$$

$$Var(\tilde{\theta}_i|s_i,p) = \frac{(1+\sigma^2)+(I-2)\bar{\rho}-(I-1)\bar{\rho}^2}{(1+\sigma^2+(I-1)\bar{\rho})(1+\sigma^2-\bar{\rho})}\sigma^2_\varepsilon; \quad Var(E[\tilde{\theta}_i|s_i,p]-p) = \frac{(1-\bar{\rho})^2}{1+\sigma^2-\bar{\rho}}\frac{I-1}{I}\sigma^2_\theta.$$

Trader i gets the ex-ante utility $E[u_i] = -(1+\xi_i)^{-2}$ where

$$\xi_i = \frac{1+2\widehat{\lambda}_i}{(1+\widehat{\lambda}_i)^2}\frac{Var(E[\tilde{\theta}_i|s_i,p]-p)}{Var(\tilde{\theta}_i|s_i,p)}.$$

The liquidity effect on utility is captured by the term $\frac{1+2\widehat{\lambda}_i}{(1+\widehat{\lambda}_i)^2}$. With this closed-form solution of inference parameters and price impact,

$$\frac{1+2\widehat{\lambda}_i}{(1+\widehat{\lambda}_i)^2} = 1-(\frac{\widehat{\lambda}_i}{1+\widehat{\lambda}_i})^2 = 1-\Big(\frac{(1+\sigma^2-\bar{\rho})(1+(I-1)\bar{\rho})}{(I-1)(1-\bar{\rho})(1+\sigma^2+(I-1)\bar{\rho})}\Big)^2.$$

The liquidity term increases as I increases or $\bar{\rho}$ decreases. Larger market size and/or more negative correlation with others on average results in more liquidity, and thus higher utility for traders. When the information precision $\phi = 1/\sigma^2$ increases, the endogenous liquidity of the market increases if $\bar{\rho} > 0$, and decreases if $\bar{\rho} < 0$.

[9] In asymmetric markets, in the sense that the profiles of correlations $\{\rho_{ij}\}_{j\neq i}$ are heterogeneous across traders, as well as, across pairs (i,j) and/or that information precision ϕ_i are heterogeneous, traders' optimal trading strategies are asymmetric. The effects of characteristics in traders' behavior and utilities in this section and the comparative statics on endogenous market structures in Section 4 can also be applied to asymmetric markets. This section focuses on the symmetric market, in order to avoid technical complexities.

The effect of learning from the price on utility is measured by

$$\frac{Var(E[\tilde{\theta}_i|s_i,p]-p)}{Var(\tilde{\theta}_i|s_i,p)} = \frac{I-1}{I}\frac{(1-\bar{\rho})^2(1+\sigma^2+(I-1)\bar{\rho})}{\sigma^2((1+\sigma^2)+(I-2)\bar{\rho}-(I-1)\bar{\rho}^2)},$$

which is increasing in information precision $\phi = 1/\sigma^2$. *The effect of average correlation* $\bar{\rho}$ *is ambiguous. The utility component due to learning is decreasing with respect to* $\bar{\rho}$, *if and only if,* $(1+\sigma^2)+(2I-3)(1+\sigma^2)\bar{\rho}+(I-3)(I-1)\bar{\rho}^2-(I-1)^2\bar{\rho}^3>0$. □

Proposition 2 characterizes the effects of each characteristic - market size, correlations, and information precision - on traders' expected utilities through learning and liquidity, when the other characteristics are fixed.

Proposition 2 (Liquidity and Learning Effects) *For a given market, the equilibrium utility of a trader* i *increases as*

(i) the number of traders in market N *is larger; or*

(ii) asset values are more negatively correlated to price, i.e., $corr(\tilde{\theta}_i, p)$ *are more negative.*

The equilibrium utility is non-monotone in the average information precision, i.e.,

(iii) information precision $\phi^*_{-i} \in (0,\infty]$ *of other traders maximizes trader* i*'s utility.*

The effects of the number of traders (part (i)) on learning and liquidity have been studied in literature. In a sufficiently symmetric market, with more traders participating in the market, price reveals more accurate information. Moreover, price impact can be small in large markets, when other characteristics are fixed (See Rostek and Weretka (2012)). Proposition 2 (ii), however, suggests that the benefits of large markets in learning and liquidity are not necessarily true if traders in each market, large or small, have heterogeneous asset valuations. Suppose that the equilibrium price in a small market exhibits greater negative correlations with trader i's asset valuation. Price provides new information that is not captured in trader i's private information, and with larger correlation in the absolute sense implies that the information is more relevant to his asset value. This learning effect is captured by the decrease in conditional variance $V(\tilde{\theta}_i|s_i,p)$. The correlation structure also affects the liquidity through the endogenous price impacts λ_i. The price impact is characterized by the slope of the residual supply curve, $\lambda_i = -\big(\sum_{j\neq i}\partial q_j(\cdot)/\partial p\big)^{-1}$, that is an inverse of the aggregate reaction of other traders when price increases. With more negative correlations, trader $j \neq i$ would rely more on the price for his inference, in the sense that $c_{p,j}$ is more negative. This makes his demand more elastic to price change and thus trader i's price impact is smaller. Hence, his equilibrium utility increases due to both learning and liquidity as the correlation between his asset values and price is more negative. From the arguments, more negative correlations between traders' asset values and/or more traders in the market are beneficial to both learning and liquidity. These conditions of two characteristics jointly determine traders' incentives to choose a market, which will be examined in the next Section 4.

In addition to the joint condition of market size and interdependent asset valuation, the last characteristic that affects equilibrium is the precision of traders' private information. Information precision has an ambiguous influence on traders' expected utilities. The value of learning increases when the trader's own information precision is lower or when the (weighted) average of other traders' information precision is higher. At the same time, the price impact increases, and thus, the liquidity decreases. It creates a *trade-off* between learning and liquidity when the precision of information from the price changes. The trade-off between learning and liquidity over traders' information precision is shown in Figure 3.1. As a result, trader i's utility is non-monotone with respect to the other other traders' information precision. If his precision is sufficiently low, the learning effect dominates the liquidity effect, so that his utility is monotonically increasing in others' precision.

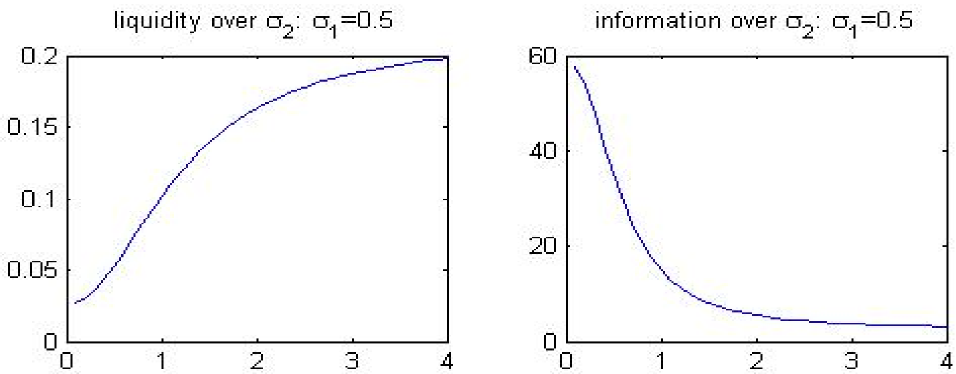

Figure 3.1: Liquidity and learning effects with respect to other traders' information precision $1/\sigma_2$. A trader i has a information precision $\phi_1 = 1/\sigma_1^2 = 1/0.25 = 4$. The liquidity effect and the learning effect in his expected utility (5) are shown in each graph respectively. Two effects are monotone over the other traders' information precision $\phi_2 = 1/\sigma_2 \equiv avg(1/\sigma_j^2)_{j\neq 1}$.

The comparative statics show the types of traders who would enter an over-the-counter market based on the interdependence of asset values and the precision of their information. The ambiguity in the influence of information precision on traders' utilities will be considered as a key determinant of traders' market and counterparty choices.

4 Endogenous Market Structure

Traders' individual choice for markets and counterparties forms a distribution of traders' types in centralized and over-the-counter markets and also an over-the-counter matching. This is called a *market structure*. This section characterizes endogenous market structures by using the comparative statics developed in the previous section. There are three types of market structure: only centralized market opens, only the over-the-counter market opens, and both markets co-exist. Example 2 with a competitive centralized market (i.e. perfectly liquid market with $\lambda_i = 0$ for all i) provides some intuition on endogenous market structures. The competitive centralized market maximizes the difference in market sizes. The example shows that traders can still be attracted to the over-the-counter market with certain condition for asset and trader characteristics.

Example 1 & 2 - Cont'd *Equation (5) provides the explicit formula of expected utility when there are N symmetric traders. The utility from the centralized trading is derived by taking N to infinity, while the utility from a bilateral trades in the over-the-counter market is by setting $N = 2$. With these ex-ante utilities in two exchanges, trader i chooses which exchange he wants to participate in. Under the equilibrium existence, the necessary and sufficient condition for him to enter the over-the-counter exchange is as follows:*

$$E[u_i^{CM}] < E[u_i^{OTC}] \quad \Leftrightarrow \quad \frac{1 - \bar{\rho}_{CM}}{\sigma^2} < \frac{-2\rho_{OTC}}{1 + \sigma^2 + \rho_{OTC}} \mathbf{1}_{\{\rho_{OTC}<0\}}.$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function. Here, $\bar{\rho}_{CM} = \frac{\sigma_{cv}^2}{\sigma_{cv}^2+\sigma_{iv}^2}$ is the average correlation in the centralized market and $\rho_{OTC} = \frac{\sigma_{cv}^2-\sigma_{iv}^2\rho}{\sigma_{cv}^2+\sigma_{iv}^2}$ is the correlation between two traders who are matched in the over-the-counter market, which are sufficient statistics on $Corr(\tilde{\theta}_i, p)$ in two markets. Consider the following canonical assumptions on traders' asset valuation:

- *Independent Private Value: $\sigma_{cv}^2 = 0, \sigma_{iv}^2 = 1$, and $\rho = 0$. With independent private (idiosyncratic) value, traders get no gain-to-trade in the over-the-counter market, i.e., $\frac{-2\rho_{OTC}}{1+\sigma^2+\rho_{OTC}}\mathbf{1}_{\rho_{OTC}<0} = 0$, while they get strictly positive equilibrium utility $1/\sigma^2$. Therefore, all traders choose the centralized market, and the over-the-counter market does not open. Independent private value structure implies that the market has no valuable information to any trader (no learning occurs), so all traders choose the centralized market to benefit from the liquidity.*

- *Fundamental Value (Vives (2011)): $\sigma_{cv}^2 = 1 - a, \sigma_{iv}^2 = a$ with a constant $a \in (0, 1)$, and $\rho = 0$. Taking $a \to 0$, the fundamental value model includes the common value: $\tilde{\theta}_i = \theta$ for all i. With the fundamental value model, the over-the-counter trade provides no gain-to-trade since $\rho_{OTC} > 0$. On the other hand, traders get strictly positive equilibrium utility ϵ/σ^2, and thus, all traders choose the centralized market. The over-the-counter market does not exist. Fundamental value model does not incorporate heterogeneous correlations across pairs of traders, in the sense that $Corr(\tilde{\theta}_i, \tilde{\theta}_j) = \frac{\sigma_{cv}^2}{\sigma_{cv}^2+\sigma_{iv}^2}$ for any pair (i, j). Hence, it concludes that the heterogeneous correlation is necessary for a trader to choose the over-the-counter market.*

- *Two-Sided Market with Buyers and Sellers (Example 1): $\sigma_{cv}^2 = 1-a, \sigma_{iv}^2 = a$, and $\rho = \pm 1$.*

Traders choose the over-the-counter market if and only if

$$\frac{a}{\sigma^2} < \frac{4a-2}{2-2a+\sigma^2} \quad \Leftrightarrow \quad \underbrace{a > \frac{2-3\sigma^2+\sqrt{(2+3\sigma^2)^2-8\sigma^2}}{4} > \frac{1}{2}}_{\text{iv is sufficiently large}} \quad \text{and} \quad \underbrace{\frac{2a(1-a)}{3a-2} < \sigma^2}_{\text{precision is sufficiently low}}. \tag{6}$$

Traders prefer to trade in the over-the-counter market if the idiosyncratic value component in asset values is sufficiently large and the information precision is sufficiently small.

When the centralized exchange is competitive, the liquidity incentive strongly derives traders to avoid higher price impacts in over-the-counter bilateral trades. Hence, participation to the over-the-counter market occurs only when the benefit of learning from the market is high enough to dominate the loss from illiquidity. When the aggregated correlation $\bar{\rho}_{CM} = \frac{1}{I-1}\sum_{j\neq i}\rho_{ij}$ *satisfies* $|\bar{\rho}_{CM}| < |\rho|$*, the price informativeness is higher in the over-the-counter exchange and thus the benefit from learning is higher.* □

The predictions from the above example hold generally in the model with a non-competitive centralized market and/or with asymmetric traders. (i) Traders choose the over-the-counter market over the centralized market if the idiosyncratic value component in asset values is sufficiently large and the information precision is sufficiently small; (ii) The over-the-counter market does not exist if there is no heterogeneity across traders' asset valuation.

Proposition 2 showed that equilibrium utility of a trader increases as the number of traders who are participating in the market increases or as the correlation between his asset value and the market price is more negative. With the presence of $L \geq 2$ liquidity traders in the centralized market the number of traders (hereafter, market size) is larger in the centralized market and its effect encourages traders to enter into the centralized market. However, the effect of correlation between asset value and price is the opposite. In the over-the-counter market, a trader can target a counterparty that has the most negative correlation, while the correlation with the CM price is determined by the average correlation over all participants. Hence, the effect of correlation $Corr(\tilde{\theta}_i, p)$ supports traders choosing the over-the-counter market. It is worth emphasizing that from the comparative statics in Proposition 2, the correlation effect reduces the price impact, as well as, improves learning. Hence, the benefit of more negative correlation in the over-the-counter market is not only from better learning but also from low price impact.

The effects of market size and correlation create a trade-off in the traders' choice of market. Theorem 1 examines when the correlation effect would dominate and provides a sufficient and necessary condition such that the trader prefers to trade in the over-the-counter market and endogenous market structure following the traders' individual market choices.

	OTC		CM
# of trader	2	<	$L+I$
$corr(\tilde{\theta}_i, p)$	$corr(\tilde{\theta}_i, \tilde{\theta}_j)$	<	$\frac{1}{I-1}\sum_{j\neq i} corr(\tilde{\theta}_i, \tilde{\theta}_j)$

Table 1: Traders' market choices and trade-off between market sizes and correlations

Theorem 1 (Endogenous Market Structure) *The over-the-counter market opens in equilibrium by some traders entering into the market, if and only if*

$$\sigma_{iv}^2/(\sigma_{cv}^2+\sigma_{iv}^2) \;>\; \hat{\kappa}(\{\phi_i\}_i, \Sigma) \quad \text{and} \quad \phi_i = 1/\sigma_{i,\varepsilon}^2 \;<\; \hat{\phi}_i(\sigma_{cv}, \sigma_{iv}, \Sigma)$$

for some bounds $\hat{\kappa} < \infty$ and $\hat{\phi}_i > 0$.

The first inequality in Theorem 1 implies that traders choose the over-the-counter market when their asset values are sufficiently heterogeneous by having a large variance of idiosyncratic value component $\sigma_{iv}^2/(\sigma_{cv}^2+\sigma_{iv}^2)$. It implies that the targeted bilateral trades in the over-the-counter market can benefit a trader, in terms of both learning and price impact. Suppose that a trader (say, a buyer) has a negative correlation with sellers but has a positive correlation with another buyer. When both buyers and sellers participate in the centralized market, the price that aggregates values of all traders mitigates the correlation so that the average correlation $\bar{\rho}$ gets closer to zero. In a bilateral trade, the correlation with a single counterparty (i.e. seller) can demonstrate a greater negative value than the average correlation in the centralized market. The difference between these correlations can be enhanced when the asset valuation relies more on the idiosyncratic value component than the common value component. This helps explain why assets that have a strong common value component tend to be traded in the centralized market, while assets that are heterogeneously valued tend to be traded in the over-the-counter market. Furthermore, the second inequality condition in Theorem 1 shows that the dominance of correlation effect is strengthened by low information precision. This is because the trader's inference depends more on the price, so the difference between correlations is emphasized more in the trade-off between the effects of market size and correlations. The low information precision, *a characteristic of traders*, creates a joint condition with the sufficiently heterogeneous asset valuation, *a characteristic of assets*, for the over-the-counter market to be chosen by traders in endogenous market structure.

The predictions in Theorem 1 are consistent with financial markets. Many over-the-counter products, such as forward contracts or corporate bonds, are traded for the purpose of hedging. The portfolio they need to diversify is idiosyncratic so that asset values are valued by idiosyncratic value components. On the other hand, stocks or options that are often traded by speculators are valued by the common values, which are the future prices in the market.[10] It should be noted that the condition for over-the-counter markets to exist is a joint condition on asset valuation and information precision. Bonds with low credit rating, which traders would not have precise information due to its volatile value, are often traded in over-the-counter markets. Alternatively, treasuries and high ranked bonds are also traded in centralized futures markets, even though they are valued idiosyncratically.

The following two corollaries show some properties of traders' market choice and endogenous market structure, other than learning and liquidity in this paper. These properties have been studied in literature. Namely, the literature has shown that the over-the-counter markets can be beneficial for (i) providing an additional trading opportunity to traders who could not fully clear their trading needs in centralized markets, and for (ii) allowing traders to search for better prices than the centralized market price. Despite these benefits of trading in over-the-counter markets being present in this model, Corollary 1 and 2 show that the over-the-counter market can be still chosen by traders even without the illiquidity in centralized markets or the price difference between trading venues.

Corollary 1 (OTC with a Competitive CM) *Suppose that the centralized market is competitive, i.e., the number of liquidity traders is large, $L \to \infty$. There exists a set of correlation Σ and precision ϕ, satisfying the inequalities (6), such that all strategic traders choose the over-the-counter market over the competitive centralized market.*

Corollary 1, as seen in Example 2 - Cont'd, shows that traders may prefer to trade in the over-the-counter market even when the centralized market is perfectly liquid, and thus, all traders choose to trade over-the-counter and only liquidity traders participate in the centralized market. Recent studies (e.g., Bessembinder and Venkataraman (2014), Ready (2014), and Degryse, Jong, and Kervel (2015)) show that the over-the-counter market or off-exchange trading venues can open because of the illiquidity (i.e. non-competitiveness) of centralized markets: Trading needs of institutions who have large endowments may not be fully absorbed in the centralized market, and thus, these traders would trade in all available trading venues subject to the entry costs. An over-the-counter market is a platform that provides more trading opportunities. Traders' improved learning about heterogeneous asset values can favor the choice of an over-the-counter market, even if the price impact were lower in the centralized market.

Corollary 2 (No Price Difference in OTC and CM) *There exists a set of primitives such that the expected prices are the same in two markets and traders choose the over-the-counter market.*

[10]Another canonical example on over-the-counter products is a housing market. Even though the unit demand for houses does not directly fit the divisible good model in this paper, the intuition carries over. Houses are valued in different aspects by different traders, and through brokers, the over-the-counter market matches a buyer and a seller who have highly correlated valuations.

Literature that studies an over-the-counter market mechanism with search and bargaining (e.g., Zhu (2013), Vayanos and Wang (2007), and Vayanos and Weill (2008)) shows that traders choose to trade in over-the-counter markets when searching for better prices. Traders can choose an over-the-counter market even when the prices in the two markets are the same (Corollary 2). This is because, for certain traders and asset characteristics, trading in over-the-counter markets offers improved learning and lower price impact.

5 Market Structure with Asymmetric Traders

Theorem 1 examined a trader's choice between the over-the-counter and centralized markets and the endogenous market structures. The key characteristics is the heterogeneous correlations in asset valuation $Corr(\tilde{\theta}_i, \tilde{\theta}_j)$ across pairs of traders (i, j). This section considers the effects of other types of heterogeneities: namely, heterogeneous correlations across traders, not only across pairs, and heterogeneous information precision. A model with such heterogeneities across traders is called an *asymmetric* model, and a model without the heterogeneities a *symmetric* model (See Definition 2).

With symmetric traders, all bilateral matches in the over-the-counter market are identical in the sense that the correlation between two matched traders is the same for all pairs and $Corr(\tilde{\theta}_i, \tilde{\theta}_j) = \min_{j \neq i} Corr(\tilde{\theta}_i, \tilde{\theta}_j)$ for any i. It leads all traders to have equal incentives to choose either market by fixing the correlation difference between the two markets to be symmetric for all traders. All strategic traders are symmetric in the market choice. On the other hand, the incentives to choose either market can differ by traders in asymmetric markets, and thus, the incentives of traders in the choice of the market are functions of distribution of traders in two markets as well as of endogenized market structure. The next theorem shows that the asymmetry across traders is necessary for the coexistence of two trading venues: the over-the-counter and centralized markets.

Theorem 2 (Coexistence of CM and OTC) *The centralized and over-the-counter market coexist in equilibrium only if traders are asymmetric.*

Suppose that a trader gains from deviating from the over-the-counter to centralized market. His deviation increases the market size in the centralized market. With the fixed correlation difference as discussed above, the incentive to deviate from the over-the-counter market to the centralized market for the next trader is even stronger. Recursively, all traders end up choosing the centralized market. The deviation from the centralized market to the over-the-counter market follows the similar argument, by recursively increasing incentives to deviate due to decreasing market size in the centralized market. It concludes that the endogenous distribution of traders in two trading venues has a *corner* solution when traders are symmetric. Equivalently, the over-the-counter and centralized markets can coexist in equilibrium only if traders are asymmetric: in the sense that either (i) the profile of correlations $\{\rho_{ij}\}_{j\neq i}$ is asymmetric or (ii) the information precision ϕ_i is asymmetric across traders.

In an endogenous market structure in which both trading venues coexist, some strategic traders choose the over-the-counter market while other traders choose the centralized market. The next sections examine which types of traders are more attracted to the over-the-counter market versus which are more attracted to the centralized market. Section 5.1 explores the effect of heterogeneous correlation profiles and Section 5.2 examines the heterogeneous information precision in both endogenous over-the-counter matching and market structure.[11]

5.1 Asymmetric Interdependence of Asset Valuations

Suppose that traders are asymmetric in the sense that their profiles of correlation $\{\rho_{ij}\}_{j\neq i}$ differ. In the over-the-counter market, each trader can choose his counterparty based on the type, i.e. correlation structure. The matching fails if the choice of the counterparty is not mutual. In addition, even when two traders are matched, the trade does not occur if the traders have positively correlated asset values. In that, if $Corr(\theta_i, \theta_j) = \rho_0 > 0$, traders optimal bid function becomes inelastic so that there is no trade. No trader chooses a counterparty whose values are positively correlated with his. Consequently, if a trader i's asset value satisfies $Corr(\tilde{\theta}_i, \tilde{\theta}_j) > 0$ for any $j \neq i$, i.e., his asset valuations are positively correlated with all other traders, then he gets zero gains-to-trade in the over-the-counter market which makes him enter the centralized market.

Recall that the over-the-counter matching is determined by traders' ranking on counterparties and that a trader prefers to trade with the one whose value is more negatively correlated with his. Traders who have relatively less negative correlations may not be matched with whom he wants, or he may not be matched with anyone. These traders will not be chosen by the ones that they want to be matched with. This lowers the benefit of the over-the-counter market and makes them choose the centralized market.

Proposition 3 (OTC Matching with Heterogeneous Correlations) *Suppose that traders are asymmetric in correlation structures but symmetric in information precision. There exists a pairwise stable over-the-counter matching determined by the ranking in the negative correlations* $(-Corr(\tilde{\theta}_i, \tilde{\theta}_j))$ *of pairs of traders.*

The pair-wise stable over-the-counter matching is determined by the following algorithm:

[11] Since their incentives in the choice of the market are functions of endogenized market structure, traders' market choice and endogenous market structure create a fixed point problem in equilibrium. This complexity makes it difficult to analyze endogenous market structure in general. Hence, the two-dimensional asymmetry - heterogeneous correlation profiles and heterogeneous information precision - will be considered separately. The results from each heterogeneity can be jointly studied in a model with an arbitrary correlation structure and information precisions.

STEP-1. Two traders, who have the most negative correlation among all pairs, are matched and the matching is denoted by (i_1, j_1). If there are multiple pairs that have the most negative correlation, select one pair randomly.

STEP-2. Eliminating the selected traders, select the most negative correlation among the remaining paper and create another pair: called (i_2, j_2).

STEP-3. Repeating this procedure until there is at most one remaining trader in the over-the-counter market, and then the over-the-counter matching is determined $\{(i_t, j_t)\}_{t=1,2,\ldots}$.

In general, a stable equilibrium may fail to exist in one-sided matching problems. However, if traders are asymmetric only in correlation structure, a pairwise stable over-the-counter matching always exists in endogenous market structure with the endogenized choice of market and counterparty. First, the ranking based on negative correlations in Proposition 3 guarantees the transitive property on ranking. It is shown by the fact that the rankings of traders do not create any circular preferences. Suppose that there exists traders i, j, and k whose rankings are circular: i prefers j to k, j prefers k to i, and k prefers i to j. By Proposition 3, it implies that $Corr(\tilde{\theta}_i, \tilde{\theta}_j) \leq Corr(\tilde{\theta}_i, \tilde{\theta}_k) < 0$, $Corr(\tilde{\theta}_j, \tilde{\theta}_k) \leq Corr(\tilde{\theta}_j, \tilde{\theta}_i) < 0$, and $Corr(\tilde{\theta}_k, \tilde{\theta}_i) \leq Corr(\tilde{\theta}_k, \tilde{\theta}_j) < 0$. Hence, the correlations between the three traders satisfy $Corr(\tilde{\theta}_i, \tilde{\theta}_j) = Corr(\tilde{\theta}_j, \tilde{\theta}_k) = Corr(\tilde{\theta}_k, \tilde{\theta}_i)$, and thus, each trader is indifferent between the other traders in his counterparty choice. In addition to the stability, the individual market choice keeps a trader from not being matched in the over-the-counter market. A trader who fails to find a counterparty in the over-the-counter market deviates to the centralized market. Therefore, a stable over-the-counter matching exists in equilibrium due to the ranking mechanism in Proposition 3 and the endogenized market choice between over-the-counter and centralized markets.

5.2 Asymmetric Information Precision

In order to understand the effect of heterogeneity in information precision on endogenous market structure, suppose that traders are asymmetric only in information precisions but symmetric in the correlation structure, i.e., ϕ_i is different across i but the profile $\{\rho_{ij}\}_{j\neq i}$ is the same for all i. Proposition 2 (iii) shows that a trader's equilibrium utility is non-monotone over other traders' information precision. A trader prefers an informed counterparty to improve learning, but he would prefer an uninformed counterparty to have a lower price impact. Concerning this trade-off between learning and price impact over the information precision of counterparty, Proposition 4 shows how a trader ranks the counterparty depending on his own information precision.

Proposition 4 (Trade-Off in Learning and Price Impact over Precision) *In a bilateral matching between traders i and j, the equilibrium utility (5) of trader i satisfies that* *(i) the liquidity effect* $\frac{1+2\widehat{\lambda}_i}{(1+\widehat{\lambda}_i)^2}$ *decreases in* ϕ_j*; and*

(ii) the learning effect $\frac{Var(E[\tilde{\theta}_i|s_i,p]-p)}{Var(\tilde{\theta}_i|s_i,p)}$ *increases in* ϕ_j*.*

The liquidity effect dominates the learning incentive if and only if the trader's own precision ϕ_i *is sufficiently high.*

In the trade-off between learning and price impact, which effect dominates depends on the trader's own precision. If trader i's own information precision is already high, lower price impact is more valuable than better learning so that he chooses a relatively less informed counterparty. On the other hand, if his precision is low he prefers a more informed counterparty for better learning. The trade-off between learning and liquidity gives a non-monotone preference on the counterparty's information precision, in the sense that the optimal counterparty's information precision $(\phi_{-i})_i^* = arg\max_{\phi_{-i}=1/\sigma_{-i}^2} E[u_i; OTC(i,j)]$ is determined in the interior of support $(\phi_{-i})_i^* \in (0, \infty)$ for traders with sufficiently large ϕ_i. Furthermore, the optimal counterparty's precision $(\phi_{-i})_i^*$ is decreasing in trader i's own precision ϕ_i.

It is useful to consider a model with two types of information precisions, informed and uninformed types. The information precisions of two types are assumed by $\phi_U = 1/\sigma_U^2 < \phi_I = 1/\sigma_I^2$. With two informational types, the over-the-counter matching is either positive assortative matching (i.e., *same-type* matching) or negative assortative matching (i.e., *cross-type* matching).[12] The same-type matching is when traders are matched with other traders with the same type, and cross-type matching is when informed and uninformed traders are matched to each other. Figure 5.1 presents regions of $\{\sigma_I, \sigma_U^2\}$ for the same-type and cross-type matching in equilibrium. First, consider the case where informed traders have sufficiently high precision (small σ_I) and uninformed traders have sufficiently low precision (large σ_U). Both types of traders choose to trade with the opposite type of counterparty, with different incentives: informed traders get benefit from liquidity incentive, and uninformed traders benefit from learning incentive. Hence, the cross-type matching occurs in equilibrium. Outside of this region, equilibrium shows the same-type matching. When the precision levels for both traders are high (small σ_I and σ_U), liquidity incentives dominate and all traders want to be matched with an uninformed counterparty. As a result, some informed traders can not be matched with their preferred counterparty. Since there is no more matching opportunity, the informed traders optimally shift their counterparty choice to a less preferred counterparty, informed traders. Hence, the same-type matching occurs in equilibrium. Proposition 6 in Appendix shows a sufficient and necessary condition for the cross-type matching equilibrium, in terms of information precision (σ_I, σ_U) and correlation structure Σ.

[12] The non-monotone ranking on counterparty's precision may prevent an assortative matching in over-the-counter markets. However, I am expecting that there exists an endogenous statistics in which the matching is assortative.

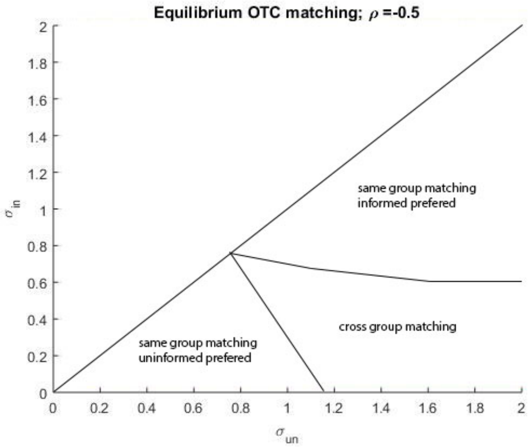

Figure 5.1: Equilibrium matching in OTC subgame: Each region shows an endogenous over-the-counter matching, either same-type matching or cross-type matching between informed and uninformed traders. All traders are assumed to be in the over-the-counter market. The x-axis is the noise variance of uninformed traders while the y-axis is of informed traders. $\sigma_{cv}^2 = 0.25, \sigma_{iv}^2 = 0.75$ and $\min_{j \neq i} \rho_{ij} = -1$.

With the presence of the centralized market, an over-the-counter market in which two informational types match does not exist. When the informed type follows lower price impact rather than better learning, he can be better off trading in the centralized market than in the over-the-counter trading with uninformed counterparties.

Proposition 5 (No Cross-Type OTC Matching) *Suppose that there are two precision types $\sigma_{\varepsilon,i}^2 \in \{\sigma_{\varepsilon,in}^2 < \sigma_{\varepsilon,un}^2\}$. With $\sigma_c^2 > \hat{\sigma}^2 > 0$, there is no over-the-counter trade between informed and uninformed traders (i.e., cross-type matching).*

Figure 5.2 presents an example of endogenous market structures through traders' market and counterparty choice. The left panel three types of equilibrium: all traders choose the centralized market (when both σ_I and σ_U are small, and learning is not sufficiently valuable to either of them); only uninformed traders choose to trade in the over-the-counter market (large σ_U but small σ_I); and all traders choose the over the counter market (both types of traders have inaccurate information). Since learning incentive is a dominant incentive for uninformed traders, there is no equilibrium where only informed traders enter the over-the-counter market.

Endogenizing the market choice, the over-the-counter market will attract either the uninformed or both.[13] Furthermore, traders in the over-the-counter market always trade with the same-type counterparty. The non-existence of matching between informed and uninformed attributes to aggravating information asymmetry in over-the-counter markets. With a random

[13] The conventional wisdom that informed traders are more likely to trade in the over-the-counter market for keeping their private information from the public. Since this model considers only a static trading, such privacy incentive does not exist. The incentives to trade in the over-the-counter market due to the heterogeneity across traders is a separate effect. In dynamic models, I expect that the privacy incentives interact with the effects of heterogeneities.

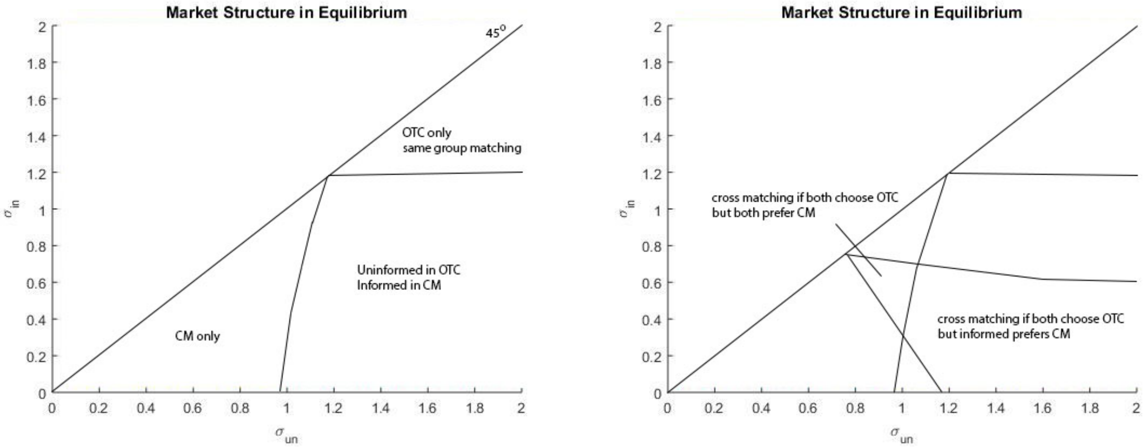

Figure 5.2: Endogenous Market Structure: The x-axis is the noise variance of uninformed traders while the y-axis is of informed traders. $\sigma_{cv}^2 = 0.25, \sigma_{iv}^2 = 0.75$ and $\min_{j \neq i} \rho_{ij} = -1$.

match mechanism, information can be transmitted from the informed to the uninformed trader when they met and thus information asymmetry disappears or diminishes over time. However, when traders choose their own counterparty based on information precision, the informed traders do not want to be matched with uninformed traders. Information is shared only within each type, and the asymmetry between types increases after trades take place. Consequently, the informational inefficiency by allowing an over-the-counter market into the economy.

6 Discussion

Connection to Markets. I show that an over-the-counter market opens when the size of the centralized market is small, the asset values are closer to idiosyncratic than common, and private information of traders is less precise. Many financial derivatives such as forwards contracts, interest rate swaps, or equity or credit linked securities are traded in over-the-counter markets, even though their trading volumes (liquidity) are large. When these products are required to be held by traders until the maturity, it suggests that the purpose of trading can be hedging of traders' outside portfolios. This paper suggests that idiosyncratically valued assets tend to be traded in the over-the-counter markets. On the other hand, centralized markets attract assets traded mostly by speculators, such as stocks or bonds with short maturity, which are valued by future prices that are common to all traders. High-yield bonds that have low credit ranking are often traded in the over-the-counter markets (e.g., Hendershott and Madhavan (2015)). The volatile return prevents the traders' access to quality information and hence the information precision is low. This is consistent with this paper's prediction that low information precision encourages traders to choose over-the-counter markets.

Alternative Over-the-Counter Designs. I have endogenized centralized and over-the-counter markets assuming their prices and allocations based on an uniform-price double auction. This allows us to focus on the effects of the characteristics of market, asset, or traders, rather than the difference between mechanisms. With random matching, searching with frictions, or other mechanisms in literature introduced in over-the-counter markets, the effects in this paper continue to be present. For instance, suppose that the over-the-counter market is operated by random matching instead of traders' counterparty choice. Traders' expected utilities in the over-the-counter market would strengthen the effect of the asymmetric interdependence of traders' asset values and heterogeneous information precisions. Uninformed traders have a chance to meet an informed traders and to learn more precise information, while informed traders' liquidity can be improved with a higher chance of meeting uninformed counterparty. Introducing random matching mechanism in the over-the-counter market does not affect the endogenous market structure qualitatively, but it can increase traders' incentive to enter the over-the-counter market when the heterogeneity across traders is present. Exogenous frictions in the over-the-counter markets - a probability that a trader does not trade, cost of waiting, etc. - can decrease traders' incentive to trade in over-the-counter markets.

In this paper, traders' individual market choice occurs ex-ante, in that traders choose where to trade before their private information is realized. This model is appropriate when traders' individual asset values θ_i contains future returns of the asset in the markets and/or future returns of traders' individual portfolios, as interpreted in Section 2. The future returns from certain assets are often unobservable or costly to observe to those who are not in the market in order to keep market participants privacy. If the traders asset values or their private information are interdependent by other sources - for example, traders' endowments before markets, pre-trades, macroeconomic information, cheap talk, etc. - the model may incorporate *interim* market choice, in that, traders choose a market to participate in after they observe the private information. Interim market choice increases the dimension of heterogeneity, in addition to the correlation structure and information precision. A trader whose realized signal is high can be more attracted to the over-the-counter market since the difference between asset values and price, $|E[\theta_i|s_i, p] - p|$, is larger in a bilateral trades. [14] The additional heterogeneity of realized private information with interim market choice influence equilibrium distribution of traders' types in each market and the distribution of equilibrium prices, but the trade-off between learning and liquidity for each trader continues to shape trader's incentives.

[14] Boyarchenko, Lucca, and Veldkamp (2015) study the effect of realized private signals in market structure in a different market mechanism from this paper. They consider an inter-dealer market and dealer-customer market. Traders face a choice to be either a dealer or a customer. They show that traders who have a high private signal choose to trade directly in the inter-dealer market to keep their information private. Although the conjecture on the interim market choice in my model is related to the intuitions in Boyarchenko, Lucca, and Veldkamp (2015), the determinants for main results are different. In their paper, traders reveal their private information truthfully to the dealers, and thus, there is no learning incentive in over-the-counter markets. This paper considers traders' learning on the values due to demand schedule conditioning on equilibrium prices.

References

Acharya, V. and A. Bisin (2010): "Centralized versus Over-The-Counter Markets," working paper.

Atakan, A. and M. Ekmekci (2016): "Market Selection and the Information Content of Prices," working paper.

Atkeson, A., A. Eisfeldt, and P. Weill (2014): "Entry and Exit in OTC Derivatives Markets," working paper.

Attanasi, G., S. Centorrino, and I. Moscati (2016): "Over-the-counter Markets vs. Double Auctions: A Comparative Experimental Study," Journal of Behavioral and Experimental Economics, 63, 22-35.

Babus, A. and T. Hu (2016): "Endogenous Intermediation in Over-the-Counter Markets " working paper.

Babus, A. and P. Kondor (2016): "Trading and Information Diffusion in Over-The-Counter Markets," working paper.

Babus, A. and C. Parlatore (2017): "Strategic Fragmented Markets," working paper.

Back, K., R. Liu, and A. Teguia(2016): "Signaling in Over-the-Counter Markets: Benefits and Costs of Transparency," working paper.

Barclay, M., T. Hendershott, and D. T. McCormick (2003): "Competition among Trading Venues: Information and Trading on Electronic Communications Networks," Journal of Finance, 58, 6, 2637-2665.

Bessembinder, H. and H. Kaufman (1997): "A Comparison of Trade Execution Costs for NYSE and NASDAQ-Listed Stocks," Journal of Financial and Quantitative Analysis, 32, 3, 287-310.

Biais, B. and R. Green (2007): "The Microstructure of the Bond Market in the 20th Century," working paper.

Bolton, P., T. Santos, and J. Scheinkman (2015): "Cream Skimming in Financial Markets," working paper.

Bessembinder H. and K. Venkataraman (2014): "Does an Electronic Stock Exchange Need an Upstairs Markets?" Journal of Financial Economics, 73, 1, 3-36.

Chang, B. and S. Zhang (2016): "Endogenous Market Making and Network Formation," working paper.

Conrad, J., K. Johnson, and S. Wahal (2003): "Institutional trading and alternative trading systems," Journal of Financial Economics, 70, 1, 99-134.

Degryse, H., F. Jong, and V. Kervel (2015): "The Impact of Dark Trading and Visible Fragmentation on Market Quality," Review of Finance, 19, 4, 1587-1622.

Duffie, D., L. Qiao, and Y. Sun (2017): "Dynamic Directed Random Matching," working paper.

Duffie, D., N. Gârleanu, and L.H. Pedersen (2005): "Over-the-Counter Markets," Econometrica, 73, 6, 1815-1847.

Duffie, D., S. Malamud, and G. Manso (2014): "Information Percolation in Segmented Markets," Journal of Economic Theory, 153, 1-32.

Duffie, D. and C. Wang (2016): “Efficient Contracting in Network Financial Markets,” working paper.
Farboodi, M. (2015): “Intermediation and Voluntary Exposure to Counterparty Risk,” working paper.
Farboodi, M., G. Jarosch, and G. Menzio (2016): “Intermediation as Rent Extraction,” working paper.
Farboodi, M., G. Jarosch, and R. Shimer (2017): “The Emergence of Market Structure,” working paper.
Glode, V. and C. Opp (2017): “Can Decentralized Markets Be More Efficient?,” working paper.
Golosov, M., G. Lorenzoni, and A. Tsyvinski (2014): “Decentralized Trading with Private Information,” Econometrica, 82, 3, 1055-1091.
Hendershott, T. and A. Madhavan (2015): “Click or Call? Auction versus Search in the Over-the-Counter Market,” Journal of Finance, 70, 1, 419-447.
Hugonnier, J., B. Lester, and P. O. Weill (2016): “Heterogeneity in Decentralized Asset Markets,” working paper.
Kirilenko, A. (2000): “On the Endogeneity of Trading Arrangements,” Journal of Financial Markets, 3, 287-314.
Malamud, S. and M. Rostek (2014): “Decentralized Exchange,” working paper.
Maurin, V. (2015): “Liquidity Fluctuations in Over the Counter Markets,” working paper.
Praz, R. (2015): “Equilibrium Asset Pricing with Both Liquid and Illiquid Markets,” working paper.
Ready, M. (2014): “Determinants of Volume in Dark Pool Crossing Networks,” working paper.
Rostek, M. and M. Weretka (2012): “Price Inference in Small Markets,” Econometrica, 80, 2, 687-711.
Rostek, M. and J. H. Yoon (2017): “Inference Design,” working paper.
Vayanos, D. and T. Wang (2007): “A Search-Based Theory of the On-the-Run Phenomenon,” Journal of Finance, 63, 3, 1361-1398.
Vayanos, D. and P. O. Weill (2008): “Search and Endogenous Concentration of Liquidity in Asset Markets,” Journal of Economic Theory, 136, 66-104.
Viswanathan, S. and J. Wang (2002): “Market Architecture: Limit-Order Books versus Dealership Markets,” Journal of Financial Markets, 5, 127-167.
Vives, X. (2011): “Strategic Supply Function Competition with Private Information,” Econometrica, 79, 6, 1919-1966.
Vohra, R. and A. Peivandi (2017): “Instability of Centralized Markets,” working paper.
Wang, C. (2017): “Core-Periphery Trading Networks,” working paper.
Weill, P. O. (2008): “Liquidity Premia in Dynamic Bargaining Markets,” Journal of Economic Theory, 140, 1, 66-96.
Zong, Z. (2014): “The Risk Sharing Benefit versus the Collateral Cost: The Formation of the Inter-Dealer Network in Over-the-Counter Trading, ” working paper.
Zhu, H. (2012): “Finding a Good Price in Opaque Over-the-Counter Markets,” Review of Financiel Studies, 25, 4, 1255-1285.

A Proofs

Proof of Proposition 1 (Equilibrium Representation in a Market). For a given price impact $\lambda_i > 0$ and inference $E[\theta_i|s_i, p] = c_{\theta,i}E[\theta_i] + c_{s,i}s_i + c_{p,i}p$, trader i's first order condition gives his best response, i.e. demand schedule.

$$q_i = \frac{E[\theta_i|s_i, p] - p}{\mu Var(\theta_i|s_i, p) + \lambda_i} = \frac{c_{\theta,i}E[\theta_i] + c_{s,i}s_i - (1 - c_{p,i})p}{\mu Var(\theta_i|s_i, p) + \lambda_i}.$$

With positive price impacts, the second order condition holds for all i:

$$-\mu Var(\theta_i|s_i, p) - 2\lambda_i < 0.$$

The market clearing condition $\sum_i q_i(\cdot) = 0$ determines equilibrium price from the demand function.

$$p = (\sum_i \frac{1 - c_{p,i}}{\mu Var(\theta_i|s_i, p) + \lambda_i})^{-1} \sum_i \frac{c_{\theta,i}E[\theta_i] + c_{s,i}s_i}{\mu Var(\theta_i|s_i, p) + \lambda_i}.$$

Since the price is a linear function of traders' private information $\{s_i\}_i$, it follows a normal distribution as well as the signals. This Gaussian-linear structure allows us to use the Projection Theorem in order to derive traders' conditional expectation on asset value. First, the unconditional expectation of price is equal to $E[\theta_i]$ which is same across traders. It results in $c_{\theta,i} + c_{s,i} + c_{p,i} = 1$ for any i. The inference coefficient $\{c_{s,i}, c_{p,i}\}$ is

$$\begin{bmatrix} c_{s,i} \\ c_{p,i} \end{bmatrix} = \begin{bmatrix} Var(s_i) & Cov(s_i, p) \\ Cov(s_i, p) & Var(p) \end{bmatrix}^{-1} \begin{bmatrix} Cov(\theta_i, s_i) \\ Cov(\theta_i, p) \end{bmatrix}, \tag{7}$$

and the conditional variance of θ_i over (s_i, p) is

$$\begin{aligned} Var(\theta_i|s_i, p) &= Var(\theta_i) - \begin{bmatrix} Cov(\theta_i, s_i) \\ Cov(\theta_i, p) \end{bmatrix} \cdot \begin{bmatrix} Var(s_i) & Cov(s_i, p) \\ Cov(s_i, p) & Var(p) \end{bmatrix}^{-1} \begin{bmatrix} Cov(\theta_i, s_i) \\ Cov(\theta_i, p) \end{bmatrix} \\ &= Var(\theta_i) - (Cov(\theta_i, s_i)c_{s,i} + Cov(\theta_i, p)c_{p,i}). \end{aligned}$$

We denote $\sigma_i^2 = \sigma_{i,\varepsilon}^2/\sigma_\theta^2$, the relative variance of noise in private information compared to variance of asset values. By plugging the following variance and covariance of (s_i, p) into equation (7),

$$Var(p) = (\sum_i \frac{1 - c_{p,i}}{\mu Var(\theta_i|s_i, p) + \lambda_i})^{-2} \Big(\frac{c_{s,i}}{\mu Var(\theta_i|s_i, p) + \lambda_i}\Big)_i \cdot (\sigma_\theta^2 \Sigma + diag(\sigma_{i,\varepsilon}^2)_i) \Big(\frac{c_{s,i}}{\mu Var(\theta_i|s_i, p) + \lambda_i}\Big)_i,$$

$$Cov(s_i, p) = (\sum_i \frac{1 - c_{p,i}}{\mu Var(\theta_i|s_i, p) + \lambda_i})^{-1} \Big(\frac{c_{s,i}}{\mu Var(\theta_i|s_i, p) + \lambda_i}\Big)_i \cdot \Big(\sigma_\theta^2 \rho_{ij} + \sigma_{i,\varepsilon}^2 \mathbf{1}_{j=i}\Big)_j,$$

we get a fixed point problem for the inference coefficients $\{c_{s,i}, c_{p,i}\}_i$,

$$c_{s,i} = \frac{\sum_{j,k} \frac{c_{s,j}c_{s,k}(\rho_{jk}+\sigma_j^2\mathbf{1}_{j=k})-c_{s,j}c_{s,k}\rho_{ij}(\rho_{ik}+\sigma_i^2\mathbf{1}_{i=k})}{(\mu Var(\theta_j|s_j,p)+\lambda_j)(\mu Var(\theta_k|s_k,p)+\lambda_k)}}{\sum_{j,k} \frac{(1+\sigma_i^2)c_{s,j}c_{s,k}(\rho_{jk}+\sigma_j^2\mathbf{1}_{j=k})-c_{s,j}c_{s,k}(\rho_{ij}+\sigma_i^2\mathbf{1}_{i=j})(\rho_{ik}+\sigma_i^2\mathbf{1}_{i=k})}{(\mu Var(\theta_j|s_j,p)+\lambda_j)(\mu Var(\theta_k|s_k,p)+\lambda_k)}}, \quad \forall i, \tag{8}$$

$$c_{p,i} = \frac{\sum_i \frac{1-c_{p,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i} \sum_j \frac{(1+\sigma_i^2)c_{s,j}\rho_{ij}-c_{s,j}(\rho_{ij}+\sigma_i^2\mathbf{1}_{i=j})}{\mu Var(\theta_j|s_j,p)+\lambda_j}}{\sum_{j,k} \frac{(1+\sigma_i^2)c_{s,j}c_{s,k}(\rho_{jk}+\sigma_j^2\mathbf{1}_{j=k})-c_{s,j}c_{s,k}(\rho_{ij}+\sigma_i^2\mathbf{1}_{i=j})(\rho_{ik}+\sigma_i^2\mathbf{1}_{i=k})}{(\mu Var(\theta_j|s_j,p)+\lambda_j)(\mu Var(\theta_k|s_k,p)+\lambda_k)}} \quad \forall i. \tag{9}$$

In addition, the price impacts are characterized by

$$\lambda_i = \Big(\sum_{j\neq i} \frac{1-c_{p,j}}{\mu Var(\theta_j|s_j,p)+\lambda_j}\Big)^{-1}, \quad \forall i, \tag{10}$$

for a given inference coefficients $\{c_{p,j}\}_j$ and the conditional variance of asset values,

$$Var(\theta_i|s_i,p) = \sigma_\theta^2\Big(1-c_{s,i}-c_{p,i}(\sum_j \frac{1-c_{p,j}}{\mu Var(\theta_j|s_j,p)+\lambda_j})^{-1}\sum_j \frac{c_{s,j}\rho_{ij}}{\mu Var(\theta_j|s_j,p)+\lambda_j}\Big), \quad \forall i. \tag{11}$$

Equations (8) - (11) solves $\{c_{s,i}, c_{p,i}, \lambda_i, Var(\theta_i|s_i,p)\}_i$, and thus, characterizes equilibrium.

With the equilibrium characterization, a trader's indirect interim utility is

$$\begin{aligned} E[u_i|s_i,p] &= -\exp\Big(-\mu(-pq_i + E[\theta_i|s_i,p]q_i - \frac{\mu}{2}Var(\theta|s_i,p)q_i^2)\Big) \\ &= -\exp\Big(-\mu(\frac{\mu Var(\theta|s_i,p)+2\lambda_i}{2(\mu Var(\theta_i|s_i,p)+\lambda_i)^2}(E[\theta_i|s_i,p]-p)^2)\Big), \end{aligned}$$

while his ex-ante utility is

$$E[u_i] = E\Big[-\exp\Big(-\mu(\frac{\mu Var(\theta|s_i,p)+2\lambda_i}{2(\mu Var(\theta_i|s_i,p)+\lambda_i)^2}(E[\theta_i|s_i,p]-p)^2)\Big)\Big].$$

Considering that the difference of individual expected asset value from equilibrium price, $(E[\theta_i|s_i,p]-p)$, follows a normal distribution that is generated by Gaussian structure of $\{\theta_i, s_i\}_i$, the expectation on the right hand side of the above equation is in form of the moment generating function for χ_k^2 distribution. It provides an explicit formula for the ex-ante indirect utility:

$$
\begin{aligned}
E[E[u_i|s_i,p]] &= E\Big[-\exp\Big(-\mu(\frac{\mu Var(\theta|s_i,p)+2\lambda_i}{2(\mu Var(\theta_i|s_i,p)+\lambda_i)^2}(E[\theta_i|s_i,p]-p)^2)\Big)\Big] \\
&= E\Big[-\exp\Big(-\mu(\frac{\mu Var(\theta|s_i,p)+2\lambda_i}{2(\mu Var(\theta_i|s_i,p)+\lambda_i)^2}Var(E[\theta_i|s_i,p]-p)\chi^2_{k=1}\Big)\Big] \\
&= -\Big(1-2\Big\{-\mu(\frac{\mu Var(\theta|s_i,p)+2\lambda_i}{2(\mu Var(\theta_i|s_i,p)+\lambda_i)^2}Var(E[\theta_i|s_i,p]-p))\Big\}\Big)^{-1/2} \\
&= -\Big(1+2\mu\frac{\mu Var(\theta|s_i,p)+2\lambda_i}{2(\mu Var(\theta_i|s_i,p)+\lambda_i)^2}Var(E[\theta_i|s_i,p]-p)\Big)^{-1/2}
\end{aligned}
$$

We introduce a measure for the ex-ante utility. For each i,

$$
\tau_i \equiv \big(\frac{1}{E[u_i]^2}-1\big) = \frac{\mu(\mu Var(\theta|s_i,p)+2\lambda_i)}{(\mu Var(\theta_i|s_i,p)+\lambda_i)^2}Var(E[\theta_i|s_i,p]-p).
$$

The ex-ante utility is strictly increasing and strictly concave in τ_i. ■

Proof of Proposition 2 (Benefits of Learning and Liquidity). In equilibrium for a given market, subject to existence, the ex-ante indirect utility of a trader i is strictly increasing and strictly concave in the following measure:

$$
\xi_i = \frac{\mu(\mu Var(\theta|s_i,p)+2\lambda_i)}{(\mu Var(\theta_i|s_i,p)+\lambda_i)^2}Var(E[\theta_i|s_i,p]-p) = \frac{1+2\widehat{\lambda}_i}{(1+\widehat{\lambda}_i)^2}\frac{Var(E[\theta_i|s_i,p]-p)}{Var(\theta_i|s_i,p)}.
$$

Intuitively, more negative asset correlation increases the variance of difference between individual asset value and price, $Var(E[\theta_i|s_i,p]-p)$; the higher information precision of other traders, on average, improves trader i's learning from price so that decreases $Var(\theta_i|s_i,p)$; and the larger market size decreases price impact λ_i. Suppose that the correlation of traders' asset values and information precision is symmetric across traders: when the average correlation is defined by $\bar{\rho}_i \equiv (\frac{c_{s,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i})^{-1}\frac{1}{I-1}\sum_{j\neq i}\frac{c_{s,j}}{\mu Var(\theta_j|s_j,p)+\lambda_j}\rho_{ij}$ for each i and $\sigma_i^2 = \sigma_{i,\varepsilon}^2/\sigma_\theta^2$ for each i, the symmetric market assumes $\bar{\rho}_i = \bar{\rho}$ and $\sigma_i^2 = \sigma^2$ for all i. In general, the inference coefficients are

$$
\begin{aligned}
c_{s,i} &= \frac{\frac{1}{I-1}\sum_{j\neq i}(\frac{c_{s,j}}{(\mu Var(\theta_j|s_j,p)+\lambda_j)})^2(1+\sigma_j^2+(I-1)\bar{\rho}_j) - (\frac{c_{s,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i})^2\bar{\rho}_i(1+\sigma_i^2+(I-1)\bar{\rho}_i)}{(1+\sigma_i^2)\frac{1}{I-1}\sum_{j\neq i}(\frac{c_{s,j}}{\mu Var(\theta_j|s_j,p)+\lambda_j})^2(1+\sigma_j^2+(I-1)\bar{\rho}_j) - (\frac{c_{s,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i})^2\bar{\rho}_i(1+\sigma_i^2+(I-1)\bar{\rho}_i)}, \\
c_{p,i} &= \frac{(\frac{1-c_{p,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i}+\frac{1}{\lambda_i})\ \frac{c_{s,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i}\bar{\rho}_i\sigma_i^2}{(1+\sigma_i^2)\frac{1}{I-1}\sum_{j\neq i}(\frac{c_{s,j}}{\mu Var(\theta_j|s_j,p)+\lambda_j})^2(1+\sigma_j^2+(I-1)\bar{\rho}_j) - (\frac{c_{s,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i})^2\bar{\rho}_i(1+\sigma_i^2+(I-1)\bar{\rho}_i)}.
\end{aligned}
$$

In symmetric markets, for each i,

$$c_{s,i} = \frac{1-\bar\rho}{1+\sigma_i^2-\bar\rho}, \qquad c_{p,i} = \frac{I\bar\rho\sigma^2}{(1-\bar\rho)(1+\sigma^2+(I-1)\bar\rho)+\bar\rho\sigma^2},$$
$$\lambda_i = \Big(\sum_{j\neq i}\frac{1-c_{p,j}}{\mu Var(\theta_j|s_j,p)+\lambda_j}\Big)^{-1} = \frac{\mu Var(\theta_i|s_i,p)}{I-2}.$$

Furthermore, the inference coefficients characterize the learning effect in expected utility.

$$\begin{aligned} & Var(E[\theta_i|s_i,p]-p) = \big(c_{s,i}^2\sigma_\theta^2(1+\sigma_i^2)+(1-c_{p,i})^2Var(p)-2c_{s,i}(1-c_{p,i})Cov(s_i,p)\big) \\ = \; & \sigma_\theta^2\big(c_{s,i}^2(1+\sigma_i^2)+(1-c_{p,i})^2(\sum_i\frac{1-c_{p,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i})^{-2}\sum_{j,k}\frac{c_{s,j}c_{s,k}(\rho_{jk}+\sigma_j^2\mathbf{1}_{j=k})}{(\mu Var(\theta_j|s_i,p)+\lambda_j)(\mu Var(\theta_j|s_i,p)+\lambda_j)} \\ & -2c_{s,i}(1-c_{p,i})(\sum_i\frac{1-c_{p,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i})^{-1}\sum_j\frac{c_{s,j}(\rho_{ij}+\sigma_i^2\mathbf{1}_{j=i})}{\mu Var(\theta_j|s_j,p)+\lambda_j}\big). \end{aligned}$$

$$\frac{Var(E[\theta_i|s_i,p]-p)}{Var(\theta_i|s_i,p)} = \frac{\left(\begin{array}{c} c_{s,i}^2(1+\sigma_i^2)+(1-c_{p,i})^2(\sum_i\frac{1-c_{p,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i})^{-2}\sum_{j,k}\frac{c_{s,j}c_{s,k}(\rho_{jk}+\sigma_j^2\mathbf{1}_{j=k})}{(\mu Var(\theta_j|s_i,p)+\lambda_j)(\mu Var(\theta_j|s_i,p)+\lambda_j)} \\ -2c_{s,i}(1-c_{p,i})(\sum_i\frac{1-c_{p,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i})^{-1}\sum_j\frac{c_{s,j}(\rho_{ij}+\sigma_i^2\mathbf{1}_{j=i})}{\mu Var(\theta_j|s_j,p)+\lambda_j}\end{array}\right)}{1-c_{s,i}-c_{p,i}(\sum_j\frac{1-c_{p,j}}{\mu Var(\theta_j|s_j,p)+\lambda_j})^{-1}\sum_j\frac{c_{s,j}\rho_{ij}}{\mu Var(\theta_j|s_j,p)+\lambda_j}}.$$

In symmetric market, these equations are simplified into

$$Var(E[\theta_i|s_i,p]-p) = \sigma_\theta^2c_s^2\frac{I-1}{I}(1+\sigma^2-\bar\rho).$$

The above characterization for the symmetric markets provides the following comparative statics of the three characteristics (i) the market size, (ii) the average correlation $\bar\rho_i$, and (iii) information precision $\phi_i = 1/\sigma^2$: The liquidity effect on utility is captured by the term $\frac{1+2\widehat\lambda_i}{(1+\widehat\lambda_i)^2}$. With this closed-form solution of inference parameters and price impact,

$$\frac{1+2\widehat\lambda_i}{(1+\widehat\lambda_i)^2} = 1-(\frac{\widehat\lambda_i}{1+\widehat\lambda_i})^2 = 1-\Big(\frac{(1+\sigma^2-\bar\rho)(1+(I-1)\bar\rho)}{(I-1)(1-\bar\rho)(1+\sigma^2+(I-1)\bar\rho)}\Big)^2.$$

The liquidity term increases as I increases or $\bar\rho$ decreases.

$$\frac{\partial}{\partial I}\frac{1+2\widehat\lambda_i}{(1+\widehat\lambda_i)^2} = 2\frac{\widehat\lambda_i}{1+\widehat\lambda_i}\frac{1+\sigma^2-\bar\rho}{1-\bar\rho}\frac{\sigma^2+(1+(I-1)\bar\rho)^2}{(I-1)^2(1+\sigma^2+(I-1)\bar\rho)^2} > 0$$

Larger market size and/or more negative correlation with others on average results in more liquidity, and thus higher utility for traders. When the information precision $\phi = 1/\sigma^2$ increases, the endogenous liquidity of the market increases if $\bar{\rho} > 0$, and decreases if $\bar{\rho} < 0$.

$$\frac{\partial}{\partial \sigma^2} \frac{1 + 2\widehat{\lambda}_i}{(1 + \widehat{\lambda}_i)^2} = -2 \frac{\widehat{\lambda}_i}{1 + \widehat{\lambda}_i} \frac{(1 + (I-1)\bar{\rho})}{(I-1)(1-\bar{\rho})} \frac{I\bar{\rho}}{(1 + \sigma^2 + (I-1)\bar{\rho})^2}.$$

The effect of learning from the price on utility is measured by

$$\frac{Var(E[\tilde{\theta}_i|s_i, p] - p)}{Var(\tilde{\theta}_i|s_i, p)} = \frac{I-1}{I} \frac{(1-\bar{\rho})^2(1 + \sigma^2 + (I-1)\bar{\rho})}{\sigma^2(\sigma^2 + (1-\bar{\rho})(1 + (I-1)\bar{\rho}))},$$

which is increasing in information precision $\phi = 1/\sigma^2$. The effect of average correlation $\bar{\rho}$ is ambiguous. The utility component due to learning is decreasing with respect to $\bar{\rho}$, if and only if, $(1 + \sigma^2) + (2I-3)(1+\sigma^2)\bar{\rho} + (I-3)(I-1)\bar{\rho}^2 - (I-1)^2\bar{\rho}^3 > 0$.

In asymmetric markets, we can heuristically explore influences of the three characteristics on trader i's expected utility $E[u_i]$. We impose another simplicity on information precision: $\sigma_j^2 = \sigma_{-i}^2$ for any $j \neq i$. It implies that all other traders $j \neq i$ except for trader i have symmetric strategy. This assumption is to make the derivation simpler, but it does not affect conclusions.

$$\widehat{\lambda}_i = \frac{1}{(I-1)(1 - c_{p,-i})} \frac{\mu Var(\theta_{-i}|s_{-i}, p) + \lambda_{-i}}{\mu Var(\theta_i|s_i, p)}$$

$$c_{s,i} = \frac{1 - \bar{\rho}_i}{1 + \sigma_i^2 - \bar{\rho}_i}, \qquad c_{p,i} = \frac{\bar{\rho}_i \sigma_i^2}{(1-\bar{\rho}_i)(1 + \sigma_i^2 + (I-1)\bar{\rho}_i) + \bar{\rho}_i\sigma_i^2} \frac{1 + 2\widehat{\lambda}_i}{\widehat{\lambda}_i}.$$

$$\frac{Var(E[\theta_i|s_i, p] - p)}{Var(\theta_i|s_i, p)} = \frac{c_{s,i}^2 Var(s_i) + (1 - c_{p,i})^2 Var(p) - 2c_{s,i}(1 - c_{p,i})Cov(s_i, p)}{1 - c_{s,i} - c_{p,i}(\sum_j \frac{1 - c_{p,j}}{\mu Var(\theta_j|s_j,p) + \lambda_j})^{-1} \sum_j \frac{c_{s,j}\rho_{ij}}{\mu Var(\theta_j|s_j,p)+\lambda_j}}. \tag{12}$$

(i) Market size, equivalently, the number of traders I: Suppose that $\bar{\rho}_i$ and σ_i^2 are fixed. We can see that I affects utility only through the normalized price impact $\widehat{\lambda}_i$ and the inference coefficient on price $c_{p,i}$. As I increases, $\widehat{\lambda}_i$ decreases and $|c_{p,i}|$ increases for sufficiently large I. The liquidity effect in utility, $(1 + 2\widehat{\lambda}_i)/(1 + \widehat{\lambda}_i)^2$, increases by the decrease of $\widehat{\lambda}_i$. The learning effect in equation (12) increases when $\bar{\rho}_i > 0$ and decreases when $\bar{\rho}_i < 0$. With sufficiently symmetric market, the effect of liquidity dominates the learning effect, so that the expected utility $E[u_i]$ increases in the market size I.

(ii) Average correlation $\bar{\rho}_i = \bar{\rho}$: As more negative correlation $\bar{\rho}_i$, i.e., as $\bar{\rho}_i$ decreases, the inference coefficient on private information $c_{s,i}$ decreases and the absolute value of the coefficient on price $|c_{p,i}|$ increases:

$$\frac{\partial c_{p,i}}{\partial \bar{\rho}_i} = \frac{\sigma_i^2(1 + \sigma_i^2 + (I-1)\bar{\rho}_i^2)}{((1-\bar{\rho}_i)(1 + \sigma_i^2 + (I-1)\bar{\rho}_i) + \bar{\rho}_i\sigma_i^2)^2} \frac{1 + 2\widehat{\lambda}_i}{\widehat{\lambda}_i} > 0.$$

It implies that the price impact $\widehat{\lambda}_i$ and the conditional variance $Var(\theta_i|s_i, p)$ both decrease. In addition, it increases $Var(E[\theta_i|s_i, p] - p)$ by decreasing $Cov(s_i, p)$. Hence, the more negative correlation $\bar{\rho}$ increases both liquidity and learning effects and thus increases traders' expected utility.

(iii) Information precision σ^2_{-i} of other traders: as the information precision $1/\sigma^2_{-i}$ increases (i.e., σ^2_{-i} decreases), trader $j \neq i$'s inference coefficient on private information $c_{s,-i}$ increases and the absolute value of the coefficient on price $|c_{p,-i}|$ decreases.

$$\frac{\partial c_{p,-i}}{\partial \sigma^2_{-i}} = \frac{\bar{\rho}(1-\bar{\rho})(1+(I-1)\bar{\rho})}{((1-\bar{\rho})(1+\sigma^2_{-i}+(I-1)\bar{\rho})+\bar{\rho}\sigma^2_{-i})^2} \frac{1+2\widehat{\lambda}_{-i}}{\widehat{\lambda}_{-i}}.$$

It makes $\widehat{\lambda}_i$ decreasing if $\bar{\rho}_{-i} > 0$ and increasing $\widehat{\lambda}_i$ otherwise. Hence, the liquidity effect of trader i's utility changes depending on the correlation of other traders: as σ^2_{-i} decreases, the liquidity effect $(1+2\widehat{\lambda}_i)/(1+\widehat{\lambda}_i)^2$ increases when $\bar{\rho}_{-i} > 0$ and decreases when $\bar{\rho}_{-i} > 0$. The learning effect in trader i's utility is increasing in other traders' information precision by the decrease of $Var(\theta_i|s_i, p)$. The liquidity and learning can create a trade-off with respect to others' information precision. ∎

Proof of Theorem 1 (Endogenous Market Structure). This proof is under the condition that equilibrium exists. Suppose that only a centralized market opens in equilibrium but no over-the-counter market does. Since no trader has an incentive to switch his market choice to the over-the-counter market, his expected utilities $E[u_i^{CM}; \text{all in CM}]$, when all traders are in the centralized market, is higher than a potential utility in the over-the-counter market. The potential utility in the over-the-counter market $E[u_i; OTC(i, j)]$ is pair-specific, i.e., it depends on the pair (i, j). A sufficient and necessary condition on the endogenous market structure consists of only the centralized market is that there is no trader who has a positive incentive to deviate to the over-the-counter market with his best counterparty. This condition is equivalent to that the utility in centralized market is higher than the maximum utility trader i would get in over-the-counter market:

$$E[u_i^{CM}; \text{all in CM}] > \max_{j \neq i} E[u_i; OTC(i, j)], \quad \forall i.$$

Under the symmetry assumption, $\frac{1}{I-1}\sum_{j\neq i} \rho_{ij} = \bar{\rho}$ and $\phi_i = \phi = 1/\sigma^2_\varepsilon$ for all i, the equilibrium utility in each market is characterized as follows. Suppose that $\sigma = \sigma_\varepsilon/\sigma_\theta$. In the centralized market,

$$c_s = \frac{1-\bar{\rho}}{1+\sigma^2-\bar{\rho}}, \qquad c_p = \frac{I(1-c_p)\bar{\rho}\sigma^2}{(1-\bar{\rho})(1+\sigma^2+(I-1)\bar{\rho})} = \frac{I\bar{\rho}\sigma^2}{(1+\sigma^2-\bar{\rho})(1+(I-1)\bar{\rho})},$$

$$\widehat{\lambda}_i = \frac{\lambda_i}{\mu Var(\tilde{\theta}_i|s_i, p)} = \frac{(1+(I-1)\bar{\rho})(1+\sigma^2-\bar{\rho})}{(I-2)(1+\sigma^2)+((I-1)^2+1-2(I-1)(1+\sigma^2))\bar{\rho}-(I-1)(I-2)\bar{\rho}^2},$$

$$Var(\tilde{\theta}_i|s_i,p) = \frac{(1+\sigma^2)+(I-2)\bar{\rho}-(I-1)\bar{\rho}^2}{(1+\sigma^2+(I-1)\bar{\rho})(1+\sigma^2-\bar{\rho})}\sigma_\varepsilon^2; \quad Var(E[\tilde{\theta}_i|s_i,p]-p) = \frac{(1-\bar{\rho})^2}{1+\sigma^2-\bar{\rho}}\frac{I-1}{I}\sigma_\theta^2.$$

From these equilibrium variables, the equilibrium utility is characterized by

$$\xi_i^{cm} \equiv \big(\frac{1}{E[u_i]^2}-1\big) = \frac{(I-1)^2(1-\bar{\rho})^2(1+\sigma^2+(I-1)\bar{\rho})^2-(1+\sigma^2-\bar{\rho})^2(1+(I-1)\bar{\rho})^2}{(I-1)I\sigma^2(1+\sigma^2+(I-1)\bar{\rho})((1+\sigma^2)+(I-2)\bar{\rho}-(I-1)\bar{\rho}^2)}.$$

On the other hand, the over-the-counter market provides the equilibrium characterization with the following endogenous parameters:

$$c_s = \frac{1-\rho_{ij}}{1+\sigma^2-\rho_{ij}}, \qquad c_p = \frac{I(1-c_p)\rho_{ij}\sigma^2}{(1-\rho_{ij})(1+\sigma^2+\rho_{ij})} = \frac{2\rho_{ij}\sigma^2}{(1+\sigma^2-\rho_{ij})(1+\rho_{ij})},$$

$$\widehat{\lambda}_i = \frac{\lambda_i}{\mu Var(\tilde{\theta}_i|s_i,p)} = \frac{(1+\rho_{ij})(1+\sigma^2-\rho_{ij})}{-2\sigma^2\rho_{ij}},$$

$$Var(\tilde{\theta}_i|s_i,p) = \frac{1+\sigma^2-\rho_{ij}^2}{(1+\sigma^2)^2-\rho_{ij}^2}\sigma_\varepsilon^2; \quad Var(E[\tilde{\theta}_i|s_i,p]-p) = \frac{(1-\rho_{ij})^2}{1+\sigma^2-\rho_{ij}}\frac{1}{2}\sigma_\theta^2.$$

From these equilibrium variables, the equilibrium utility is characterized by

$$\xi_i^{otc} \equiv \big(\frac{1}{E[u_i]^2}-1\big) = \frac{(1-\rho_{ij})^2(1+\sigma^2+\rho_{ij})^2-(1+\sigma^2-\rho_{ij})^2(1+\rho_{ij})^2}{2\sigma^2(1+\sigma^2+\rho_{ij})(1+\sigma^2-\rho_{ij}^2)}.$$

Comparing the equilibrium utility in two markets,

$$\xi_i^{CM} > \xi_i^{OTC} \quad \Leftrightarrow \quad \frac{1+2\widehat{\lambda}_i^{cm}}{(1+\widehat{\lambda}_i^{cm})^2}\frac{Var(E[\theta_i|s_i,p]-p)}{Var(\theta_i|s_i,p_{cm})} > \frac{1+2\widehat{\lambda}_i^{otc}}{(1+\widehat{\lambda}_i^{otc})^2}\frac{Var(E[\theta_i|s_i,s_j]-p)}{Var(\theta|s_i,s_j)}.$$

$$\frac{(I-1)^2(1-\bar{\rho})^2(1+\sigma^2+(I-1)\bar{\rho})^2-(1+\sigma^2-\bar{\rho})^2(1+(I-1)\bar{\rho})^2}{(I-1)I(1+\sigma^2+(I-1)\bar{\rho})((1+\sigma^2)+(I-2)\bar{\rho}-(I-1)\bar{\rho}^2)} > \frac{(1-\rho_{ij})^2(1+\sigma^2+\rho_{ij})^2-(1+\sigma^2-\rho_{ij})^2(\cdots}{2(1+\sigma^2+\rho_{ij})(1+\sigma^2-\rho_{ij}^2)}$$

Here, without loss of generality, the correlations between asset values $(\tilde{\theta}_i)_i$ can be written by $\bar{\rho} = \frac{\sigma_{cv}^2}{\sigma_{cv}^2+\sigma_{iv}^2}$ and $\rho_{ij} = \min_{j\neq i} Corr(\tilde{\theta}_i,\tilde{\theta}_j) = 2\bar{\rho}-1 = \frac{\sigma_{cv}^2-\sigma_{iv}^2}{\sigma_{cv}^2+\sigma_{iv}^2}$.

$$\frac{(I-1)^2(1-\bar{\rho})^2(\sigma^2+1+(I-1)\bar{\rho})^2-(\sigma^2+1-\bar{\rho})^2(1+(I-1)\bar{\rho})^2}{(I-1)I(\sigma^2+1+(I-1)\bar{\rho})(\sigma^2+(1-\bar{\rho})(1+(I-1)\bar{\rho}))} > \frac{(2(1-\bar{\rho}))^2(\sigma^2+2\bar{\rho})^2-(\sigma^2+2(1-\bar{\rho}))^2(2\bar{\rho}\cdots}{2(\sigma^2+2\bar{\rho})(\sigma^2+2\bar{\rho}\cdot 2(1-\bar{\rho}))} \tag{13}$$

The inequality can be rewritten as

$$1-\bar{\rho} < \frac{K+\sqrt{K^2+(I^2-1)(\sigma^2+z)^2(\sigma^2+y)^2+2y^2(\sigma^2+z)^2-(4I(I-1)(\sigma^2+y)^2z^2)L}}{(I^2-1)(\sigma^2+z)^2(\sigma^2+y)^2+2y^2(\sigma^2+z)^2-(4I(I-1)(\sigma^2+y)^2z^2)}\sigma^2 \equiv \hat{\kappa}(\sigma^2),$$

where $K = ((I-1)Iz^2(\sigma^2+y)^2-(\sigma^2+z)^2y^2) > 0, L = ((I-1)Iz^2(\sigma^2+y)^2-2(\sigma^2+z)^2y^2) > 0$, $y = 1+(I-1)\bar{\rho} > 0$, and $z = 2\bar{\rho} < 1$. It shows that the sufficient and necessary condition on

$E[u_i^{CM}; \text{all in CM}] > \max_{j\neq i} E[u_i; OTC(i,j)]$ is

$$\frac{\sigma_{iv}^2}{\sigma_{cv}^2+\sigma_{iv}^2} = 1-\bar{\rho} < \hat{\kappa}(\sigma^2).$$

In addition, the inequality (13) can be also written in terms of signal variance $\sigma^2 = \frac{\sigma_\varepsilon^2}{\sigma_{cv}^2+\sigma_{iv}^2}$:

$$\sigma^2 < \frac{-2K+\sqrt{2K^2+L(I^2-1)(\sigma^2+z)^2(\sigma^2+y)^2+2y^2(\sigma^2+z)^2-(4I(I-1)(\sigma^2+y)^2z^2)}}{((I-1)Iz^2(\sigma^2+y)^2-2(\sigma^2+z)^2y^2)} \equiv \frac{1}{\hat{\phi}(\sigma_{cv}^2+\sigma_{iv}^2)}$$

Hence, traders choose the centralized market if and only if the information precision satisfies

$$\phi_i = \frac{1}{\sigma_\varepsilon^2} = \frac{\sigma_{cv}^2+\sigma_{iv}^2}{\sigma^2} > \hat{\phi}(\sigma_{cv}^2, \sigma_{iv}^2, \Sigma).$$

The over-the-counter market is chosen by a trader if these inequalities are violated, i.e., $\frac{\sigma_{iv}^2}{\sigma_{cv}^2+\sigma_{iv}^2} > \hat{\kappa}(\sigma^2)$ and $\phi_i < \hat{\phi}(\sigma_{cv}^2, \sigma_{iv}^2, \Sigma)$. The proof is complete for symmetric markets.

With asymmetric traders, the equilibrium utility in each market is characterized as follows. In the centralized market:

$$c_{s,i} = \frac{\frac{1}{I-1}\sum_{j\neq i}(\frac{c_{s,j}}{(\mu Var(\theta_j|s_j,p)+\lambda_j)})^2(1+\sigma_j^2+(I-1)\bar{\rho}_j) - (\frac{c_{s,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i})^2\bar{\rho}_i(1+\sigma_i^2+(I-1)\bar{\rho}_i)}{(1+\sigma_i^2)\frac{1}{I-1}\sum_{j\neq i}(\frac{c_{s,j}}{\mu Var(\theta_j|s_j,p)+\lambda_j})^2(1+\sigma_j^2+(I-1)\bar{\rho}_j) - (\frac{c_{s,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i})^2\bar{\rho}_i(1+\sigma_i^2+(I-1)\bar{\rho}_i)},$$

$$c_{p,i} = \frac{(\frac{1}{I-1}\sum_j\frac{1}{\lambda_j})\ \frac{c_{s,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i}\bar{\rho}_i\sigma_i^2}{(1+\sigma_i^2)\frac{1}{I-1}\sum_{j\neq i}(\frac{c_{s,j}}{\mu Var(\theta_j|s_j,p)+\lambda_j})^2(1+\sigma_j^2+(I-1)\bar{\rho}_j) - (\frac{c_{s,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i})^2\bar{\rho}_i(1+\sigma_i^2+(I-1)\bar{\rho}_i)}.$$

$$\begin{aligned} & Var(E[\theta_i|s_i,p]-p) = \big(c_{s,i}^2\sigma_\theta^2(1+\sigma_i^2)+(1-c_{p,i})^2Var(p)-2c_{s,i}(1-c_{p,i})Cov(s_i,p)\big) \\ = \ & \sigma_\theta^2\big(c_{s,i}^2(1+\sigma_i^2)+(1-c_{p,i})^2(\sum_i\frac{1-c_{p,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i})^{-2}\sum_{j,k}\frac{c_{s,j}c_{s,k}(\rho_{jk}+\sigma_j^2\mathbf{1}_{j=k})}{(\mu Var(\theta_j|s_i,p)+\lambda_j)(\mu Var(\theta_j|s_i,p)+\lambda_j)} \\ & -2c_{s,i}(1-c_{p,i})(\sum_i\frac{1-c_{p,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i})^{-1}\sum_j\frac{c_{s,j}(\rho_{ij}+\sigma_i^2\mathbf{1}_{j=i})}{\mu Var(\theta_j|s_j,p)+\lambda_j}\big) \end{aligned}$$

$$\lambda_i = \Big(\sum_{j\neq i}\frac{1-c_{p,j}}{\mu Var(\theta_j|s_j,p)+\lambda_j}\Big)^{-1}.$$

$$Var(\theta_i|s_i,p) = \sigma_\theta^2\Big(1-c_{s,i}-c_{p,i}(\sum_j\frac{1-c_{p,j}}{\mu Var(\theta_j|s_j,p)+\lambda_j})^{-1}\sum_j\frac{c_{s,j}\rho_{ij}}{\mu Var(\theta_j|s_j,p)+\lambda_j}\Big).$$

$$\xi_i^{cm} \equiv (\frac{1}{E[u_i]^2}-1) = \frac{\mu(\mu Var(\theta|s_i,p)+2\lambda_i)}{(\mu Var(\theta_i|s_i,p)+\lambda_i)^2}Var(E[\theta_i|s_i,p]-p).$$

On the other hand, the over-the-counter market provides the equilibrium characterization with the following endogenous parameters:

$$c_{s,i} = \frac{(\frac{c_{s,j}}{(\mu Var(\theta_j|s_j,s_i)+\lambda_j)})^2(1+\sigma_j^2+\rho) - (\frac{c_{s,i}}{\mu Var(\theta_i|s_i,s_j)+\lambda_i})^2\rho(1+\sigma_i^2+\rho)}{(1+\sigma_i^2)(\frac{c_{s,j}}{\mu Var(\theta_j|s_j,s_i)+\lambda_j})^2(1+\sigma_j^2+\rho) - (\frac{c_{s,i}}{\mu Var(\theta_i|s_i,s_j)+\lambda_i})^2\rho(1+\sigma_i^2+\rho)},$$
$$c_{p,i} = \frac{(\frac{1}{\lambda_j}+\frac{1}{\lambda_i})\ \frac{c_{s,i}}{\mu Var(\theta_i|s_i,s_j)+\lambda_i}\rho\sigma_i^2}{(1+\sigma_i^2)(\frac{c_{s,j}}{\mu Var(\theta_j|s_j,s_i)+\lambda_j})^2(1+\sigma_j^2+\rho) - (\frac{c_{s,i}}{\mu Var(\theta_i|s_i,s_j)+\lambda_i})^2\rho(1+\sigma_i^2+\rho)}.$$

$$\begin{aligned}
&Var(E[\theta_i|s_i,p]-p) = \big(c_{s,i}^2\sigma_\theta^2(1+\sigma_i^2) + (1-c_{p,i})^2 Var(p) - 2c_{s,i}(1-c_{p,i})Cov(s_i,p)\big) \\
= \ & \sigma_\theta^2\big(c_{s,i}^2(1+\sigma_i^2) + (1-c_{p,i})^2(\sum_i \frac{1-c_{p,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i})^{-2}\sum_{j,k}\frac{c_{s,j}c_{s,k}(\rho_{jk}+\sigma_j^2\mathbf{1}_{j=k})}{(\mu Var(\theta_j|s_i,p)+\lambda_j)(\mu Var(\theta_k|s_k,p)+\lambda_k)} \\
&-2c_{s,i}(1-c_{p,i})(\sum_i \frac{1-c_{p,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i})^{-1}\sum_j \frac{c_{s,j}(\rho_{ij}+\sigma_i^2\mathbf{1}_{j=i})}{\mu Var(\theta_j|s_j,p)+\lambda_j}\big) \\
&Var(E[\theta_i|s_i,s_j]-p) = Var\Big(\frac{(1+\sigma_j^2-\rho^2)s_i+\rho\sigma_i^2 s_j}{(1+\sigma_i^2)(1+\sigma_j^2)-\rho^2} - p\Big)
\end{aligned}$$

$$\lambda_i = \frac{\mu Var(\theta_j|s_j,s_i)+\lambda_j}{1-c_{p,j}}; \quad Var(\theta_i|s_i,s_j) = \sigma_\theta^2\frac{\sigma_i^2(1+\sigma_j^2-\rho^2)}{(1+\sigma_i^2)(1+\sigma_j^2)-\rho^2}$$
$$\xi_i^{otc} \equiv \big(\frac{1}{E[u_i]^2}-1\big) = \frac{\mu(\mu Var(\theta|s_i,p)+2\lambda_i)}{(\mu Var(\theta_i|s_i,p)+\lambda_i)^2}Var(E[\theta_i|s_i,p]-p).$$

Comparing the equilibrium utility in two markets,

$$\xi_i^{CM} > \xi_i^{OTC} \quad \Leftrightarrow \quad \frac{1+2\widehat{\lambda}_i^{cm}}{(1+\widehat{\lambda}_i^{cm})^2}\frac{Var(E[\theta_i|s_i,p]-p)}{Var(\theta_i|s_i,p_{cm})} > \frac{1+2\widehat{\lambda}_i^{otc}}{(1+\widehat{\lambda}_i^{otc})^2}\frac{Var(E[\theta_i|s_i,s_j]-p)}{Var(\theta|s_i,s_j)}.$$

If CM is worse than the minimum utility that a trader can get in OTC, in the sense that $E[u_i^{CM};\text{no one in CM}] < \min_{j;\rho_{ij}<0} E[u_i;OTC(i,j)]$, for all traders. Remark that it is a sufficient condition. When $\min_{j;\rho_{ij}<0} E[u_i;OTC(i,j)] < E[u_i^{CM}] < \max_j E[u_i;OTC(i,j)]$ for some traders, it can result in the extreme market structure, such as only centralized market or only over-the-counter market opens, depending on the over-the-counter matching outcome. ■

Proof of Corollary 1 (OTC Existence with a Competitive CM). From the proof of Theorem 1, the inequality (13) is a sufficient and necessary condition for a trader to prefer the centralized market to the over-the-counter market. Taking the number of traders in the centralized market I to infinity, the inequality is written as

$$(1-\bar{\rho}) > \frac{(2(1-\bar{\rho}))^2(\sigma^2+2\bar{\rho})^2 - (\sigma^2+2(1-\bar{\rho}))^2(2\bar{\rho})^2}{2(\sigma^2+2\bar{\rho})(\sigma^2+2\bar{\rho}\cdot 2(1-\bar{\rho}))},$$

and it is simplified into

$$0 > \sigma^2 - (3\sigma^2 + 2)\bar{\rho} + 2\bar{\rho}^2.$$

The inverse of this inequality is satisfied, so that the trader choose to enter to the over-the-counter market rather than to the centralized market, if and only if

$$1 - \bar{\rho} = \frac{\sigma_{iv}^2}{\sigma_{cv}^2 + \sigma_{iv}^2} > \frac{2 - 3\sigma^2 + \sqrt{(3\sigma^2 + 2)^2 - 8\sigma^2}}{4} > \frac{1}{2}, \tag{14}$$

which is equivalent to $\sigma^2 > \frac{2\bar{\rho}(1-\bar{\rho})}{3\bar{\rho}-2}$. The proof is complete. ■

Proof of Corollary 2 (No Price Difference in OTC and CM). Suppose that traders' asset values ($\tilde{\theta}_i = \theta + \delta_i$) follow the distributions $\theta \sim \mathcal{N}(E[\theta], \sigma_{cv}^2)$ and $\delta_i \sim \mathcal{N}(0, \sigma_{iv}^2 \Sigma)$ with $\Sigma = \left[\begin{array}{c|c} \mathbf{1} & -\mathbf{1} \\ \hline -\mathbf{1} & \mathbf{1} \end{array}\right]$ where $\mathbf{1} = (1)_{I/2\times I/2}$ is a $(\frac{I}{2} \times \frac{I}{2})$-matrix with all elements being one, as in Example 1. In this model, the equilibrium price in the centralized market with a sufficiently large number of traders is $p_{cm} = \frac{1}{I}\sum_{i\in I} E[\theta+\delta_i|s_i, p] \approx \frac{c_\theta E[\theta]+c_s\theta}{c_\theta+c_s}$, since the average correlation of the idiosyncratic component (δ_i) is zero. On the other hand, the equilibrium price is determined in each over-the-counter matching between two traders whose correlation is $Corr(\delta_i, \delta_j) = -1$.

$$p_{otc} = \frac{c_\theta E[\theta_i] + c_s\frac{1}{2}(\theta + \delta_i + \varepsilon_i + \theta + \delta_j + \varepsilon_j)}{1 - c_p} = \frac{c_\theta E[\theta] + c_s\theta}{c_\theta + c_s} + \frac{c_s}{c_\theta + c_s}\big((\delta_i + \delta_j) + (\varepsilon_i + \varepsilon_j)\big) = p_{cm} + (\text{noise}).$$

Hence, the price p_{otc} in each over-the-counter market follows a normal distribution with the mean equal to the centralized market price p_{cm}. It shows that there exists a model where the over-the-counter and centralized market prices are same in expectation. Even in this case, a trader choose to trade in the over-the-counter market if the relative variance of idiosyncratic value component, $\frac{\sigma_{iv}^2}{\sigma_{cv}^2+\sigma_{iv}^2}$, satisfies the inequality (14). ■

Proof of Theorem 2 (Coexistence of OTC and CM). Suppose that traders are symmetric in the sense that the profile of correlation in each row is same and that the information precision is same across traders (See Definition 2).

(i) With the symmetric correlation structure, a trader is matched with the counterparty whose asset valuations has the minimum correlation, i.e., the over-the-counter matching (i, j) occurs such that $Corr(\tilde{\theta}_i, \tilde{\theta}_j) = \min_{k\neq i} Corr(\tilde{\theta}_i, \tilde{\theta}_k) = \rho_{min}$, upon traders i and j's participation in the over-the-counter market. It can be shown as a contradiction on the symmetricity of traders. Suppose that there is an over-the-counter matching (i, j) such that $Corr(\tilde{\theta}_i, \tilde{\theta}_j) \gneq \rho_{min}$. Since the profile of correlations are same across traders, there exists another trader $k \neq i, j$ who has the minimum correlation with trader i, $Corr(\tilde{\theta}_i, \tilde{\theta}_k) = \rho_{min}$. If trader k's current matching in the over-the-counter market is not with the minimum correlation, traders i and k have a positive incentive to deviate from their current matchings, and thus the current matchings are not pair-wise stable.Therefore, trader k's current matching (k, l) has to be such that $Corr(\tilde{\theta}_k, \tilde{\theta}_l) = \rho_{min}$.

It implies that trader k has two other traders i and l that provides the minimum correlation ρ_{min}. By the symmetry, the profile of correlations of trader i, $\{Corr(\tilde{\theta}_i, \tilde{\theta}_m)\}_{i,m}$, contains two or more ρ_{min}, in that there exists another trader $m \neq i, j, k$ such that $Corr(\tilde{\theta}_i, \tilde{\theta}_m) = \rho_{min}$. With the same argument, the current matching for trader m has the minimum correlation, while implies that there are three or more ρ_{min} in the correlation profile. Recursively, the symmetricity of traders and pair-wise stable matching concludes that all pairs of traders have $Corr(\tilde{\theta}_i, \tilde{\theta}_j) = \rho_{min}$, which is a contradiction to the assumption that there exists (i, j)-match with $Corr(\tilde{\theta}_i, \tilde{\theta}_j) \gneq \rho_{min}$. Therefore, all over-the-counter matching in equilibrium satisfies $Corr(\tilde{\theta}_i, \tilde{\theta}_j) = \min_{k \neq i} Corr(\tilde{\theta}_i, \tilde{\theta}_k) = \rho_{min}$, $\forall (i, j)$.

(ii) By the part (i), the utility comparison between centralized and over-the-counter market is same for all traders. More formally, ρ_{ij}^{otc} and $\bar{\rho}^{cm}$ in two markets, and thus, the difference of these two correlations are fixed and symmetric for all traders. With the fixed correlation difference and the symmetric information precision, the incentive to enter either market is determined only by the different market size between the over-the-counter ($N = 2$) and centralized ($L + I$) markets. Suppose that a trader i who is currently in the over-the-counter market has a profitable deviation to the centralized market. After trader i's deviation, the market size in the centralized market increases. With the fixed correlation difference, the incentive to deviate from the over-the-counter to centralized market for other traders $j \neq i$ with $m_j = OTC$, i.e. for those who are currently in the over-the-counter market, is even stronger than the trader i. Applying this argument recursively, all traders choose the centralized market with endogenized market choice. The opposite direction of deviation also concludes that if there exists a trader who is currently in the centralized market and has a strictly positive incentive to deviate to the over-the-counter market, then all traders have the same incentive by the symmetricity. Therefore, it completes the proof for that the endogenous distribution of traders in the over-the-counter and centralized market has a corner solution if traders are symmetric. Equivalently, it concludes that two trading venues coexist only if traders are asymmetric. ■

Proof of Proposition 3 (OTC Matching with Heterogeneous Correlations). The pair-wise stable over-the-counter matching is determined by the following algorithm: (i) Upon traders' entry to the over-the-counter market, two traders, who have the most negative correlation among all pairs, are matched: we call this pair (i_1, j_1). (ii) If there are multiple pairs that have the most negative correlation, select one pair randomly. (iii) Eliminating the selected traders, select the most negative correlation among the remaining paper and create another pair: called (i_2, j_2). (iv) Repeating this procedure until there is at most one remaining trader in the over-the-counter market, and then the over-the-counter matching is determined $\{(i_t, j_t)\}_{t=1,2,\cdots}$.

It suffices to show that the matching $\{(i_t, j_t)\}_{t=1,2,\cdots}$ from this algorithm is pairwise stable. Suppose that there exists two traders who have a strictly positive incentive to deviate from their current matching and create their own matching. Formally, there exists i_t and j_s, for some $t \lneq s$, such that $E[u_{i_t}; OTC, (i_t, j_s)] \gneq E[u_{i_t}; OTC, (i_t, j_t)]$ and $E[u_{j_s}; OTC, (i_t, j_s)] \gneq$

$E[u_{j_s}; OTC, (i_s, j_s)]$. From Proposition 2, it implies that $Corr(i_t, j_s) \lneq Corr(i_t, j_t)$ and $Corr(i_t, j_s) \lneq Corr(i_s, j_s)$, since traders are symmetric in information precision and any bilateral trade has the equal market size $N = 2$. It is a contradiction to the algorithm: at the step (i) for t, the matching at t is created between (i_t, j_t), and hence, $Corr(i_t, j_t) \leq Corr(i_t, j_s)$ for any $s > t$. It is contradicted to the assumption that trader i_t has a profitable deviation by having another counterparty j_s, $Corr(i_t, j_s) \lneq Corr(i_t, j_t)$. The proof is complete. ■

Proof of Proposition 4 (Trade-Off in Learning and Price Impact over Precision). First, we characterize the equilibrium in a given over-the-counter bilateral matching between two traders i and j. Here, the correlation between two traders' asset values are ρ and information precision is $\phi_i = 1/\sigma_i^2$ and $\phi_j = 1/\sigma_j^2$.

$$c_{s,i} = \frac{\Gamma_i^2(1+\sigma_j^2+\rho) - \rho(1+\sigma_i^2+\rho)}{(1+\sigma_i^2)\Gamma_i^2(1+\sigma_j^2+\rho) - \rho(1+\sigma_i^2+\rho)}; \quad c_{p,i} = \frac{(\frac{1-c_{p,i}}{c_{s,i}} + \frac{1-c_{p,j}}{c_{s,j}}\Gamma_i)\rho\sigma_i^2}{(1+\sigma_i^2)\Gamma_i^2(1+\sigma_j^2+\rho) - \rho(1+\sigma_i^2+\rho)},$$

where $\Gamma_i = \frac{c_{s,j}}{\mu Var(\theta_j|s_j,p)+\lambda_j} / \frac{c_{s,i}}{\mu Var(\theta_i|s_i,p)+\lambda_i}$ and $\Gamma_j = 1/\Gamma_i$. Moreover,

$$Var(E[\theta_i|s_i,p] - p) = \sigma_\theta^2 c_{s,i}^2 \Big((1+\sigma_i^2) + \frac{(1+\sigma_i^2) + \Gamma_i^2(1+\sigma_j^2) + 2\Gamma_i\rho}{(1+\frac{1-c_{p,j}}{1-c_{p,i}}\frac{c_{s,i}}{c_{s,j}}\Gamma_i)^2} - 2\frac{(1+\sigma_i^2) + \Gamma_i\rho}{(1+\frac{1-c_{p,j}}{1-c_{p,i}}\frac{c_{s,i}}{c_{s,j}}\Gamma_i)}\Big),$$

$$\lambda_i = \frac{\mu Var(\theta_j|s_j,p) + \lambda_j}{1-c_{p,j}}; \quad \hat{\lambda}_i = \frac{Var(\theta_j|s_j,p)}{Var(\theta_i|s_i,p)}\frac{1+\hat{\lambda}_j}{1-c_{p,j}}; \quad Var(\theta_i|s_i,p) = \sigma_\theta^2 \frac{\sigma_i^2(1+\sigma_j^2-\rho^2)}{(1+\sigma_i^2)(1+\sigma_j^2)-\rho^2}.$$

The equilibrium utility is derived as follows:

$$\xi_i(i,j) = \Big(\frac{1}{E[u_i;(i,j)]^2} - 1\Big) = \frac{1+2\hat{\lambda}_i}{(1+\hat{\lambda}_i)^2}\frac{Var(E[\theta_i|s_i,p]-p)}{Var(\theta_i|s_i,p)}.$$

By taking a derivative of equilibrium utility of trader i with respect to the information precision ϕ_j of trader j, the optimal counterparty $\phi_j^* = 1/(\sigma_j^2)^* \equiv arg\max_{\phi_j=1/\sigma_j^2} \xi_i(i,j)$ of trader i is determined by the first-order condition.

$$\Big(\frac{\partial}{\partial\sigma_j^2}\frac{1+2\hat{\lambda}_i}{(1+\hat{\lambda}_i)^2}\Big)\frac{Var(E[\theta_i|s_i,p]-p)}{Var(\theta_i|s_i,p)} + \frac{1+2\hat{\lambda}_i}{(1+\hat{\lambda}_i)^2}\Big(\frac{\partial}{\partial\sigma_j^2}\frac{Var(E[\theta_i|s_i,p]-p)}{Var(\theta_i|s_i,p)}\Big) = 0.$$

Here, the partial derivatives of liquidity and learning effect with respect to the counterparty j's noise variance σ_j^2 satisfy the following equations.

$$\frac{\partial}{\partial \sigma_j^2}\frac{1+2\hat{\lambda}_i}{(1+\hat{\lambda}_i)^2} = \frac{-2\hat{\lambda}_i}{(1+\hat{\lambda}_i)^3}\frac{\partial}{\partial \sigma_j^2}\frac{V(\theta_j|s_j,p)(1-c_{p,i})+1}{(1-c_{p,i})(1-c_{p,j})-1}$$

$$= \frac{-2\hat{\lambda}_i}{(1+\hat{\lambda}_i)^3}\frac{\frac{\partial V(\theta_j|s_j,p)}{\partial \sigma_j^2}(1-c_{p,i})((1-c_{p,i})(1-c_{p,j})-1)+\frac{\partial c_{p,i}}{\partial \sigma_j^2}(1-c_{p,j})+\frac{\partial c_{p,j}}{\partial \sigma_j^2}(1-c_{p,i})+V(\theta_j|s_j,p)\frac{\partial c_{p,j}}{\partial \sigma_j^2}((1-c_{p,i})^2+1)}{((1-c_{p,i})(1-c_{p,j})-1)^2},$$

$$\frac{\partial}{\partial \sigma_j^2}\frac{Var(E[\theta_i|s_i,p]-p)}{Var(\theta_i|s_i,p)} = \frac{\frac{\partial Var(E[\theta_i|s_i,p]-p)}{\partial \sigma_j^2}Var(\theta_i|s_i,p) - \frac{\partial Var(\theta_i|s_i,p)}{\partial \sigma_j^2}Var(E[\theta_i|s_i,p]-p)}{(Var(\theta_i|s_i,p))^2}$$

$$= \frac{\partial}{\partial \sigma_j^2}\Big(\frac{(1+\sigma_i^2)(1+\sigma_j^2)-\rho^2}{\sigma_i^2(1+\sigma_j^2-\rho^2)}c_{s,i}^2\Gamma_i^2\frac{(1+\sigma_i^2)(\frac{1-c_{p,j}}{1-c_{p,i}}\frac{c_{s,i}}{c_{s,j}})^2-2\rho\frac{1-c_{p,j}}{1-c_{p,i}}\frac{c_{s,i}}{c_{s,j}}+(1+\sigma_j^2)}{(1+\frac{1-c_{p,j}}{1-c_{p,i}}\frac{c_{s,i}}{c_{s,j}}\Gamma_i)^2}\Big),$$

where $\frac{\partial c_{p,i}}{\partial \sigma_j^2} = \frac{(\frac{1-c_{p,i}}{c_{s,i}}+\frac{1-c_{p,j}}{c_{s,j}}\Gamma_i)\rho\sigma_i^2(1+\sigma_i^2)\Gamma_i^2}{((1+\sigma_i^2)\Gamma_i^2(1+\sigma_j^2+\rho)-\rho(1+\sigma_i^2+\rho))^2} > 0$, $\frac{\partial c_{p,j}}{\partial \sigma_j^2} = \frac{(\frac{1-c_{p,i}}{c_{s,i}}+\frac{1-c_{p,j}}{c_{s,j}}\Gamma_i)\rho(\Gamma_j^2(1+\sigma_i^2+\rho)-\rho(1+\rho))}{((1+\sigma_j^2)\Gamma_j^2(1+\sigma_i^2+\rho)-\rho(1+\sigma_j^2+\rho))^2} < 0$, $\frac{\partial Var(\theta_j|s_j,p)}{\partial \sigma_j^2} = \sigma_\theta^2\frac{(1+\sigma_i^2-\rho^2)^2}{((1+\sigma_j^2)(1+\sigma_i^2)-\rho^2)^2} > 0$, and $\frac{\partial Var(E[\theta_i|s_i,p]-p)}{\partial \sigma_j^2} = \frac{1}{(1+\frac{1-c_{p,j}}{1-c_{p,i}}\frac{c_{s,i}}{c_{s,j}}\Gamma_i)^2} > 0$. Hence, $\frac{\partial}{\partial \sigma_j^2}\frac{1+2\hat{\lambda}_i}{(1+\hat{\lambda}_i)^2} > 0$ so that the liquidity effect of trader i decreases as the precision of trader j increases (i.e., the noise variance σ_j^2 of trader j decreases). In addition, $\frac{\partial}{\partial \sigma_j^2}\frac{Var(E[\theta_i|s_i,p]-p)}{Var(\theta_i|s_i,p)} < 0$ if $\frac{Var(E[\theta_i|s_i,p]-p)}{Var(\theta_i|s_i,p)}$ is sufficiently large, in the sense that the precision of both traders i and j is sufficiently small. Under this condition, the learning effect i increases in the precision of the other trader j. This proves the trade-off between liquidity and learning effects with respect to the counterparty j's information precision ϕ_j. ■

Lemma 1 *There is no over-the-counter matching between two players who have positively correlated asset values. In that, if $Corr(\tilde{\theta}_i, \tilde{\theta}_j) = \rho_0 > 0$, then neither of trader i nor j choose the other as his counterparty.*

Proof. What we want to prove is that there is no equilibrium in the bilateral trade. From the assumption that no equilibrium leads to no trade, the incentive to choose the other as a counterparty is zero. The equilibrium price is

$$p^* = \left(\frac{1-c_{p1}}{\mu+\lambda_1}+\frac{1-c_{p2}}{\mu+\lambda_2}\right)^{-1}\left(\frac{c_{s1}s_1}{\mu+\lambda_1}+\frac{c_{s2}s_2}{\mu+\lambda_2}+(\frac{c_{\theta1}}{\mu+\lambda_1}+\frac{c_{\theta2}}{\mu+\lambda_2})\theta\right).$$

Therefore, the equilibrium price impact is characterized by

$$\lambda_i = \frac{\mu+\lambda_j}{1-c_{pj}} = \frac{\mu(2-c_{pi})}{(1-c_{pi})(1-c_{pj})-1} > 0, \qquad i \neq j \in \{1,2\}.$$

The characterization implies that the positivity condition for price impact $\lambda_i > 0, i = 1,2$ implies that $c_{pi} < 1$ for both $i = 1,2$. By the projection theorem, the endogenous coefficients $(c_{si}, c_{pi}, c_{\theta i}, \lambda_i)_{i=1,2}$ are the fixed-point solution of the following system of equations: with $c_{si}+$

$c_{pi} + c_{\theta i} = 1,$

$$c_{si} = \frac{((\frac{c_{sj}}{\mu+\lambda_j})(1-\rho^2+\frac{\sigma^2_{\varepsilon,j}}{\sigma^2_\theta}) - \frac{c_{si}}{\mu+\lambda_i}\rho\frac{\sigma^2_{\varepsilon,i}}{\sigma^2_\theta})}{(\frac{c_{sj}}{\mu+\lambda_j})((1+\frac{\sigma^2_{\varepsilon,i}}{\sigma^2_\theta})(1+\frac{\sigma^2_{\varepsilon,j}}{\sigma^2_\theta})-\rho^2)}, \quad c_{pi} = \frac{\left(\frac{1-c_{pi}}{\mu+\lambda_i}+\frac{1-c_{pj}}{\mu+\lambda_j}\right)\rho\frac{\sigma^2_{\varepsilon,i}}{\sigma^2_\theta}}{(\frac{c_{sj}}{\mu+\lambda_j})((1+\frac{\sigma^2_{\varepsilon,i}}{\sigma^2_\theta})(1+\frac{\sigma^2_{\varepsilon,j}}{\sigma^2_\theta})-\rho^2)}, \quad i \neq j \in \{1,2\}.$$

From the further calculation, we get the following explicit solution[15]

$$\begin{aligned} c_{si} &= \frac{(1-\rho^2+\sigma_i^2)(1-\rho^2+\sigma_j^2)-\rho^2\sigma_i^2\sigma_j^2}{(1-\rho^2+\sigma_i^2+\frac{\mu+\lambda_j}{\mu+\lambda_i}\rho\sigma_i^2)((1+\sigma_i^2)(1+\sigma_j^2)-\rho^2)} = \frac{1-\rho^2}{1-\rho^2+\sigma_i^2+\frac{\mu+\lambda_j}{\mu+\lambda_i}\rho\sigma_i^2} \\ c_{pi} &= \frac{4\rho\sigma_i^2(1+\rho+\sigma_j^2)}{(1-\rho^2+\sigma_i^2)(1+\rho+\sigma_j^2)+(1-\rho^2+\sigma_j^2)(1+\rho+\sigma_i^2)+2\rho\sigma_i^2(1+\rho+\sigma_j^2)} \end{aligned}$$

$$\begin{aligned} \lambda_i &= -\mu\frac{(1-\rho^2+\sigma_i^2)(1+\rho+\sigma_j^2)+(1-\rho^2+\sigma_j^2)(1+\rho+\sigma_i^2)+2\rho\sigma_j^2(1+\rho+\sigma_i^2)}{2\rho\left\{\sigma_i^2(1+\rho+\sigma_j^2)+\sigma_j^2(1+\rho+\sigma_i^2)\right\}} \\ &= -\mu\left\{\frac{1}{2\rho}-\frac{(1-\rho^2)(2+2\rho+\sigma_i^2+\sigma_j^2)}{2\rho\{(1+\rho)(\sigma_i^2+\sigma_j^2)+2\sigma_i^2\sigma_j^2\}}-\frac{\sigma_j^2(1+\rho+\sigma_i^2)}{\sigma_i^2(1+\rho+\sigma_j^2)+\sigma_j^2(1+\rho+\sigma_i^2)}\right\}. \end{aligned}$$

Here, the positivity condition for the price impacts, $\lambda_i > 0, \lambda_j > 0$, is equivalent to $\rho < 0$. Hence, when two traders' asset values have a non-negative correlation $Corr(\tilde{\theta}_i, \tilde{\theta}_j) < 0$, there is no equilibrium. ■

Proposition 6 (OTC matching equilibrium) *The over-the-counter market is in form of cross-type matching, if and only if*

$$\begin{aligned} &\frac{\left\{\sigma_i^2(1+\rho+\sigma_j^2)+\sigma_j^2(1+\rho+\sigma_i^2)\right\}(1+\rho+\sigma_i^2)(1-\rho^2+\sigma_j^2)(L-2\rho\sigma_j^2(1+\rho+\sigma_i^2))^2}{4(1+\rho)^2\{(1+\sigma_i^2)(1+\sigma_j^2)-\rho^2\}^2(L-2\rho\sigma_i^2(1+\rho+\sigma_j^2))^2} \\ &\times\left\{\begin{array}{c} \frac{(1+\sigma_i^2)(L+2\rho\sigma_i^2(1+\rho+\sigma_j^2))^2}{[(1+\rho+\sigma_i^2)\{2(1+\rho+\sigma_j^2)(1-\rho+\sigma_i^2)+\rho(\sigma_i^2-\sigma_j^2)\}]^2}+\frac{(1+\sigma_j^2)(L+2\rho\sigma_j^2(1+\rho+\sigma_i^2))^2}{[(1+\rho+\sigma_j^2)\{2(1+\rho+\sigma_i^2)(1-\rho+\sigma_j^2)+\rho(\sigma_j^2-\sigma_i^2)\}]^2} \\ -\frac{2\rho(L+2\rho\sigma_j^2(1+\rho+\sigma_i^2))(L+2\rho\sigma_i^2(1+\rho+\sigma_j^2))}{[(1+\rho+\sigma_i^2)\{2(1+\rho+\sigma_j^2)(1-\rho+\sigma_i^2)+\rho(\sigma_i^2-\sigma_j^2)\}][(1+\rho+\sigma_j^2)\{2(1+\rho+\sigma_i^2)(1-\rho+\sigma_j^2)+\rho(\sigma_j^2-\sigma_i^2)\}]} \end{array}\right\} \\ &\geq \frac{\sigma_i^2(1-\rho^2+\sigma_i^2)}{(1+\rho+\sigma_i^2)^2(1-\rho+\sigma_i^2)}, \end{aligned}$$

for all $i \neq j$, when $L = (1-\rho^2+\sigma_i^2)(1+\rho+\sigma_j^2)+(1-\rho^2+\sigma_j^2)(1+\rho+\sigma_i^2)$.

Proof. The same-group matching gives a trader with σ_i^2 his expected utility as follows:

$$E[u_i^{same}] = \frac{\mu+2\lambda}{(\mu+\lambda)^2}\frac{c_s^2}{4}(1+\sigma_i^2-\rho) = -\frac{\rho\sigma_i^2(1-\rho^2+\sigma_i^2)}{\mu(1+\rho+\sigma_i^2)^2(1-\rho+\sigma_i^2)}$$

[15] As special cases, the above fixed-point problem provides that (a) in a symmetric case, $c_s = \frac{1-\rho}{1-\rho+\sigma^2}, c_p = \frac{2\rho\sigma^2}{(1+\rho)(1-\rho+\sigma^2)}$, and that (b) if $\sigma^2_{\varepsilon,j} = \infty$, $c_{si} = \frac{1}{1+\sigma_i^2}, c_{pi} = 0$ and $c_{sj} = 0, c_{pj} = \frac{2\rho}{1+\rho/2}$.

where $\lambda = -\mu\frac{(1+\rho)(1-\rho+\sigma_i^2)}{2\rho\sigma_i^2}$, $c_s = \frac{1-\rho}{1-\rho+\sigma_i^2}$. In addition, the cross-group matching gives equilibrium utility as follows:

$$E[u_i^{cross}] = \frac{\mu+2\lambda_i}{2(\mu+\lambda_i)^2}\left(\frac{1}{\lambda_i}+\frac{1}{\lambda_j}\right)^{-2}\left((\frac{c_{si}}{\lambda_i})^2(1+\sigma_i^2)+(\frac{\mu+\lambda_i}{\mu+\lambda_j}\frac{c_{sj}}{\lambda_j})^2(1+\sigma_j^2)-2\frac{c_{si}}{\lambda_i}\frac{\mu+\lambda_i}{\mu+\lambda_j}\frac{c_{sj}}{\lambda_j}\rho\right).$$

Hence, the condition for that the cross-group matching is the equilibrium in OTC,

$$E[u_i^{cross}] \geq E[u_i^{same}], \quad \forall\, i \in \{in, un\},$$

can be characterized with the equilibrium parameters of cross-type matching. When $L = (1-\rho^2+\sigma_i^2)(1+\rho+\sigma_j^2)+(1-\rho^2+\sigma_j^2)(1+\rho+\sigma_i^2)$, the parameters $\{\lambda_i, \lambda_j\}$ have a closed form solution.

$$\mu+2\lambda_i = \mu\frac{\rho(1+\rho)(\sigma_i^2-\sigma_j^2)-L}{\rho\left\{\sigma_i^2(1+\rho+\sigma_j^2)+\sigma_j^2(1+\rho+\sigma_i^2)\right\}}.$$

This provides the closed-form solution of equilibrium utility in the cross-type matching, from the following derivations.

$$\begin{aligned}
\frac{\mu+2\lambda_i}{2(\mu+\lambda_i)^2} &= \frac{2\rho\left\{\sigma_i^2(1+\rho+\sigma_j^2)+\sigma_j^2(1+\rho+\sigma_i^2)\right\}(\rho(1+\rho)(\sigma_i^2-\sigma_j^2)-L)}{\mu(2\rho\sigma_i^2(1+\rho+\sigma_j^2)-L)^2} \\
\frac{1}{\lambda_i}+\frac{1}{\lambda_j} &= -\frac{4\rho\left\{\sigma_i^2(1+\rho+\sigma_j^2)+\sigma_j^2(1+\rho+\sigma_i^2)\right\}(L+\rho\left\{\sigma_j^2(1+\rho+\sigma_i^2)+\sigma_i^2(1+\rho+\sigma_j^2)\right\})}{\mu(L+2\rho\sigma_j^2(1+\rho+\sigma_i^2))(L+2\rho\sigma_i^2(1+\rho+\sigma_j^2))} \\
\frac{\mu+2\lambda_i}{2(\mu+\lambda_i)^2}\left(\frac{1}{\lambda_i}+\frac{1}{\lambda_j}\right)^{-2} &= \frac{(\rho(1+\rho)(\sigma_i^2-\sigma_j^2)-L)}{(L-2\rho\sigma_i^2(1+\rho+\sigma_j^2))^2}\frac{\mu(L+2\rho\sigma_j^2(1+\rho+\sigma_i^2))^2(L+2\rho\sigma_i^2(1+\rho+\sigma_j^2))^2}{8\rho\left\{\sigma_i^2(1+\rho+\sigma_j^2)+\sigma_j^2(1+\rho+\sigma_i^2)\right\}(L+\rho\left\{\sigma_j^2(1+\rho+\sigma_i^2)+\sigma_i^2(1+\rho\right.}} \\
\frac{\mu+\lambda_i}{\mu+\lambda_j}\frac{c_{sj}}{\lambda_j} &= \frac{-2\rho(1-\rho^2)\left\{\sigma_i^2(1+\rho+\sigma_j^2)+\sigma_j^2(1+\rho+\sigma_i^2)\right\}(L-2\rho\sigma_j^2(1+\rho+\sigma_i^2))}{\mu(L+2\rho\sigma_i^2(1+\rho+\sigma_j^2))[(1-\rho^2+\sigma_j^2)(L-2\rho\sigma_i^2(1+\rho+\sigma_j^2))+\rho\sigma_j^2(L-2\rho\sigma_j^2(1+\rho+\sigma_i^2))]}.
\end{aligned}$$

Hence, we get

$$\begin{aligned}
E[u_i^{cross}] &= \frac{-\rho\left\{\sigma_i^2(1+\rho+\sigma_j^2)+\sigma_j^2(1+\rho+\sigma_i^2)\right\}(1+\rho+\sigma_i^2)(1-\rho^2+\sigma_j^2)(L-2\rho\sigma_j^2(1+\rho+\sigma_i^2))^2}{4\mu(1+\rho)^2\{(1+\sigma_i^2)(1+\sigma_j^2)-\rho^2\}^2(L-2\rho\sigma_i^2(1+\rho+\sigma_j^2))^2} \\
&\times\left\{\begin{array}{c}\frac{(1+\sigma_i^2)(L+2\rho\sigma_i^2(1+\rho+\sigma_j^2))^2}{[(1+\rho+\sigma_i^2)\{2(1+\rho+\sigma_j^2)(1-\rho+\sigma_i^2)+\rho(\sigma_i^2-\sigma_j^2)\}]^2}+\frac{(1+\sigma_j^2)(L+2\rho\sigma_j^2(1+\rho+\sigma_i^2))^2}{[(1+\rho+\sigma_j^2)\{2(1+\rho+\sigma_i^2)(1-\rho+\sigma_j^2)+\rho(\sigma_j^2-\sigma_i^2)\}]^2} \\ -\frac{2\rho(L+2\rho\sigma_j^2(1+\rho+\sigma_i^2))(L+2\rho\sigma_i^2(1+\rho+\sigma_j^2))}{[(1+\rho+\sigma_i^2)\{2(1+\rho+\sigma_j^2)(1-\rho+\sigma_i^2)+\rho(\sigma_i^2-\sigma_j^2)\}][(1+\rho+\sigma_j^2)\{2(1+\rho+\sigma_i^2)(1-\rho+\sigma_j^2)+\rho(\sigma_j^2-\sigma_i^2)\}]}\end{array}\right\}.
\end{aligned}$$

The sufficient and necessary condition on $E[u_i^{some}] \leq E[u_i^{cross}]$, subject to existence, is as follows:

$$
\begin{aligned}
&\frac{\left\{\sigma_i^2(1+\rho+\sigma_j^2)+\sigma_j^2(1+\rho+\sigma_i^2)\right\}(1+\rho+\sigma_i^2)(1-\rho^2+\sigma_j^2)(L-2\rho\sigma_j^2(1+\rho+\sigma_i^2))^2}{4(1+\rho)^2\{(1+\sigma_i^2)(1+\sigma_j^2)-\rho^2\}^2(L-2\rho\sigma_i^2(1+\rho+\sigma_j^2))^2} \\
&\times\left\{\begin{array}{c}
\frac{(1+\sigma_i^2)(L+2\rho\sigma_i^2(1+\rho+\sigma_j^2))^2}{[(1+\rho+\sigma_i^2)\{2(1+\rho+\sigma_j^2)(1-\rho+\sigma_i^2)+\rho(\sigma_i^2-\sigma_j^2)\}]^2}+\frac{(1+\sigma_j^2)(L+2\rho\sigma_j^2(1+\rho+\sigma_i^2))^2}{[(1+\rho+\sigma_j^2)\{2(1+\rho+\sigma_i^2)(1-\rho+\sigma_j^2)+\rho(\sigma_j^2-\sigma_i^2)\}]^2} \\
-\frac{2\rho(L+2\rho\sigma_j^2(1+\rho+\sigma_i^2))(L+2\rho\sigma_i^2(1+\rho+\sigma_j^2))}{[(1+\rho+\sigma_i^2)\{2(1+\rho+\sigma_j^2)(1-\rho+\sigma_i^2)+\rho(\sigma_i^2-\sigma_j^2)\}][(1+\rho+\sigma_j^2)\{2(1+\rho+\sigma_i^2)(1-\rho+\sigma_j^2)+\rho(\sigma_j^2-\sigma_i^2)\}]}
\end{array}\right\} \\
\geq\;&\frac{\sigma_i^2(1-\rho^2+\sigma_i^2)}{(1+\rho+\sigma_i^2)^2(1-\rho+\sigma_i^2)}
\end{aligned}
$$

Comparing the expected utilities $E[u_i^{some}]$ and $E[u_i^{cross}]$ from same-type and cross-type matching, the sufficient and necessary condition for over-the-counter matching to be between informed and uninformed traders in equilibrium. ■