Corporate Debt Illiquidity and Agency Costs

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Abstract

We propose a theoretical framework to study the impact of exogenous illiquidity shock in the secondary corporate debt market on the agency costs (asset substitution) between equity and debt holders. Taking advantage of the closed-form solutions for debt and equity values, we find that liquidity risk increases agency costs, especially for a firm with weak fundamental. Empirically, we use implied asset volatility and earning volatility as proxies for a firm's risk taking and confirm the positive relationship between illiquidity and agency costs. Further, using TRACE dissemination as an exogenous event, we verify the causality between illiquidity and agency cost proxies.

Keywords: Illiquidity; agency costs; asset substitution; asset volatilities

JEL Classification: G12, G13, G32, G33

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1. Introduction

Corporate bond illiquidity, reflecting market friction in the secondary bond market, draws academics, policy makers and media's attention, especially after a recent financial recession starting from 2008. Corporate bond illiquidity affects a firm's behaviour mainly through direct (Ericsson and Reault, 2006) and rollover (He and Xiong, 2012) channels. In the context of structural model in which a firm's debt and equity are treated as contingent claims on unlevered assets[2], He and Xiong (2012) point out that the costs because of rolling over short term debt to keep a firm alive increase endogenous default boundary[3] and also the costs of debt. Because of the limitation of their model in which it is assume a constant asset volatility, they did not study and quantify the effect of liquidity risk on the agency costs between the debt and equity holders, so-called "*Asset substitution*" (Jensen and Meckling, 1976).

In this paper, we present a theoretical framework to extent He and Xiong (2012)'s framework by incorporating the time-varying asset volatility. Specifically, we introduce low- and high- asset volatility regimes and allow a firm to switch between these two regimes, similar to Leland (1998)'s regime switch framework. We use Laplace transform approach to derive the analytical solution for corporate debt and equity values. By the simulated calibration, we find that equity holders optimally choose to increase the regime-switching boundary and switch to high-volatility regime earlier when corporate debt illiquidity is worse, which amplifies the agency costs between equity and debt holders significantly[4], especially for a firm with weak fundamental[5]. Empirically, we

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[2] The structural model is originated from Merton (1973)'s seminal work. A firm's equity (debt) is treated as longing a call option (shorting a put option) whose underlying asset is unlevered asset.
[3] He and Milbrat (2013) build a theoretical model to show the feedback of credit risk on the liquidity risk and create a spiral circle between the liquidity and credit risk.
[4] We define the agency cost as the percentage decrease of asset value to maximize the equity value ex post compared to the case to maximize the total firm's value ex ante.
[5] Custodio, Ferreira and Laureano (2012) examined the debt maturities in U.S. during last four decades and found that the average debt maturity decreases recently. It documents that around 45.6% firms have a 3-year average debt

employ the implied asset volatilities and earning volatilities as proxies for a firm's risk taking and document a positive relationship between illiquidity and the risk taking measures. Further, by adopting the TRACE dissemination as an exogenous event that increase the liquidity of the secondary corporate debt market, we confirming the causality between liquidity risk and a firm's risk taking that is positively related to the agency costs between equity and debt holders.

Our model is built on the structural model of Leland (1998) and exogenous liquidity risk framework of Amihud and Mendelson (1986)[6]. Following Leland (1998), we adopt two regimes with low and high asset volatilities, respectively, and allow equity holders to switch between these two regimes by altering the risky level of assets. Initially, by anticipating equity holders' risk-shifting in the future, debt holders choose risk-shifting boundary and corresponding optimal endogenous default boundary to maximize a firm's asset value. Based on such information set, all shareholders determine the debt and capital structure ex-ante to maximize the asset value. The equity holders cannot change debt structure once it is in place but can manipulate optimal endogenous default boundary and risk-shifting boundary to maximize equity value only ex post. Leland (1998) assumes that there is no liquidity risk in the bond market and all new debts can always be issued at par. In this paper, we incorporate exogenous liquidity risk framework similar to that in Amihud and Mendelson (1986). In particular, it assumes that each bond holder is exposed to the idiosyncratic liquidity shocks that hit the secondary corporate debt market randomly. Upon the arrival of liquidity shocks, bond holders have to exit by selling bonds at a lower price, suggesting a cost for exiting. The difference between the market price of newly issued debt and

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maturity in their sample in 2008. We refer the short term debt maturity to the maturity less than 1 year, the intermediate debt maturity to the maturity great than 1 year but less than 5 years and the long-term maturity to the maturity equal to or greater than 5 years.

[6] He and Xiong (2012) adopt Amihud and Mendelson (1986)'s illiquidity setting to examine the impact of liquidity shocks on the default component of corporate credit spread.

face value of retired debt is the rollover costs that have to be absorbed by equity holders, which deteriorates the value of outstanding debts, as shown in He and Xiong (2012). In our framework, when a liquidity shock hits the secondary bond market as a surprise, equity holders can only adjust the anticipated risk-shifting boundary and corresponding endogenous default boundary to maximize equity value ex-post, but not the debt structure, such as coupon payment and face value. As long as the underlying asset value touches risk-shifting boundary, equity holders switch to high-volatility regime to take advantage of asset substitution, which enlarges agency costs and reduces a firm's total asset value.

We adopt the Laplace transform approach to derive the analytical solution for both debt and equity values and calibrated numerical results for the firms with different fundamentals. First, we find a significant increase of risk-shifting boundary when an illiquidity shock hits corporate debt market compared to that without illiquidity shock. To maximize equity value ex post, equity holders shift to high-volatility regime earlier by sacrificing a firm's total asset value, which enlarges the agency costs between equity and debt holders. With calibrations, we find that agency costs increase approximately 78% and 103% for *A* and *BB* rated debts in the presence of liquidity shocks compare to these without liquidity risk, respectively.

Empirically, we adopt a firm's implied asset volatility using Merton (1973)'s model and earning volatility to proxy for a firm's risk taking. We document a significant positive relationship between illiquidity of corporate debt and a firm's risk-taking, which is consistent with our calibrated results and confirms our theoretical results. Further, using the TRACE dissemination as an exogenous event that increase corporate bond liquidity, we find an decreasing of a firm's risk taking after being included in TRACE, which indicates the causality between liquidity risk and a firm's risk taking. Moreover, we perform the difference-in-difference analysis to examine the impact of rollover risk and a firm's fundamental on the relationship between liquidity risk and agency costs. We find that rollover risk enhances but a firm's profit and size mitigates the impact of illiquidity on a firm's risk taking, which is consistent with our calibrated results.

This paper proposes a theoretical framework to quantify and provides an empirical evidence about the influence of liquidity risk on agency costs. It contributes to the strand of existing literatures on both liquidity risk and agency costs. On one hand, originated from the seminal work of Jensen and Meckling (1976) and Myers (1977), a sizable amount of literature emerge to study the impact of agency costs on corporate financing and investment decisions in the context of contingent claim models, such as Mello and Parsons (1992), Leland (1998), Goldstein, Ju and Leland (2001), Moyen (2002), Titman and Tsyplakov (2002) and Mauer and Sarkar (2005). None of these models considers the liquidity risk in the corporate debt market or address how does the liquidity risk affects the agency costs. On the other hand, there is a large volume of literature to study liquidity risk independently[7]. Until recent, especially after the recent financial crisis, Ericsson and Renault (2005) firstly propose a model to show a direct impact of liquidity risk on the default component of credit spreads and find the empirical evidence to support positive correlation. He and Xiong (2012) identify the rollover channel through which the liquidity risk increases the default component of credit spread in the context of the structural model (Leland (1994, 1996)). Later on, He and Milbradt (2013) propose a model to build a default and liquidity

[7] As there are a large volume of literature to study the liquidity risk and credit risk independently, we only list the most cited ones, such as Amihud and Mendelson (1986), Morris and Shin (2004), Frank and Driessen (2006), Bessembinder, Maxwell and Venkataraman (2006), Chen, Lesmond and Wei (2007), Mahanti, Nashikkar, Subrahmanyam, Chacko and Mallik (2008), Beber, Brandt and Kavajecz (2009), Bao, Pan and Wang (2011), Acharya, Amihud and Bharath (2013) for the corporate bond liquidity, Collin-Dufresne and Goldstein (2001), Collin-Dufresne, Goldstein and Martin (2001), Duffie and Singleton (1999), Duffie and Lando (2001), Longstaff, Mithal and Neis (2005), Hackbarth, Miao and Morellec (2006), Acharya, Gale and Yorulmazer (2011), Huang and Huang (2012), Huang and Zhou (2008), Zhang, Zhou and Zhu (2009), Elkamhi, Ericsson, Jiang and Du (2013) Perrakis and Zhong (2014a, b) for the credit risk.

loop to show the interaction between them. Nonetheless, none of these models allow the risk shifting that induces the agency costs. To our knowledge, this paper is the first to study the impact of liquidity risk on the agency costs both theoretically and empirically.

This paper is organized as follows. In Section 2, we show the economic setup for our model and derive the analytical solutions for both debt and equity value. In Section 3, we illustrate calibrations and present results. In Section 4, we construct empirical models and report the results. Section 5 concludes. The proofs for lemmas and propositions are showed in appendix A and the detailed definitions of variables are reported in appendix B.

2. Economic Setup and Debt and Equity Valuation

2.1 Economic Setup

a. Unlevered asset dynamic

We consider a firm whose assets are financed by equity and debt with a tax-deductible coupon. As in most related literature, the values of the components of a firm's balance sheet are estimated as contingent claims of the state variable *V,* a firm's unlevered asset value, representing its economics activities.[8] Following Leland (1998), under risk-neutral measure, we assume unlevered asset value, V, follows a process,

$$\begin{cases} \dfrac{dV_t}{V_t} = (r-q)dt + \sigma_L dW^Q, \; for\ V_S \le V \\ \dfrac{dV_t}{V_t} = (r-q)dt + \sigma_H dW^Q, \; for\ K \le V \le V_S \end{cases} \tag{2.1}$$

[8] Goldstein, Ju and Leland (2001) and He and Milbradt (2013) use a firm's cash flow instead of V; the two approaches are strictly equivalent under strong capital market assumptions.

Where r is risk free rate; q is a firm's payout rate; K is default boundary; and W^Q is a Brownian Motion under a risk-neutral measure. Compared to Leland (1994a, b) and Leland and Toft (1996), there are low- and high-asset-volatility regimes, denoting by subscript L and H, respectively, where $\sigma_L \leq \sigma_H$. When unlevered asset value V is greater than or equals to the risk switching boundary, denoted by V_S, a firm stays in a low-volatility regime. Once unlevered asset value touches risk switching boundary but is still above default boundary, a firm switch to high-volatility regime σ_H by replacing low-risky assets with high-risky ones, such as adopting high-risky projects. Due to the option nature of an equity, the increase of asset volatility increases anticipated equity value at the costs of reducing debt values, which is so-called *asset substitution* (or *agency cost*)[9]. If asset value hits or is below default boundary K for the first time, this firm goes bankrupt and liquidation occurs immediately[10].

b. Stationary debt structure

We adopt an exponential stationary debt structure, proposed by Leland (1994b), under which a firm keeps issuing and retiring pieces of corporate debt at a proportional rate g. Denote face value by p and continuous coupon payments by c per unit time for a piece of debt. Since all pieces of debts outstanding are identical, the total outstanding principals and coupon payments are $P = p/g$ and $C = c/g$, respectively. Debt is initially issued at $t = 0$ with principal P and coupon payment C. As time goes by, the remaining principal at time t is $e^{-gt}P$ and the debt holders receive a cash flow $e^{-gt}(C + gP)$, provided a firm remains solvent. Thus, the average

[9] See Ericsson (1997), Leland and Toft (1994) and Leland (1998) for asset substitution. Jensen and Meckling (1976) firstly discussed agency cost systematically.
[10] It is assumed that absolute priority rule is fully respected.

maturity of this debt is the reciprocal of proportional retirement rate, $g = 1/T$ [11]. Compared to the finite-maturity debt structure proposed by Leland and Toft (1996), the exponential stationary debt structure provides similar results for debt and equity valuations, even in the presence of liquidity risk in the secondary debt market[12] and a simpler analysis[13].

c. Liquidity shocks in the secondary debt market

When a firm keeps retiring debts at face value and issuing new debts at market price given predetermined stationary debt structure, a deviation between market price and face value induces the so-called "*Rollover risk*". To consider rollover risk in our model, we incorporate a secondary debt structure that is similar to Amihud and Mendelson (1996). It is assumed each bond holder is exposed to the idiosyncratic liquidity shocks that hit market randomly. Upon the arrival of a liquidity shock, bond holders have to sell their shares at discount and exit market. For computational convenience, we assume that the arrival of a liquidity shock follows a Possion distribution with arrival intensity, ξ. The costs for selling bonds are proportional to the market value of bonds with a proportional rate, k. As pointed out in He and Xiong (2012), this setup only focuses on analyzing the effect of external market liquidity that is alleviated by improving the internal liquidity of a firm through accumulating cash holding and enlarging available credit lines.

2.2 Corporate Debt and Equity Valuation

a. Debt valuation

[11] See equation (7) in Leland (1994b).
[12] See Perrakis and Zhong (2014) for the comparison between the exponential stationary debt structure (Leland, 1994b) and finite-maturity stationary debt structure (Leland and Toft, 1996) in term of debt and equity valuation.
[13] Exponential stationary debt structure has been used in Ericsson (1997), Mauer and Ott (1996), Leland (1998), He and Milbrat (2014), and Perrakis and Zhong (2014a, b).

Given asset dynamic (2.1) and exponential stationary debt structure, the total outstanding debt value satisfies two differential equations.

For $V_S \le V$,

$$rD_L(V,g) = C + gP - gD_L(V,g) - \xi k D_L(V,g) + (r-q)V\frac{\partial D_L}{\partial V} + \frac{1}{2}\sigma_L{}^2 V^2 \frac{\partial D_L^2}{\partial^2 V} \qquad (2.2)$$

and for $K \le V \le V_S$,

$$rD_H(V,g) = C + gP - gD_H(V,g) - \xi k D_H(V,g) + (r-q)V\frac{\partial D_H}{\partial V} + \frac{1}{2}\sigma_H{}^2 V^2 \frac{\partial D_H^2}{\partial^2 V} \qquad (2.3)$$

with four boundary conditions,

$$D_L\left(V \to +\infty\right) = \frac{C+gP}{r+g+\xi k} \qquad (2.4)$$

$$D_H\left(K\right) = \left(1-\alpha\right)K \qquad (2.5)$$

$$D_H\left(V_S\right) = D_L\left(V_S\right) \qquad (2.6)$$

$$\frac{\partial D_H}{\partial V_{|V=V_S}}\left(V_S\right) = \frac{\partial D_L}{\partial V_{|V=V_S}}\left(V_S\right) \qquad (2.7)$$

The first and second terms on the right side of (2.2) and (2.3) are continuous coupon payments and retired principles received by debt holders respectively. The third term is continuously issued new debts at market price. The liquidity shocks in corporate debt markets induce expected costs to debt holders, which is represented by the fourth term. While the last two terms shows a change of debt value with respect to the asset dynamics in each volatility regime.

When asset value approaches positive infinity, the default risk of corporate debt is close to zero, which is captured by the first boundary condition. On the other hand, when asset value hits default boundary, a default event occurs and debt holders only receive $(1-\alpha)K$, as showed in the second boundary condition. In addition, the third and fourth boundary conditions maintain the continuity of debt dynamics when asset value crosses risk-switching boundary.

Proposition 1*: When an asset dynamic follows (2.1), the analytical solution for corporate debt value in (2.2) and (2.3) with boundary conditions (2.4), (2.5), (2.6) and (2.7), is,*

$$D(V)=\begin{cases} D_L(V)=\dfrac{C+gP}{r+g+\xi k}+a_{1L}V^{y_{1L}}+a_{2L}V^{y_{2L}}, & V_S\le V\le V_U \\ D_H(V)=\dfrac{C+gP}{r+g+\xi k}+a_{1H}V^{y_{1H}}+a_{2H}V^{y_{2H}}, & V_B\le V\le V_S \end{cases} \tag{2.8}$$

Where,

$$\begin{aligned} & a_{1L}=0, \qquad a_{1H}=\frac{A}{K^{y_{1H}}}\left[1-\frac{1}{B}\right] \\ & a_{2H}=\frac{A}{K^{y_{2H}}B}, \qquad a_{2L}=a_{1H}V_S^{y_{1H}-y_{2L}}+a_{2H}V_S^{y_{2H}-y_{2L}} \\ & A=(1-\alpha)K-\frac{C+gP}{r+g+\xi k}, \quad B=1-\frac{y_{2H}-y_{2L}}{y_{1H}-y_{2L}}\left(\frac{V_S}{K}\right)^{y_{2H}-y_{1H}} \end{aligned} \tag{2.9}$$

b. Equity valuation and endogenous default boundary

The equity holders have a right to claim for residual cash flows in the form of dividends after coupon payments and rollover costs if the market price of debt deviates from face value. Leland (1998) solves for equity value by deducting debt value and bankruptcy costs, and adding tax shields to the unlevered asset value[14], without solving above differential equations directly. This method

[14] See equation (21) in Leland (1998).

cannot be applied to solve for equity value after incorporating the liquidity shocks in the secondary debt market because rollover costs themselves are a function of debt value. In our case, the following two differential equations have to be satisfied by equity.

For $V_S \leq V$

$$rE_L = qV - (1-w)C + gD_L(V) - gP + (r-q)V\frac{\partial E_L}{\partial V} + \frac{1}{2}\sigma_L{}^2V^2\frac{\partial E_L{}^2}{\partial^2 V} \tag{2.10}$$

For $K \leq V \leq V_S$,

$$rE_H = qV - (1-w)C + gD_H(V) - gP + (r-q)V\frac{\partial E_H}{\partial V} + \frac{1}{2}\sigma_H{}^2V^2\frac{\partial E_H^2}{\partial^2 V} \tag{2.11}$$

with boundary conditions,

$$E_H(K) = 0 \tag{2.12}$$

$$\frac{\partial E_H}{\partial V_{|V=K}} = l \tag{2.13}$$

$$E_L(V_S) = E_H(V_S) \tag{2.14}$$

$$\frac{\partial E_L}{\partial V_{|V=V_S}} = \frac{\partial E_H}{\partial V_{|V=V_S}} \tag{2.15}$$

The first and second terms in (2.10) and (2.11) show the cash flows in the form of dividends and after tax coupon payments, respectively. The difference between the third and fourth terms represents rollover costs, a function of debt value that satisfies (2.2) and (2.3), to equity holders. As debt structure is determined ex ante, when a liquidity shock hits debt market as a surprise, the debt value decreases, suggesting a significant increase of rollover costs for equity holders[15]. While the last two terms show a fluctuation of equity value with respect to asset dynamics.

When asset value hits default boundary, equity value becomes to be zero, composing the first boundary condition. We set the first derivative of equity value equal to a free parameter l that is determined by boundary condition. When asset value approaches positive infinity, equity value is linear with respect to asset value. Moreover, we add boundary conditions (2.14) and (2.15) to maintain continuity when asset value crosses risk-switching boundary. Since the solution of (2.10) and (2.11) depends on the solution of (2.2) and (2.3), we have to solve these four differential equation simultaneously for equity value.

Proposition 2: *When an asset dynamic follows (2.1), the solutions for equity value in (2.10) and (2.11) with boundary conditions (2.12), (2.13), (2.14) and (2.15), are,*

$$\begin{aligned} E_L(V) = V + \frac{2}{\sigma_L{}^2}\left[\frac{(1-w)C + gP - ga_0}{\eta_L+\gamma_L}\left(\frac{\phi_{1L}-1}{\eta_L} - \frac{1-\phi_{2L}}{\gamma_L}\right) - \frac{qV_S}{\eta_L+\gamma_L}\left(\frac{\phi_{1L}}{\eta_L-1}+\frac{\phi_{2L}}{\gamma_L+1}\right)\right] \\ -\frac{2}{\sigma_L{}^2}\left[\frac{ga_{1L}V_S{}^{y_{1L}}}{\eta_L+\gamma_L}\left(\frac{\phi_{1L}-\phi_{3L}}{\eta_L-y_{1L}} - \frac{\phi_{3L}-\phi_{2L}}{\gamma_L+y_{1L}}\right) + \frac{ga_{2L}V_S{}^{y_{2L}}}{\eta_L+\gamma_L}\left(\frac{\phi_{1L}-\phi_{4L}}{\eta_L-y_{2L}} - \frac{\phi_{4L}-\phi_{2L}}{\gamma_L+y_{2L}}\right)\right] \\ +E_H(V_S)\frac{(\eta_L\phi_{1L}+\gamma_L\phi_{2L})}{\eta_L+\gamma_L} + \frac{2(\phi_{1L}-\phi_{2L})}{\sigma_L{}^2(\eta_L+\gamma_L)}\left(\left(r-q-\frac{1}{2}\sigma_L{}^2\right)E_H(V_S) + \frac{1}{2}\sigma_L^2\frac{\partial E_H}{\partial V_{V=V_S}}\right) \end{aligned} \quad (2.16)$$

[15] See He and Xiong (2012).

$$E_H(V) = V + \frac{2}{\sigma_H{}^2}\left[\frac{(1-w)C + gP - ga_0}{\eta_H + \gamma_H}\left(\frac{\phi_{1H}-1}{\eta_H} - \frac{1-\phi_{2H}}{\gamma_H}\right) - \frac{qK}{\eta_H+\gamma_H}\left(\frac{\phi_{1H}}{\eta_H - 1} + \frac{\phi_{2H}}{\gamma_H+1}\right)\right]$$
$$-\frac{2}{\sigma_H{}^2}\left[\frac{ga_{1H}K^{y_{1H}}}{\eta_H+\gamma_H}\left(\frac{\phi_{1H}-\phi_{3H}}{\eta_H - y_{1H}} - \frac{\phi_{3H}-\phi_{2H}}{\gamma_H + y_{1H}}\right) + \frac{ga_{2H}K^{y_{2H}}}{\eta_H+\gamma_H}\left(\frac{\phi_{1H}-\phi_{4H}}{\eta_H - y_{2H}} - \frac{\phi_{4H}-\phi_{2H}}{\gamma_H + y_{2H}}\right)\right]$$
$$+\frac{l(\phi_{1H}-\phi_{2H})}{(\eta_H+\gamma_H)}$$

(2.17)

Where,

$$l = \frac{\left[\begin{array}{l} -\frac{qV_S}{\eta_L - 1} + \frac{(1-w)C + gP - ga_0}{\eta_L} - \frac{ga_{1L}V_S{}^{y_{1L}}}{\eta_L - y_{1L}} - \frac{ga_{2L}V_S{}^{y_{2L}}}{\eta_L - y_{2L}} \\ +\left(\left(r - q - \frac{1}{2}\sigma_L{}^2(1-\eta_L)\right)H_0 + \frac{1}{2}\sigma_L^2 H_2\right) \end{array}\right]}{-\left(r - q - \frac{1}{2}\sigma_L{}^2(1-\eta_L)\right)H_1 - \frac{1}{2}\sigma_L^2 H_3} \qquad (2.18)$$

The ancillary parameters are defined in appendix.

Equity holders are allowed to choose ex-post risk-switching boundary and default boundary endogenously to maximize equity value only after debt holders put debt structure in place ex ante with the purpose of maximizing the total firm's value. Given risk-switching boundary, limited liability of equity holder and pre-determined debt structure, the endogenous default boundary essentially is the level of an asset value at which the first derivative of equity with respect to asset value is zero, which can be solved numerically by setting $l = 0$.

3. Model Calibrations

3.1 Calibration

In this section, we perform the simulated analysis with calibrated numbers to study the impact of rollover risk on agency costs. First, we construct a base case by combining empirical findings and calibrations in the literatures and show the details in Table 1. Similar to Leland (1998), we normalize initial asset value $V_0 = \$100$ and choose risk free rate $r = 8\%$. We set corporate debt tax benefit rate, $w = 27\%$, which is the same as that in He and Xiong (2012). We use rounded payout rate $q = 2\%$ based on sample firms in Huang and Zhou (2008) who document around 2.02% and 2.15% average payout rate for *A* and *BB* rated debts, respectively[16]. According to Chen (2010) who document average recovery rates, 40.1% and 41.5% with jump-risk premium and correlation within market, respectively, across nine different states, we set the proportional recovery rate, $\alpha = 40\%$. Custodio, Ferreira and Laureano (2012) examined the debt maturities in U.S. during last four decades and found that around 45.6% firms have a 3-year average debt maturity in their sample in 2008[17]. We use average maturity $T = 3$ as our base case and also check other maturities, such as $T = 1$ and $T = 5$, in our analysis.

[Please Insert Table 1 about Here]

We separate our calibration into two base cases based on rating classes. We set the proportional transaction costs in the presence of liquidity shock, $k = 1.00\%$ and $k = 0.50\%$ for the *A* and *BB* rating, respectively, followed He and Xiong (2012) who are inspired by the empirical findings in Edward, Harris and Piwowar (2007) and Bao, Pan and Wang (2011). We choose low asset volatilities, $\sigma_L = 21\%$ and $\sigma_L = 23\%$ for the *A* and *BB* rating, respectively, based on the calibrations used in Zhang, Zhou and Zhu (2009)[18]. He and Xiong (2012) choose the face value of

[16] See Table 1 in Huang and Zhou (2008).
[17] See Table 2 in Custodio, Ferreira and Laureano (2012).
[18] See Table 7 in Zhang, Zhou and Zhu (2009).

aggregated debt and coupon amount so that 1-year bonds are issued at par and the credit spreads of these bonds are consistent with the empirical evidence. In particular, they set the face value of debt $P = 61.68$ for a *BB* rating. Inspired by their work, we set $P = 60$ for our *BB* rating ex ante. Then we calculate the corresponding coupon payment and high asset volatility to make debt issued at par in the absence of liquidity shocks and generate a credit spread that equals to 330 base points for a *BB* rated bond. According to our calibration for a *BB* rated bond, we find $\sigma_H = 1.9445 * \sigma_L = 44.72\%$. By assuming a same increment magnitude of asset volatility for an *A* rated, indicating $\sigma_H = 1.9445 * 21\% = 40.83\%$, we calculate the corresponding face value of total outstanding debts to generate a credit spread that equals to 100 base points and find $P = 52.6$ for an *A* rated bond.

3.2 Risk Shifting and agency costs

a. Risk Shifting option

We allow a firm to restructure capital structure by replacing low-risky assets with high-risky assets to amplify asset volatility. The initial capital structure and debt structure of a firm are determined by all shareholders ex ante. Given information set, $\Omega_F\left(P, V_0, \sigma_L, \sigma_H, r, q\right)$, available to all shareholders ex ante, debt holders expect an increase of asset volatility manipulated by equity holders and choose corresponding coupon payments, optimal switching boundary and endogenous default boundary to maximize the total firm's value.

$$\max_{C, K, V_S} v\left(V_0, C, K, V_S\right) \tag{2.19}$$

The face value of total outstanding debts is determined exogenously and the coupon payment is selected to make the debts issued at par ex ante. Given endogenous default boundary, we solve this maximization problem by pinning down optimal risk switching boundary ex ante.

Once debt structure is in place ex ante, equity holders cannot change it but can manipulate optimal risk switching boundary (or capital restructuring point) to maximize equity value ex post. Similar to equation (26) in Leland (1998), the first derivative of equity value with respect to the risk switching boundary is,

$$z\left(V_S, K, C, P, T\right) = \frac{dE_L}{dV_S}\Big|_{V=V_S} = \frac{\partial E_L}{\partial V_S}\Big|_{V=V_S} + \frac{\partial E_L}{\partial K}\Big|_{V=V_S} \frac{\partial K}{\partial V_S} \quad (2.20)$$

The change of equity value with respect to risk-switching boundary is consisted by two parts. The first term suggests a change of equity value induced by the change of risk-switching boundary only. Meanwhile, a change of risk-switching boundary moves endogenous default boundary as well, as shown in the second term.

Due to the option nature of equity, equity holders choose optimal risk-switching boundary by balancing the anticipated equity appreciation and the increase of default probability. When asset value is high, equity is consider as a deep in-the-money option for which the volatility impact is small. When asset value is low enough, the anticipated equity appreciation dominates the costs after risk-switching, suggesting a positive value for $z\left(V_S, K, C, P, T\right)$. Hence, the optimal risk switching boundary is the asset level when equation (2.20) equals to zero[19].

[19] Leland (1998) shows that there are at most two locally optimal solutions to his problem. One is less than the initial asset value and the other one is equal to the upper boundary in his model. However, we remove the upper boundary in our model. Therefore, we only have one solution for this problem. For certain extreme calibrations, our solution could be greater than the initial asset value.

[Insert Figure 1 about Here]

Given the calibrations in the base case, Figure 1 shows that $z\left(V_S, K, C, P, T\right)$ is a decreasing function of risk-switching boundary. Compared to $z\left(V_S, K, C, P, T\right)$ for a firm with *A* rating, it's greater for a firm with *BB* rating and increases faster as asset value decreases. Further, liquidity shock shifts the curve upward and pushes the optimal risk switching boundaries upward significantly for both *A* and *BB* ratings.

b. Agency Costs

We calculate agency costs by considering the percentage of total asset value changes between the ex-ante and ex-post optimal cases,

$$AC = \frac{v_{ex\_ante} - v_{ex\_post}}{v_{ex\_ante}} \tag{2.21}$$

[Insert Table 2 about Here]

Table 2 reports the characteristics of a firm with agency costs and liquidity risk for both *A* and *BB* rated bonds. "Ex-Ante" and "Ex-Post" represents the cases of maximizing the total asset value and maximizing the total equity value, respectively, without liquidity risk. The agency costs are only 0.37% and 1.22% for *A* and *BB* rated firm, respectively, without liquidity risk. It is assumed that liquidity shock hits corporate debt market as a surprise. A firm cannot adjust its existing debt structure but can modify its expected risk-switching and endogenous default boundaries to maximize the total asset value, which is shown as "Ex-Ante-Liquidity". "Ex-Post-Liquidity" shows the case in which equity holders maximize equity value by choosing risk-switching and endogenous default boundaries ex post. Compared to the cases without liquidity risk, we find a significant increase of optimal risk switching boundary from ex ante to ex post case. We also document a significant increase of agency costs, about 78% and 103% for *A* and *BB* rated firm, with liquidity risk, compared to these without liquidity risk.

[Insert Figure 2, Figure 3 and Figure 4 about Here]

As equity holders are only allowed to manipulate risk switching and endogenous default boundary but not debt and capital structure, we plot optimal switching and endogenous default boundaries with respect to the liquidity intensities in Figure 2 and Figure 3, respectively. Consisted with the findings in He and Xiong (2012), we observe that endogenous default boundaries increases as liquidity intensity increases. Nonetheless, for a *BB* rated firm, equity holders prefer to lower the ex-post endogenous default boundary when liquidity intensity is high compared to that for the ex-ante case. While for optimal risk switching boundaries, the deviation between the ex-ante and ex-post cases becomes wider as liquidity intensity increases. Specifically, the ex-ante risk-switching boundaries are almost invariate with respect to liquidity intensity, but the ex-post risk-switching boundaries are shifted upward significantly when liquidity deteriorates, suggesting that equity holders are more likely to use asset substitution by overinvesting in high-risky projects, which enhances the confliction of interests between equity and debt holders and leads to the greater agency costs. Moreover, the incremental speed of agency costs in term of percentage is much greater for a *BB*-rated firm compared to that for an *A*- rated firm, as exhibited in Figure 4.

4. Empirical Analysis

4.1 Hypotheses

According to the calibrated results, we find that optimal risk shifting boundary is positively related to liquidity intensity, as exhibited in Figure 2. High optimal risk shifting boundary indicates an earlier switching to or a longer stay in high-volatility regime compared to that with low optimal risk shifting boundary, which suggests a positive relationship between illiquidity and asset volatility. Further, an increase of asset volatility amplifies asset substitution between equity and debt holders, inducing a higher agency cost. Therefore, we construct a hypotheses below:

Hypotheses 1: *The illiquidity in secondary corporate debt market is positively related to a firm's asset substitution between equity and debt holders.*

To test this hypotheses, we use a firm's risk-taking measures, implied asset volatility and earning volatility, to proxy for the asset substitution between equity and debt holders.

4.2 Variables and sample description

a. Illiquidity Measures

To construct the illiquidity measures used in this study, we take the following steps. We first cleaned Standard TRACE following prior literature (e.g., Harris and Piwowar, 2006; Dick-Nielsen et al., 2012). We removed trades that are canceled, corrected, and commission and the bond transactions under \$100,000 to avoid the effect of retail transactions. We then construct four illiquidity measures: Amihud ratio, Imputed roundtrip trades, Price dispersion, and Inter-quartile range. The detailed calculations for the illiquidity measures are presented below.

The Amihud ratio (Amihud) for bond *i* on day *t* is defined as the average of absolute returns of consecutive transactions divided by the trade size $Q_{i,t}^{j}$ (in million \$) within day *t*

$$Amihud_{i,t} = \frac{1}{N_{i,t}} \sum_{j=1}^{N_{i,t}} \frac{\left| P_{i,t}^{j} - P_{i,t}^{j-1} \right| / P_{i,t}^{j-1}}{Q_{i,t}^{j}} * 100,$$

where $N_{i,t}$ denotes the number of returns on day *t* for bond *i*, $Q_{i,t}^{j}$ and $P_{i,t}^{j}$ denote the trade size and price for transaction *j* respectively. A larger Amihud ratio indicates a lower bond liquidity since bond price moves more for a given trade size. We multiply the Amihud ratio by 100 to make it comparable in numerical scale with other illiquidity measures.

Imputed roundtrip trades (IRT) (Feldhutter, 2012) implicitly measure transaction costs by assuming a pre-matched arrangement trading. We defined IRT for bond *i* on day *t* as

$$IRT_{i,t} = \frac{P_{i,t}^{Max} - P_{i,t}^{Min}}{P_{i,t}^{Max}},$$

where $P_{i,t}^{Max}$ *and* $P_{i,t}^{Min}$ denote the maximum and minimum price for bond *i* on day *t*, respectively. A larger IRT implies higher roundtrip transaction costs and a lower bond liquidity.

Price dispersion (PD) (Jankowitsch, Nashikkar, and Subrahmanyam, 2011, 2012;) is defined as

$$PD_{i,t} = \sqrt{\frac{\sum_1^{K_{i,t}} Q_{i,t}^j * (P_{i,t}^j - \overline{P_{i,t}})^2}{\sum_1^{K_{i,t}} Q_{i,t}^j}},$$

where $K_{i,t}$ and $\overline{P_{i,t}}$ denote the total number of trades and the average price on day t for bond i, respectively; $Q_{i,t}^j$ and $P_{i,t}^j$ denote the trade size and price for transaction *j*. Price dispersion measures the volume-weighted price difference, reflecting the potential transaction costs for a trade.

Inter-quartile range (IQR), used by Han and Zhou (2008) and Helwege, Huang and Wang (2014), is defined as

$$IQR_{i,t} = \frac{P_{i,t}^{75th} - P_{i,t}^{25th}}{\overline{P_{i,t}}},$$

where $P_{i,t}^{25th}$, $P_{i,t}^{75th}$, and $\overline{P_{i,t}}$ indicate the 75th percentile of prices, the 25th percentile of prices and the average price for bond *i* on day *t*, respectively. The logic behind this measure is that less liquid bonds tend to have higher price volatility within a day. Compared to the maximum and minimum prices in *IRT*, the 75th percentile and the 25th percentile of the prices are less sensitive to outliers.

After calculating the trading volume weighted illiquidity measures for each bond on each day, we winsorize all the illiquidity measures at 1% and 99% to remove the potential outliers. Then we weight daily illiquidity measures using daily trading volume to calculate the monthly bond-level illiquidity measures. Next, we take the average of the monthly measures over a year to construct the annualized bond-level illiquidity measures. We finally follow Helwege, Huang, and Wang (2014) to construct the firm-level illiquidity measures using bond level measures weighted by outstanding amount of bonds. The summary statistics of the illiquidity measures are presented in Table 3.

[Please Insert Table 3 about Here]

b. Risk-taking Measures

We adopt two different measures for firms' risk-taking. Our first measure is asset volatility implied from Merton model (Merton 1974). This measure of asset volatility is a critical input to calculate *Distance to Default* and is widely used in many studies (eg. Hillegeist, Keating, Cram, and Lundstedt, 2004; Bharath and Shumway, 2008; Campell, Hilscher and Szilagyi, 2008; Mansi, Maxwell and Zhang, 2010). Asset volatility is also a well-known measure for asset substitution or risk-shifting (Jensen and Meckling, 1976, Helwege, Huang, and Wang, 2015.

Following the literature, we solve two equations below simultaneously to obtain the implied asset volatility σ_V and asset value V

$$E = VN(d_1) - e^{-rT}DN(d_2) \tag{4.1}$$

$$\sigma_E = \left(\frac{V}{E}\right)\frac{\partial E}{\partial V}\sigma_V \tag{4.2}$$

$$d_1 = \frac{Ln\left(\frac{V}{D}\right) + (r + 0.5\sigma_V^2)T}{\sigma_V\sqrt{T}}$$

$$d_2 = d_1 - \sigma_V\sqrt{T},$$

where E is the market value of a firm's equity; D is the face value of a firm's total debts that equal short-term debts plus one-half long-term debts (Crosbie and Bohn, 2001; Vassalou and Xing, 2004;

Campell, Hilscher, and Szilagyi, 2008); *r* is one-year treasury rate; *T* is the time-to-maturity of a bond that is set to 1 year (Mansi, Maxwell and Zhang, 2010; Campell, Hilscher, and Szilagyi, 2008); *N(·)* is the cumulative standard normal distribution function; σ_E is 3-month rolling sample standard deviation (Campell, Hilscher, and Szilagyi, 2008).

To solve equation (4.1) and (4.2), we follow Campell, Hilscher, and Szilagyi (2008) to set the initial value of asset volatility and asset value, and the iteration continues untill we find the solution for both asset volatility and asset value. We winsorize asset volatility at 1% and 99% to remove outliers. Table 3 reports the summary statistics of asset volatility.

The second risk-taking measure we adopt is earning volatility (John, Litov, and Yeung, 2008; Zhang, 2009). We define earning volatility as the standard deviation of quarterly income before extraordinary items deflated by total assets during the two years preceding a fiscal year end. We remove firms with missing observations during this two-year period.

c. Sample Selection

We match firm-level illiquidity measures with Compustat Fundamental Annual dataset to obtain accounting information. Observations with missing information are excluded. The final sample contains U.S. public firms with bond trading reported in Standard TRACE spanning from July 2002 to January 2015 and consists of 9,428 firm-year observations for 1,379 firms.

4.3 Empirical Results

a. OLS Specification

To assess the effect of bond illiquidity on firm's risk-taking activities, we estimate the regression below

$$\text{Asset volatility}_{\text{i,t}}\left(\text{Earning volatility}_{\text{i,t}}\right) = a + b * \text{Bond Illiquidity}_{\text{i,t}} + c' * \text{Other Controls}_{\text{i,t}} + \text{Year}_{\text{t}} + \text{FirmDummy}_{\text{ i}} + \text{Rating Dummies}_{\text{i,t}} + error_{i,t} \quad (4.3)$$

where *i* and *t* index firm and time, respectively. The dependent variables are asset volatility and earning volatility which measure a firm's risk-taking activities. We adopt Amihud ratio, Inter-quartile range, Price dispersion and Imputed roundtrip trade to proxy for the illiquidity of bonds of a firm[20]. We control for firm characteristics that may affect risk-taking activities, including firm size, leverage, sales growth, and profit (John, Litov, and Yeung, 2008).

We include the natural logarithm of firm age in the regression to control for borrowing ability since younger firms are usually associated with a lower reputation in bond market and more difficult to borrow in the bond market (Diamond, 1991; Carty, 1996; Johnson, 1997; Datta, Iskandar-Datta, and Patel, 1999; and Cai, Helwege and Warga, 2007). We argue that when illiquidity shocks hit the secondary corporate bond market unexpectedly the equity holders in younger firms prefer to take riskier projects because of higher costs to rollover their debt.

In the regression, we also control for rollover risk proxied by the percentage of debt due in one year to the total debt.

In addition, we use firm dummies to control for the possibly omitted firm characteristics that are constant during the time period and year dummies to account for the intertemporal variation that may bias the relationship between bond illiquidity and risk-taking. We also employ credit rating dummies to control for credit risk in the regression. We cluster standard errors by firms to avoid inflated t-statistics caused by the autocorrelation (Petersen, 2009).

[Please Insert Table 4 about Here]

As reported in Panel A of Table 4, we find that the estimated coefficients of four bond illiquidity measures are all positive and significant. In particular, one standard deviation increase in Amihud ratio, Imputed roundtrip trades, Price dispersion or Inter-quartile range will increase

[20] We apply four measures of bond illiquidity to confirm the robustness of the empirical results.

asset volatility by 3.720*0.01/55.3%=6.7%, 7.521*0.006/55.3%=8.2%, 3.779*0.002/55.3%=1.4% or 3.527*0.006/55.3%=3.8%, respectively. The positive relationship between asset volatility and bond illiquidity confirms the *Hypotheses 1* that when illiquidity shocks hit in the secondary bond market, a firm will transfer value from bondholders to equity holders by taking extra risk, so-called "asset substitution" which increases the volatility of the firm. Further, we re-estimate the above regressions using earning volatility as the dependent variable and report the results in Panel B of Table 4. As expected, we find that the coefficients for all illiquidity measures remain positive and significant, confirming the positive relationship between bond illiquidity and a firm's risk taking. Numerically, one standard deviation increases in Amihud ratio, Imputed roundtrip trades, Price dispersion or Inter-quartile range will amplify profit volatilities by 0.097*0.01/1.5%=6.46%, 0.254*0.006/1.5%=10.16%, 0.201*0.002/1.5%=2.68% or 0.197*0.006/1.5%=7.88%, respectively.

As discussed in our theoretical model, the illiquidity shock in the secondary corporate bond market affects a firm's risk-taking through the rolling over channel (not clear to me here, see the comment above). A high rollover risk will increase a firm's risk-taking behavior. As expected, we document significantly positive coefficients for rollover risk in both asset-volatility and earning-volatility regressions. Also, we note that the firms with a longer history are associated with relatively lower risk-taking activities, but such relationship is only significant for the asset-volatility regressions. Consistent with John, Litov, and Yeung (2008), we find that firm size and profit are negatively related to risk-taking activities, and leverage and sale's growth are positively related to risk-taking activities. The positive coefficient of sale's growth is not significant for the earning-volatility regressions.

b. Causality Test Using TRACE Dissemination Information

A potential concern in our analysis is that the relationship between bond liquidity and a firm's acquisition activity might be spurious. To address this issue we use a natural experiment around the implementation of the TRACE. Our identification strategy relies on the phase-in feature of TRACE introduction.

On July 1, 2002, Financial Industry Regulatory Agency (FINRA) began disseminating trades in investment-grade corporate bonds with issuance size of $1 billion or greater. Over time, bond coverage expanded in phases and TRACE was fully implemented by January 2006, covering essentially all publicly traded bonds. The introduction of TRACE reduced transaction costs and improved transparency leading to an increase in bond liquidity (e.g. Bessembinder, Maxwell, and Venkataraman, 2006; Edwards, Harris, Piwowar, 2007; Goldstein, Hotchkiss, and Sirri, 2007).

Since TRACE implementation is unrelated to the firms' decision to undertake acquisitions it represents an exogenous shock to firms' liquidity that allows us to isolate the impact of bond liquidity on firms' risk taking behavior.

In spirit of Bertrand and Mullainathan (1999a, 1999b, 2003), we create a variable, the POST-TRACE dummy, that equals one if a firm's bonds are covered by TRACE during year *t*, and captures the impact of an increase in liquidity in the years following TRACE introduction. In addition, for each firm, we keep the Compustat accounting information one year before and after its first bond transaction in TRACE. We then estimate the regression below,

$$\text{Asset volatility}_{\mathrm{i,t}}\left(\text{Earning volatility}_{\mathrm{i,t}}\right) = a + b * \text{Post} - \text{TRACE}_{i,t} + c' * \text{Controls}_{\mathrm{i,t}} + \text{Year}_{\mathrm{t}} + \text{Firm}_{\mathrm{i}} + \text{Ratings}_{\mathrm{i,t}} + error_{i,t} \quad (4.4)$$

where all the variable and parameters are the same as these in Equation (4.3) expect Post-TRACE dummy.

[Please Insert Table 5 about Here]

As shown in Table 5, we find that a negative and significant coefficient for the Post-TRACE dummy for both asset-volatility and earning-volatility regressions. It suggests that a positive exogenous shock of the liquidity in the secondary corporate bond market caused by external regulation change significantly reduces a firm's risk-taking activities, and confirms our theoretical conjecture that the illiquidity shocks in the second corporate bond market amplifies the asset substitution phenomenon.

c. Possible Mechanism

In our theoretical framework, we show that the presence of illiquidity shock in the secondary corporate bond market amplifies the asset substitution phenomenon through rollover channel. To test this conjecture, we introduce an interaction term that equal to the product of rollover risk proxies and illiquidity proxies, as shown below,

$$\text{Asset volatility}_{\text{i,t}}\left(\text{Earning volatility}_{\text{i,t}}\right) = a + \text{b} * \text{Amihud} + c * \text{Amihud} * \text{rollover risk} + d' * \text{Controls}_{\text{i,t}} + \text{Year}_{\text{t}} + \text{Firm}_{\text{i}} + \text{Ratings}_{\text{i,t}} + error_{i,t} \quad (4.5)$$

The regression results in Table 6 shows positive (negative) and significant coefficients for the interaction terms with rollover risk (firm age), confirms that bond illiquidity affect risk-taking activities through rollover channel.

[Please Insert Table 6 about Here]

Further, we test whether a firm's fundamental has an impact on the bond illiquidity effect on firms' risk-taking. We use profits, size and leverage to proxy for a firm's fundamental and construct the interaction terms with illiquidity measures, similar to the terms in equation (4.5).

5. Conclusion

We build a model to study the impact of liquidity risk in the corporate debt market on the agency costs, hedging benefits and default component of credit spreads through the rollover channel. The exogenous liquidity shocks decrease the debt value, which generates costs to equity holder through continuously retiring and issuing debts. Facing the rollover costs, equity holders will shift to the high asset volatility regime to take advantage of asset substitution effect, suggesting much greater agency costs and amplified the default components of credit spreads, especially for a firm with weaker fundamental. Further, we document significant hedging benefits by analyzing a wide range of hedging strategies. The hedging synergies can be created by hedging the volatility risk and liquidity risk integratedly compared to hedge them independently. Our findings highlight the economic value creation of hedging and call an attention to the risk management, especially for a firm with poor fundamental and facing high exogenous liquidity shocks in the secondary corporate debt market.

In this paper, we show a significant impact of exogenous liquidity risk on agency costs and risk management. On the flip side, agency costs decrease a firm’s fundamental, which might worsen the endogenous liquidity of corporate bond. Although this is not a concern in this paper by assuming exogenous liquidity shocks, it would be very interesting to study the interaction between the endogenous liquidity of corporate debt and the agency cost for the further research.

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Appendix A: Proofs for Proposition

Proof for Proposition 1:

The debt value D_L and D_H satisfy differential equation (2.2) and (2.3), respectively. The general solutions are,

$$\begin{aligned} D_L(V) &= a_0 + a_{1L}V^{y_{1L}} + a_{2L}V^{y_{2L}} \\ D_H(V) &= a_0 + a_{1H}V^{y_{1H}} + a_{2H}V^{y_{2H}} \end{aligned} \tag{1.1}$$

Where,

$$\begin{aligned} a_0 &= \frac{C+gP}{r+g+\xi k} \\ y_{1L} &= \frac{-\left(r-q-0.5\sigma_L{}^2\right)+\sqrt{\left(r-q-0.5\sigma_L{}^2\right)^2+2\sigma_L{}^2\left(r+g+\xi k\right)}}{\sigma_L{}^2} \\ y_{2L} &= \frac{-\left(r-q-0.5\sigma_L{}^2\right)-\sqrt{\left(r-q-0.5\sigma_L{}^2\right)^2+2\sigma_L{}^2\left(r+g+\xi k\right)}}{\sigma_L{}^2} \\ y_{1H} &= \frac{-\left(r-q-0.5\sigma_H{}^2\right)+\sqrt{\left(r-q-0.5\sigma_H{}^2\right)^2+2\sigma_H{}^2\left(r+g+\xi k\right)}}{\sigma_H{}^2} \\ y_{2H} &= \frac{-\left(r-q-0.5\sigma_H{}^2\right)-\sqrt{\left(r-q-0.5\sigma_H{}^2\right)^2+2\sigma_H{}^2\left(r+g+\xi k\right)}}{\sigma_H{}^2} \end{aligned} \tag{1.2}$$

As $V_U \to +\infty$ and $y_1 > 0, y_2 < 0$, we have $a_{1L} = 0$ in order to satisfy the boundary condition (2.4). According to the other three boundary conditions (2.5), (2.6) and (2.7), we have,

$$\begin{cases} \dfrac{C+gP}{r+g+\xi k}+a_{1H}K^{y_{1H}}+a_{2H}K^{y_{2H}}=(1-\alpha)K \\ a_{2L}V_S^{y_{2L}}-a_{1H}V_S^{y_{1H}}-a_{2H}V_S^{y_{2H}}=0 \\ y_{2L}a_{2L}V_S^{y_{2L}-1}-y_{1H}a_{1H}V_S^{y_{1H}-1}-y_{2H}a_{2H}V_S^{y_{2H}-1}=0 \end{cases} \tag{1.3}$$

Solving these linear equations, we have (2.9). QED.

<u>Proof for Proposition 2:</u>

We use Laplace transform approach to solve the differential equation (2.10) and (2.11) with four boundary conditions, simultaneously. Define $m_H=\ln\left(\frac{V}{K}\right)$ and insert into (2.11) to replace V, we have,

$$rE_H=\left(r-q-\frac{1}{2}\sigma_H{}^2\right)\frac{\partial E_H}{\partial m_H}+\frac{1}{2}\sigma_H{}^2\frac{\partial E_H{}^2}{\partial^2 m_H}+qKe^{m_H}-(1-w)C+gD_H(m_H)-gP \tag{1.4}$$

With boundary conditions,

$$E_H(0)=0 \text{ and } \frac{\partial E_H(0)}{\partial m_{H|m_H=0}}=l \tag{1.5}$$

Define the Laplace transformation of $E_H(m_H)$ as,

$$FH(s)\equiv L[E_H(m_H)]=\int_0^\infty e^{-sm_H}E_H(m_H)\,dm_H \tag{1.6}$$

Then, apply the Laplace transformation to both sides. It gives

$$rFH(s)=\left(r-q-\frac{1}{2}\sigma_L{}^2\right)L\left[\frac{\partial E_H}{\partial m_H}\right]+\frac{1}{2}\sigma^2 L\left[\frac{\partial E_H{}^2}{\partial^2 m_H}\right]+\frac{qK}{s-1}-\frac{(1-w)C+gP}{s}+gL\left[D_H(m_H)\right] \quad (1.7)$$

Note that,

$$\begin{aligned} L\left[\frac{\partial E_H}{\partial m_H}\right] &= sFH(s)-E_H(0)=sFH(s) \\ L\left[\frac{\partial E_H{}^2}{\partial^2 m_H}\right] &= s^2FH(s)-sE_H(0)-\frac{\partial E_H(0)}{\partial m_{H|m_H=0}}=s^2FH(s)-l \end{aligned} \quad (1.8)$$

Thus, we have

$$\left[r-\left(r-q-\frac{1}{2}\sigma_H{}^2\right)s-\frac{1}{2}\sigma_H{}^2s^2\right]FH(s)=\frac{qK}{s-1}-\frac{(1-w)C+gP}{s}+gL\left[D_H(m_H)\right]-\frac{1}{2}\sigma_H{}^2 l \quad (1.9)$$

Define $\eta_H>0$ and $-\gamma_H<0$ to be the two roots of the following equation with respect to s,

$$r-\left(r-q-\frac{1}{2}\sigma_H{}^2\right)s-\frac{1}{2}\sigma_H{}^2s^2=0 \quad (1.10)$$

Then we have $\eta_H=z_H-d_H>1$ and $\gamma_H=z_H+d_H>0$ where,

$$d_H=\frac{r-q-\frac{1}{2}\sigma_H{}^2}{\sigma_H{}^2} \text{ and } z_H\equiv\frac{\left(d_H{}^2\sigma_H{}^4+2r\sigma_H{}^2\right)^{1/2}}{\sigma_H{}^2} \quad (1.11)$$

Thus,

$$\frac{1}{2}\sigma_H{}^2FH(s)=-\frac{1}{(s-\eta_H)(s+\gamma_H)}\left\{\frac{qK}{s-1}-\frac{(1-w)C+gP}{s}+gL\left[D_H(m_H)\right]-\frac{1}{2}\sigma_H{}^2 l\right\} \quad (1.12)$$

According to the solutions for debt value in (1.1), we have,

$$L\left[D_H(m_H)\right]=\int_0^{\infty} e^{-sm_H}\left[a_0+a_{1H}V^{y_{1H}}+a_{2H}V^{y_{2H}}\right]dm_H=\frac{a_0}{s}+\frac{a_{1H}K^{y_{1H}}}{s-y_{1H}}+\frac{a_{2H}K^{y_{2H}}}{s-y_{2H}} \quad (1.13)$$

Inserting (1.13) into (1.12), we have

$$\frac{1}{2}\sigma_H{}^2 FH(s)=-\frac{\frac{1}{s-\eta_H}-\frac{1}{s+\gamma_H}}{\eta_H+\gamma_H}\left\{\frac{qK}{s-1}-\frac{(1-w)C+gP}{s}+\frac{ga_0}{s}+\frac{ga_{1H}K^{y_{1H}}}{s-y_{1H}}+\frac{ga_{2H}K^{y_{2H}}}{s-y_{2H}}-\frac{1}{2}\sigma_H{}^2 l\right\}$$

(1.14)

Inverting the Laplace transformation, we have the equity value,

$$\begin{aligned}
&E_H(m_H)=L^{-1}\left[FL(s)\right]\\
&=V-\frac{2qK}{(\eta_H+\gamma_H)\sigma_H{}^2}\left(\frac{1}{\eta_H-1}e^{\eta_H m_H}+\frac{1}{\gamma_H+1}e^{-\gamma_H m_H}\right)\\
&+\frac{2}{\sigma_H{}^2}\left[\frac{(1-w)C+gP-ga_0}{\eta_H+\gamma_H}\left(\frac{e^{\eta_H m_H}-1}{\eta_H}-\frac{1-e^{-\gamma_H m_H}}{\gamma_H}\right)\right]+\frac{l\left(e^{\eta_H m_H}-e^{-\gamma_H m_H}\right)}{(\eta_H+\gamma_H)} \qquad (1.15)\\
&-\frac{2}{\sigma_H{}^2}\left[\frac{ga_{1H}K^{y_{1H}}}{\eta_H+\gamma_H}\left(\frac{e^{\eta_H m_H}-e^{y_{1H}m_H}}{\eta_H-y_{1H}}-\frac{e^{y_{1H}m_H}-e^{-\gamma_H m_H}}{\gamma_H+y_{1H}}\right)\right]\\
&-\frac{2}{\sigma_H{}^2}\left[\frac{ga_{2H}K^{y_{2H}}}{\eta_H+\gamma_H}\left(\frac{e^{\eta_H m_H}-e^{y_{2H}m_H}}{\eta_H-y_{2H}}-\frac{e^{y_{2H}m_H}-e^{-\gamma_H m_H}}{\gamma_H+y_{2H}}\right)\right]
\end{aligned}$$

Denote $E_H\left(m_H\left(V_S\right)\right)=H_0+H_1 l$ and $\frac{\partial E_H\left(m_H\left(V_s\right)\right)}{\partial m_H}_{|m_H=\ln\left(\frac{V_S}{K}\right)}=H_2+H_3 l$, where

$$
\begin{aligned}
H_0 &= V_S - \frac{2qK}{(\eta_H+\gamma_H)\sigma_H^{\,2}}\left(\frac{1}{\eta_H-1}\left(\frac{V_S}{K}\right)^{\eta_H}+\frac{1}{\gamma_H+1}\left(\frac{V_S}{K}\right)^{-\gamma_H}\right)\\
&+\frac{2}{\sigma_H^{\,2}}\left[\frac{(1-w)C+gP-ga_0}{\eta_H+\gamma_H}\left(\frac{1}{\eta_H}\left(\left(\frac{V_S}{K}\right)^{\eta_H}-1\right)-\frac{1}{\gamma_H}\left(1-\left(\frac{V_S}{K}\right)^{-\gamma_H}\right)\right)\right]\\
&-\frac{2}{\sigma_H^{\,2}}\left[\frac{ga_{1H}K^{y_{1H}}}{\eta_H+\gamma_H}\left(\frac{1}{\eta_H-y_{1H}}\left(\left(\frac{V_S}{K}\right)^{\eta_H}-\left(\frac{V_S}{K}\right)^{y_{1H}}\right)-\frac{1}{\gamma_H+y_{1H}}\left(\left(\frac{V_S}{K}\right)^{y_{1H}}-\left(\frac{V_S}{K}\right)^{-\gamma_H}\right)\right)\right]\\
&-\frac{2}{\sigma_H^{\,2}}\left[\frac{ga_{2H}K^{y_{2H}}}{\eta_H+\gamma_H}\left(\frac{1}{\eta_H-y_{2H}}\left(\left(\frac{V_S}{K}\right)^{\eta_H}-\left(\frac{V_S}{K}\right)^{y_{2H}}\right)-\frac{1}{\gamma_H+y_{2H}}\left(\left(\frac{V_S}{K}\right)^{y_{2H}}-\left(\frac{V_S}{K}\right)^{-\gamma_H}\right)\right)\right]
\end{aligned}
\tag{1.16}
$$

$$
H_1 = \frac{1}{(\eta_H+\gamma_H)}\left(\left(\frac{V_S}{K}\right)^{\eta_H}-\left(\frac{V_S}{K}\right)^{-\gamma_H}\right)
\tag{1.17}
$$

$$
\begin{aligned}
H_2 &= V_S - \frac{2}{\sigma_H^{\,2}}\left[\frac{qK}{\eta_H+\gamma_H}\left(\frac{\eta_H}{\eta_H-1}\left(\frac{V_S}{K}\right)^{\eta_H}+\frac{-\gamma_H}{\gamma_H+1}\left(\frac{V_S}{K}\right)^{-\gamma_H}\right)\right]\\
&+\frac{2}{\sigma_H^{\,2}}\left[\frac{(1-w)C+gP-ga_0}{\eta_H+\gamma_H}\left(\left(\frac{V_S}{K}\right)^{\eta_H}-\left(\frac{V_S}{K}\right)^{-\gamma_H}\right)\right]\\
&-\frac{2}{\sigma_H^{\,2}}\left[\frac{ga_{1H}K^{y_{1H}}}{\eta_H+\gamma_H}\left(\frac{1}{\eta_H-y_{1H}}\left(\eta_H\left(\frac{V_S}{K}\right)^{\eta_H}-y_{1H}\left(\frac{V_S}{K}\right)^{y_{1H}}\right)-\frac{1}{\gamma_H+y_{1H}}\left(y_{1H}\left(\frac{V_S}{K}\right)^{y_{1H}}+\gamma_H\left(\frac{V_S}{K}\right)^{-\gamma_H}\right)\right)\right]\\
&-\frac{2}{\sigma_H^{\,2}}\left[\frac{ga_{2H}K^{y_{2H}}}{\eta_H+\gamma_H}\left(\frac{1}{\eta_H-y_{2H}}\left(\eta_H\left(\frac{V_S}{K}\right)^{\eta_H}-y_{2H}\left(\frac{V_S}{K}\right)^{y_{2H}}\right)-\frac{1}{\gamma_H+y_{2H}}\left(y_{2H}\left(\frac{V_S}{K}\right)^{y_{2H}}+\gamma_H\left(\frac{V_S}{K}\right)^{-\gamma_H}\right)\right)\right]
\end{aligned}
\tag{1.18}
$$

$$
H_3 = \frac{1}{(\eta_H+\gamma_H)}\left(\eta_H\left(\frac{V_S}{K}\right)^{\eta_H}+\gamma_H\left(\frac{V_S}{K}\right)^{-\gamma_H}\right)
\tag{1.19}
$$

For $V_S \le V$, define $m_L = \ln\left(\frac{V}{V_S}\right)$ and insert into differential equation (2.11),

$$rE_L = \left(r - q - \frac{1}{2}\sigma_L{}^2\right)\frac{\partial E_L}{\partial m_L} + \frac{1}{2}\sigma_L{}^2\frac{\partial E_L{}^2}{\partial^2 m_L} + qV_S e^{m_L} - (1-w)C + gD_L(m_L) - gP \quad (1.20)$$

With the boundary conditions,

$$E_L(0) = E_H\left(m_H\left(V_S\right)\right) \text{ and } \frac{\partial E_L(0)}{\partial m_{L|m_L=0}} = \frac{\partial E_H\left(m_H\left(V_S\right)\right)}{\partial m_{H_{|m_H=\ln\left(\frac{V_S}{K}\right)}}} \quad (1.21)$$

Define the Laplace transformation of $E_L\left(m_L\right)$ as,

$$FL(s) \equiv L\left[E_L(m_L)\right] = \int_0^\infty e^{-sm_L} E_L\left(m_L\right) dm_L \quad (1.22)$$

We also have,

$$\begin{aligned} L\left[\frac{\partial E_L}{\partial m_L}\right] &= sFL\left(s\right) - E_L(0) = sFL\left(s\right) - E_H\left(m_H\left(V_S\right)\right) \\ L\left[\frac{\partial E_L{}^2}{\partial^2 m_L}\right] &= s^2 FL\left(s\right) - sE_L(0) - \frac{\partial E_L(0)}{\partial m_{L\,m_L=0}} = s^2 FL\left(s\right) - sE_H\left(m_H\left(V_S\right)\right) - \frac{\partial E_H\left(m_H\left(V_s\right)\right)}{\partial m_{H_{|m_H=\ln\left(\frac{V_S}{K}\right)}}} \end{aligned} \quad (1.23)$$

Following the similar procedure, we have,

$$\frac{1}{2}\sigma_L{}^2 FL(s) = -\frac{\frac{1}{s-\eta_L} - \frac{1}{s+\gamma_L}}{\eta_L + \gamma_L}\left\{\begin{aligned} &\frac{qV_S}{s-1} - \frac{(1-w)C + gP}{s} + \frac{ga_0}{s} + \frac{ga_{1L}V_S{}^{y_{1L}}}{s - y_{1L}} + \frac{ga_{2L}V_S{}^{y_{2L}}}{s - y_{2L}} \\ &-\left(r - q - \frac{1}{2}\sigma^2\right)\left(H_0 + H_1 l\right) - \frac{1}{2}\sigma_L{}^2\left[s\left(H_0 + H_1 l\right) + H_2 + H_3 l\right] \end{aligned}\right\}$$

(1.24)

Where,

$$\eta_L = z_L - d_L > 1, \quad \gamma_L = z_L + d_L > 0$$

$$d_L = \frac{r - q - \frac{1}{2}\sigma_L^{\ 2}}{\sigma_L^{\ 2}}, \quad z_L \equiv \frac{\left(d_L^{\ 2}\sigma_L^{\ 4} + 2r\sigma_L^{\ 2}\right)^{1/2}}{\sigma_L^{\ 2}} \tag{1.25}$$

Inverting the Laplace transformation in (1.24), we have,

$$\begin{aligned}
&EL(m_L) = L^{-1}\left[FL(s)\right] \\
&= V - \frac{2qV_S}{(\eta_L + \gamma_L)\sigma_L^{\ 2}}\left(\frac{1}{\eta_L - 1}e^{\eta_L m_L} + \frac{1}{\gamma_L + 1}e^{-\gamma_L m_L}\right) \\
&+ \frac{2}{\sigma_L^{\ 2}}\left[\frac{(1-w)C + gP - ga_0}{\eta_L + \gamma_L}\left(\frac{e^{\eta_L m_L} - 1}{\eta_L} - \frac{1 - e^{-\gamma_L m_L}}{\gamma_L}\right)\right] \\
&- \frac{2}{\sigma_L^{\ 2}}\left[\frac{ga_{1L}V_S^{\ y_{1L}}}{\eta_L + \gamma_L}\left(\frac{e^{\eta_L m_L} - e^{y_{1L} m_L}}{\eta_L - y_{1L}} - \frac{e^{y_{1L} m_L} - e^{-\gamma_L m_L}}{\gamma_L + y_{1L}}\right)\right] \\
&- \frac{2}{\sigma_L^{\ 2}}\left[\frac{ga_{2L}V_S^{\ y_{2L}}}{\eta_L + \gamma_L}\left(\frac{e^{\eta_L m_L} - e^{y_{2L} m_L}}{\eta_L - y_{2L}} - \frac{e^{y_{2L} m_L} - e^{-\gamma_L m_L}}{\gamma_L + y_{2L}}\right)\right] \\
&+ \frac{2}{\sigma_L^{\ 2}}\left[\frac{1}{\eta_L + \gamma_L}\left(\left(r - q - \frac{1}{2}\sigma^2\right)(H_0 + H_1 l) + \frac{1}{2}\sigma_L^2 (H_2 + H_3 l)\right)\left(e^{\eta_L m_L} - e^{-\gamma_L m_L}\right)\right] \\
&+ (H_0 + H_1 l)\frac{\left(\eta_L e^{\eta_L m_L} + \gamma_L e^{-\gamma_L m_L}\right)}{\eta_L + \gamma_L}
\end{aligned} \tag{1.26}$$

Now we impose the boundary condition at $m_L \to \infty$. The equity value has to grow linearly when $V \to \infty$. Since $e^{\eta_L m_L} = \left(\frac{V}{V_S}\right)^{\eta_L}$ and $\eta_L > 1$, to avoid explosion we require the coefficient on $e^{\eta_L m_L}$ in $EL(m_L)$ to collapse to zero. By collecting the coefficients of $e^{\eta_L m_L}$, we have (2.18). QED.

Appendix B: Variable definitions

Panel A: Bond illiquidity measures (daily, bond-level)	
Amihud ratio (AMI)	Daily Amihud ratio is the average absolute value of bond return divided by dollar trading volume (in million dollar) within the day and then multiplied by 100.
Imputed roundtrip trade (IRT)	Daily Imputed roundtrip trade is defined as the difference between the maximum price and the minimum price for one day normalized by the maximum price on the same day.
Price Deviation (PD)	Daily price dispersion measure is defined as the root mean squared difference between the traded prices and the daily average price weighted by trading volume.
Inter-quartile range (IQR)	Daily Inter-quartile range is defined as the difference between the 75th percentile and 25th percentile of prices for one day normalized by the average price on the same day.
Panel B: Firm risk-taking measures	
Asset volatility	We back out asset volatility by solving two equations of Merton (1974) simultaneously.
Earning volatility	Standard deviation of quarterly income before extraordinary items deflated by total assets during the two year preceding the fiscal year end. To reduce the noise in calculating earnings volatility, we impose a minimum data requirement of 8 quarters of non-missing values.
Panel C: Control variables	
Firm size	
Leverage	Book value of total debt divided by the book value of total assets measured at fiscal year-end.
Profit	Earnings before interest, tax and depreciation scaled by book value of total assets at fiscal year-end.
Sales growth	Two-year average growth rate in sales. If two-year growth is not available, we use one-year growth in sales.
Rollover risk	Percentage of debt due in one year scaled by the book value of total debt measured at fiscal year-end.
Firm age	A firm's age equals to the number of years a firm existed in Compustat prior to fiscal year end.

Table 1: Calibrations

Parameters	Calibrations	Literatures
	Bond Market	
Risk free rate	$r = 8\%$	Leland (1998), He and Xiong (2012), Huang and Huang (2013)
Proportional Liquidity Costs	$k = 1.00\%$ for A rating $k = 0.50\%$ for BB rating	He and Xiong (2012)
Liquidity shock intensity	$\xi = 1$ for both	He and Xiong (2012)
	Firm Characteristics	
Initial Asset Value	$V_0 = \$100$	Leland (1998), He and Xiong (2012), etc.
Corporate Payout Rate	$q = 2\%$	Huang and Zhou (2008)
Proportional Bankruptcy Cost	$\alpha = 40\%$	Chen (2010)
	Firm's Debt Structure	
Debt Face Value	$P = \$52.6$ for A rating $P = \$60$ for BB rating	He and Xiong (2012) for BB rating
Coupon Payment	$C = \$4.35$ for A rating $C = \$5.30$ for BB rating	
Average Maturity of Debts	$T = 3$	
Debt Tax Benefit	$w = 27\%$	He and Xiong (2012)
	Firm's Asset Volatilities	
Low Asset Volatilities	$\sigma_L = 21\%$ for A rating $\sigma_L = 23\%$ for BB rating	Zhang, Zhou and Zhu (2009)
High Asset Volatilities	$\sigma_H = 40.83\%$ for A rating $\sigma_H = 44.72\%$ for BB rating	

Table 2: Characteristics of a Firm with Agency Costs and Liquidity Risk

This table reports the characteristics of a firm with A (Panel A) and BB (Panel B) rating classes, respectively, with agency costs and liquidity risk in the secondary debt market. It is assumed that the debt and capital structure are fixed ex-ante. For "Ex-Ante-Liquidity", it is assumed that a firm is aware of liquidity risk and can only adjust the endogenous default boundary and switching boundary to maximize total firm's value ex ante but not the debt and capital structure. The rest of calibrations are same as base case for A and BB rating, respectively.

	D	E	A	K	V_S	Agency Cost	%Δ Agency Cost	Credit Spread (bp)	ΔCredit Spread (bp)	%Δ Credit Spread
Panel A: A Rating										
Ex-Ante	52.60	59.94	112.54	35.22	56.30	0.37%	78%	27.87	10.50	106%
Ex-Post	52.47	59.65	112.12	35.32	62.51			38.37		
Ex-Ante-Liquidity	51.97	57.48	109.45	36.23	56.06	0.66%		78.37	21.67	
Ex-Post-Liquidity	51.70	57.02	108.72	36.29	67.52			100.04		
Panel B: BB Rating										
Ex-Ante	60.00	52.94	112.94	39.11	64.11	1.22%	103%	83.50	50.56	184%
Ex-Post	59.29	52.27	111.56	39.20	76.31			134.06		
Ex-Ante-Liquidity	58.57	47.76	106.33	41.38	63.54	2.48%		186.52	143.50	
Ex-Post-Liquidity	56.69	47.01	103.69	41.13	90.87			330.02		

Figure 1: Marginal Change of Equity Value and Switching Points

This figure depicts the marginal changes of equity value with respect to the switching boundary conditional on that the asset value equal to the switching boundaries for A (Panel A) and BB (Panel B) rating. The solid lines show the case in the absence of liquidity risk in the secondary boundary market while the dash lines show the corresponding cases with liquidity risk. The base calibrations are used for both ratings.

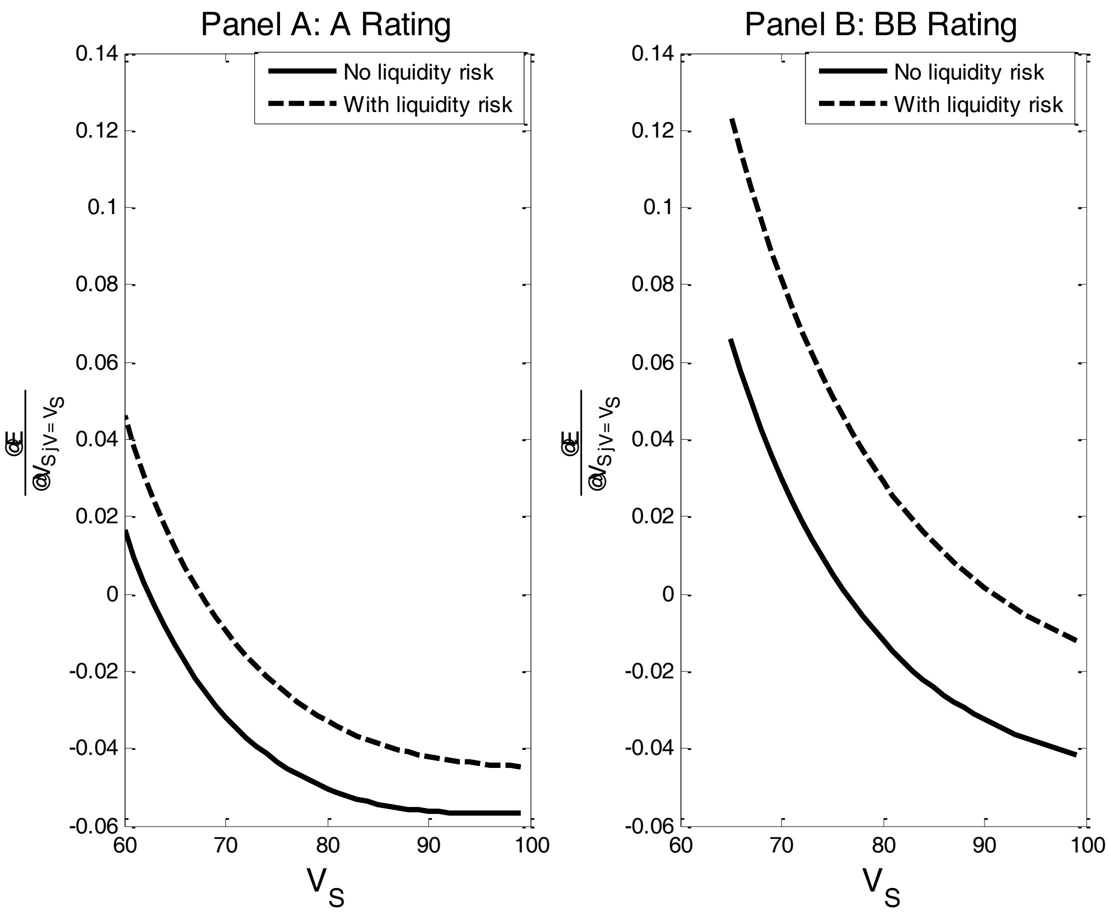

Figure 2: Optimal Switching points and Liquidity Intensity

This figure depicts the changes of ex-ante and ex-post risk switching boundaries with respect to the intensity of liquidity shocks for A (Panel A) and BB (Panel B) rating classes. The solid and dash lines shows the ex-ante and ex-post risk switching boundaries, respectively. The base calibrations are used for both ratings.

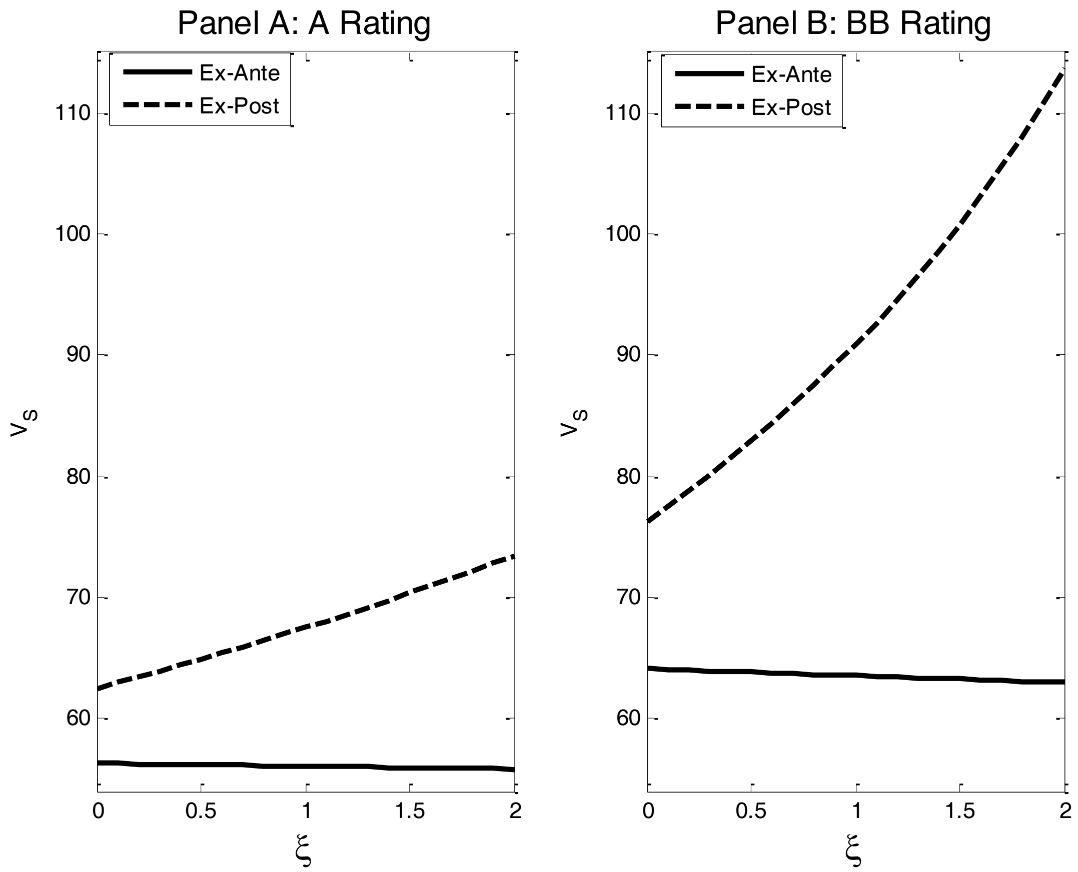

Figure 3: Liquidity Intensity and Endogenous Default Boundaries

This figure depicts the changes of ex-ante and ex-post endogenous default boundaries with respect to the intensity of liquidity shocks for A (Panel A) and BB (Panel B) rating classes. The solid and dash lines shows the ex-ante and ex-post risk switching boundaries, respectively. The base calibrations are used for both ratings.

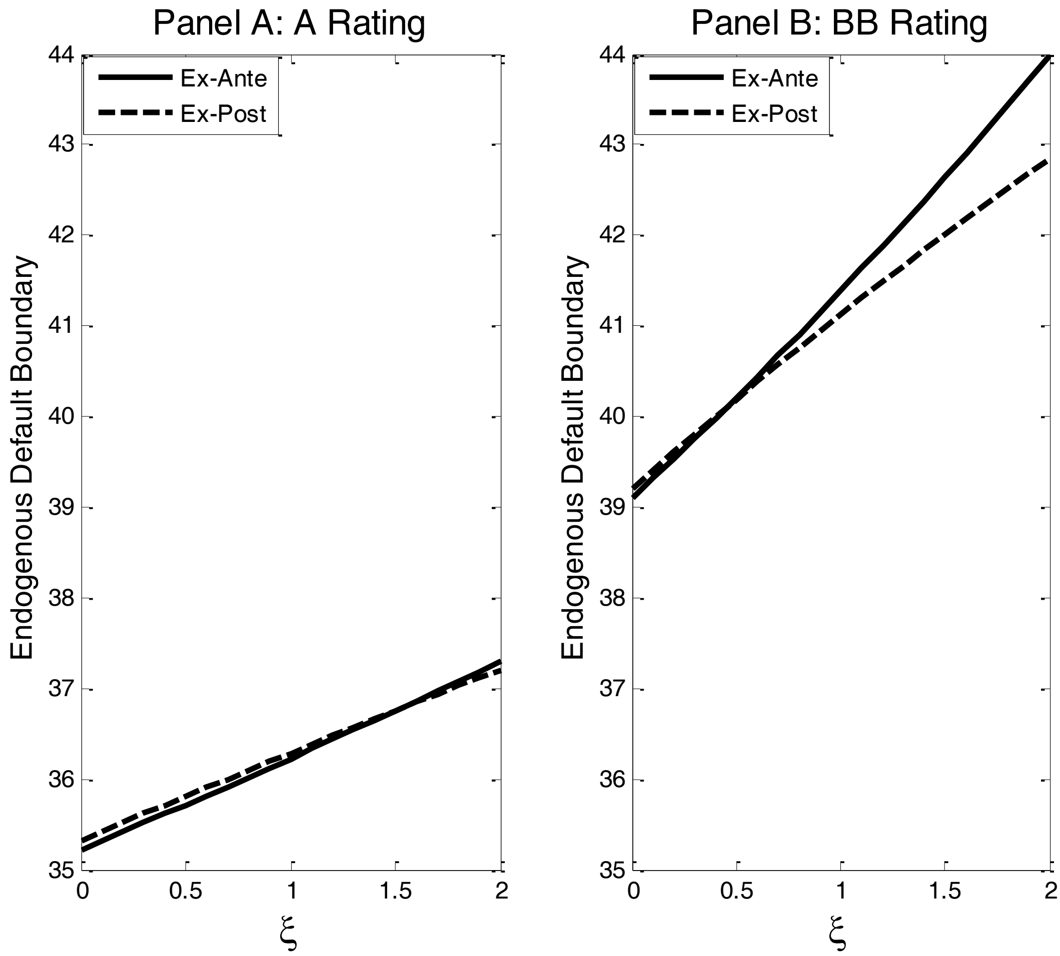

Figure 4: Liquidity Intensity and Percentage Change of Agency Costs

This figure depicts the changes of agency costs with respect to the intensity of liquidity shocks for A (dash line) and BB (solid line) rating classes. The base calibrations are used for both ratings.

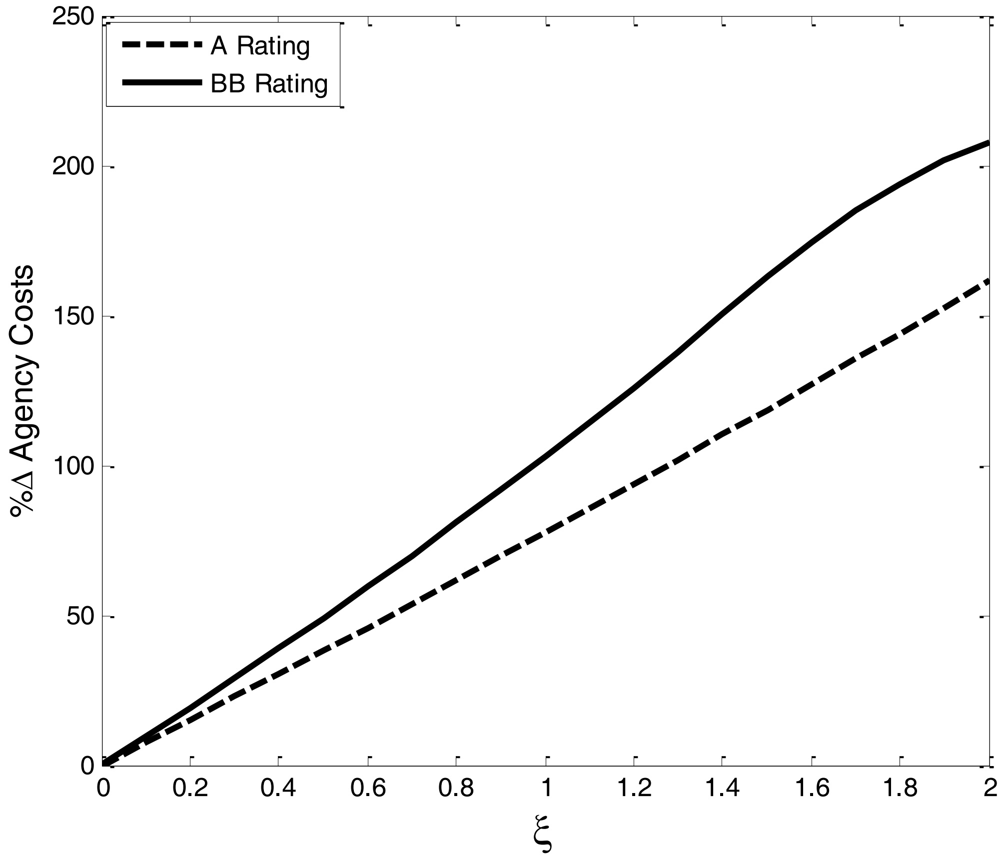

Table 3. Summary statistics

This table reports summary statistics of liquidity measures, risk-taking measures, and firm characteristics based on a sample of 1,379 firms over the period 2002-2015 (9428 firm-years). All variables are winsorized at 1% and 99%. Variable definitions are given in Table 1.

	N	Mean	Std Dev	25th Pctl	Median	75th Pctl
Illiquidity measures						
Amihud ratio (AMI)	9428	0.009	0.010	0.004	0.006	0.011
Imputed roundtrip trade (IRT)	9428	0.006	0.006	0.003	0.005	0.007
Price dispersion (PD)	9428	0.002	0.002	0.001	0.002	0.003
Inter-quartile range (IQR)	9428	0.004	0.006	0.002	0.003	0.005
Risk-raking measures						
Asset volatility	9428	0.553	0.440	0.300	0.441	0.665
Earning volatility	9428	0.015	0.023	0.003	0.007	0.015
Control variables						
Ln(asset)	9428	8.929	1.583	7.805	8.815	9.940
Leverage	9428	0.669	0.196	0.536	0.647	0.787
Profit	9428	0.119	0.081	0.076	0.116	0.163
Sales growth	9428	0.018	0.140	-0.039	0.003	0.050
Rollover risk	9428	0.028	0.050	0.000	0.007	0.035
Ln(firm age)	9428	3.213	0.594	2.819	3.287	3.774

Table 4. OLS regressions of risk-taking measures on bond iliquidity

Panel A. Asset volatility as dependent variable

Panel A tests the relationship between bond illiquidity and the asset volatility. It presents estimates from a pooled OLS regression, based on a sample of 1,379 firms over the period 2002-2015 (9428 firm-years). The dependent variable is the asset volatility. Columns (1) to (4) report the results of regressions with Amihud ratio (AMI), Imputed roundtrip trade (IRT), Price dispersion (PD), and Inter-quartile range (IQR) as illiquidity measures, respectively. Other control variables are Ln(asset), Leverage, Profit, Sales growth, Rollover risk and, Ln(firm age). All variables are winsorized at 1% and 99%. Variable definitions are in Table 1. All regressions control for year-fixed effects, firm fixed effects, and include rating dummies for each rating category. Standard errors are clustered at the firm level and are presented in parentheses. *, **, *** indicates statistical significance at 0.10, 0.05, 0.01 two-tailed levels, respectively.

Dependent variable: Asset volatility				
	(1)	(2)	(3)	(4)
AMI	3.720***			
	(0.399)			
IRT		7.521***		
		(0.677)		
PD			3.779**	
			(1.678)	
IQR				3.527***
				(0.634)
Ln(asset)	-0.025**	-0.032**	-0.026**	-0.025**
	(0.013)	(0.013)	(0.013)	(0.013)
Leverage	0.181***	0.162***	0.197***	0.191***
	(0.041)	(0.041)	(0.041)	(0.041)
Profit	-0.702***	-0.669***	-0.745***	-0.733***
	(0.075)	(0.075)	(0.075)	(0.075)
Sales growth	0.051**	0.051**	0.052**	0.051**
	(0.024)	(0.024)	(0.024)	(0.024)
Rollover risk	0.421***	0.436***	0.425***	0.427***
	(0.078)	(0.078)	(0.079)	(0.078)
Ln(firm age)	-0.323***	-0.294***	-0.309***	-0.306***
	(0.044)	(0.044)	(0.044)	(0.044)
Constant	1.393***	1.328***	1.391***	1.383***
	(0.21)	(0.21)	(0.211)	(0.211)
Year-fixed effect	Yes	Yes	Yes	Yes
Firm-fixed effect	Yes	Yes	Yes	Yes
Rating dummies	Yes	Yes	Yes	Yes
N	9428	9428	9428	9428

Table 5. OLS regressions of risk-taking measures on bond iliquidity

Panel B Earning volatility as dependent variable

Panel B tests the relationship between bond illiquidity and the earning volatility. It presents estimates from a pooled OLS regression, based on a sample of 1,379 firms over the period 2002-2015 (9428 firm-years). The dependent variable is the earning volatility. Columns (1) to (4) report the results of regressions with Amihud ratio (AMI), Imputed roundtrip trade (IRT), Price dispersion (PD), and Inter-quartile range (IQR) as illiquidity measures, respectively. Other control variables are Ln(asset), Leverage, Profit, Sales growth, Rollover risk and, Ln(firm age). All variables are winsorized at 1% and 99%. Variable definitions are in Table 1. All regressions control for year-fixed effects, firm fixed effects, and include rating dummies for each rating category. Standard errors are clustered at the firm level and are presented in parentheses. *, **, *** indicates statistical significance at 0.10, 0.05, 0.01 two-tailed levels, respectively.

Dependent variable: Earning volatility				
	(1)	(2)	(3)	(4)
AMI	0.097***			
	(0.023)			
IRT		0.254***		
		(0.039)		
PD			0.201**	
			(0.096)	
IQR				0.197***
				(0.036)
Ln(asset)	-0.009***	-0.009***	-0.009***	-0.009***
	(0.001)	(0.001)	(0.001)	(0.001)
Leverage	0.029	0.028***	0.029***	0.030***
	(0.002)	(0.002)	(0.002)	(0.002)
Profit	-0.082	-0.081***	-0.083***	-0.089***
	(0.004)	(0.004)	(0.004)	(0.004)
Sales growth	0.001	0.001	0.001	-0.002
	(0.001)	(0.001)	(0.001)	(0.001)
Rollover risk	0.010**	0.010**	0.010**	0.010**
	(0.004)	(0.004)	(0.004)	(0.005)
Ln(firm age)	0.002	0.003	0.002	0.003
	(0.003)	(0.003)	(0.003)	(0.003)
Constant	0.120***	0.118***	0.120***	0.126
	(0.012)	(0.012)	(0.012)	(0.245)
Year-fixed effect	Yes	Yes	Yes	Yes
Firm-fixed effect	Yes	Yes	Yes	Yes
Rating dummies	Yes	Yes	Yes	Yes
N	9428	9428	9428	9428

Table 6: TRACE dissmination test

This table tests the relationship between risk taking measures and the dissemination of TRACE. The dependent variables are asset volatility and earning volatility respectively. TRACE-dissemination is a dummy variable, which equals one if a firm's trading is available in Standard TRACE, otherwise zero. Other control variables are Ln(asset), Leverage, Profit, Sales growth, Rollover risk and, Ln(firm age). All variables are winsorized at 1% and 99%. Variable definitions are in Table 1. All regressions control for year-fixed effects, firm fixed effects, and include rating dummies for each rating category. Standard errors are clustered at the firm level and are presented in parentheses. *, **, *** indicates statistical significance at 0.10, 0.05, 0.01 levels, respectively.

Dependet variable	Asset volatility	Earning volatility
	(1)	(2)
TRACE-dissemination	-0.190***	-0.010***
	(0.039)	(0.002)
Ln(asset)	-0.046	0.004
	(0.095)	(0.007)
Leverage	-0.669*	0.001
	(0.397)	(0.020)
Profit	0.573**	0.098**
	(0.265)	(0.019)
Sales growth	-1.189***	-0.030
	(0.442)	(0.028)
Rollover risk	0.096	0.002
	(0.061)	(0.004)
Ln(firm age)	0.319	0.011
	(0.389)	(0.021)
Constant	-0.244	0.024
	(0.674)	(0.045)
Year-fixed effect	Yes	Yes
Firm-fixed effect	Yes	Yes
Rating dummies	Yes	Yes
N	1802	1802

Table 7: Possible mechanisms

This table presents the panel regression which includes cross-terms. It presents estimates from a pooled OLS regression, based on a sample of 1,379 firms over the period 2002-2015 (9428 firm-years). The dependent variable from Columns (1) to (3) is asset volatility, and from Columns (4) to (6) is earning volatility. Amihud* Rollover risk, Amihud* Profit, or Amihud* Ln(firm age) denotes the cross term between Amihud ratio and Rollover risk, Profit, or Ln(firm age) respectively. Other variables are Amihud ratio, Ln(asset), Leverage, Profit, Sales growth, Rollover risk and, Ln(firm age). All variables are winsorized at 1% and 99%. Variable definitions are in Table 1. All regressions control for year-fixed effects, firm fixed effects, and include rating dummies for each rating category. Standard errors are clustered at the firm level and are presented in parentheses. *, **, *** indicates statistical significance at 0.10, 0.05, 0.01 levels, respectively

Dependent variables:	Asset volatility			Earning volatility		
	(1)	(2)	(3)	(4)	(5)	(6)
Amihud* Rollover risk	20.575***			0.939***		
	(6.196)			(0.354)		
Amihud* Profit		-39.923***			-0.516**	
		(4.076)			(0.234)	
Amihud* Ln(firm age)			-6.341***			-0.198***
			(0.656)			(0.038)
Amihud	3.19***	7.23***	24.646***	0.073***	0.142***	0.751***
	(0.429)	(0.534)	(2.201)	(0.024)	(0.308)	(0.126)
Ln(asset)	-0.024*	-0.018	-0.025*	-0.009***	-0.009***	-0.009***
	(0.013)	(0.013)	(0.013)	(0.001)	(0.001)	(0.001)
Leverage	0.171***	0.156***	0.173***	0.028***	0.029***	0.029***
	(0.041)	(0.040)	(0.041)	(0.002)	(0.002)	(0.002)
Profit	-0.694***	-0.215**	-0.676***	-0.082***	-0.076***	-0.081***
	(0.075)	(0.089)	(0.074)	(0.004)	(0.005)	(0.004)
Sales growth	0.051**	0.048**	0.053**	0.001	0.001	0.001
	(0.024)	(0.024)	(0.024)	(0.001)	(0.001)	(0.001)
Rollover risk	0.225**	0.417***	0.406***	0.001	0.010**	0.009**
	(0.098)	(0.078)	(0.078)	(0.006)	(0.004)	(0.004)
Ln(firm age)	-0.325***	-0.336***	-0.313***	0.002	0.002	0.002
	(0.044)	(0.044)	(0.044)	(0.003)	(0.003)	(0.003)
Constant	1.392***	1.282***	1.300***	0.120***	0.119***	0.118***
	(0.210)	(0.209)	(0.209)	(0.012)	(0.012)	(0.012)
Year-fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Firm-fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Rating dummies	Yes	Yes	Yes	Yes	Yes	Yes
N	9428	9428	9428	9428	9428	9428