

# The Invisible Hand of Internal Markets in Mutual Fund Families

Luis Goncalves-Pinto \*      Juan Sotes-Paladino †

August, 2015

---

\* Luis Goncalves-Pinto, National University of Singapore, Department of Finance, 15 Kent Ridge Drive, MRB BIZ1 Level 7-43, Singapore 119245; Tel. +65 6516-4620; Email: [lgoncalv@nus.edu.sg](mailto:lgoncalv@nus.edu.sg)

† Juan Sotes-Paladino, University of Melbourne, Department of Finance, Level 12, 198 Berkeley St, VIC 3010, Australia; Email: [juan.sotes@unimelb.edu.au](mailto:juan.sotes@unimelb.edu.au)

# The Invisible Hand of Internal Markets in Mutual Fund Families

## Abstract

We propose a model of fund families in which trades of illiquid holdings can be crossed internally. In the model, cross-trading is the result of decentralized decisions by fund managers attempting to beat different style benchmarks, and increases with style diversity within the family. It benefits the whole family even in the absence of a family-level strategy. However, it is opportunistic: it allows managers to deviate excessively from their benchmarks, imposing heavy agency costs on investors. We provide empirical support for novel predictions of our model. Our results suggest that prior research may have understated the implications of interfund transactions.

**Keywords:** Mutual fund families, cross-trading, portfolio delegation, illiquidity.

**JEL Classification:** C61, D60, D81, G11, G12, G23.

# 1 Introduction

Mutual fund families can make up vast internal markets.<sup>1</sup> There is a substantial overlap in asset holdings among funds belonging to the same family.<sup>2</sup> Thus, funds affiliated with a family can take advantage of their internal markets and avoid having to pay the transaction costs in the open market.<sup>3</sup> However, families can also abuse internal markets to execute opportunistic trades and strategically transfer performance across funds. The motivation and consequences of strategies implemented at the family-level have been thoroughly studied by existing empirical research.<sup>4</sup> However, both the theoretical and the empirical research on decentralized decision making within fund families is scarce or non-existent. This paper is the first to take a structural approach to this question.

We argue that diversity in the investment styles offered by a mutual fund family can induce substantial opportunistic transfers of assets across family-affiliated funds. We propose a model in which the cross-fund trading responds to individual fund managers' goal of attracting fund flows by beating their respective style benchmarks, rather than a decision made centrally at the family-level. When these style benchmarks are different enough, managers can attain this goal by taking opposite idiosyncratic bets in illiquid stocks that are held in common. This in turn allows the managers to shift risk in the internal market at a lower cost, compared to having to deal with the open market. We show that this decentralized cross-trading increases with style diversity within the family. Moreover, the family stands to benefit nearly as much as if the cross-trading were centrally decided. We further show that there are heavy costs associated with internal markets which are borne by the fund investors, as cross-trading allows managers to deviate excessively from the portfolios that these investors would choose if investing directly. Lastly, we present empirical

---

<sup>1</sup> As of 2008, both the average and median mutual fund family in the U.S. domestic equity category managed more than \$4 billion in net assets.

<sup>2</sup> According to Elton, Gruber, and Green (2007), as much as 34% (17%) of total net assets of funds with the same (different) objectives consist of stocks held in common within the family, compared to 8% outside the family.

<sup>3</sup> A survey study conducted by the Bank for International Settlement (BIS (2003)) points to substantial savings on transaction costs arising from "crossing of trades" as one of the main factors behind the trend towards consolidation among investment managers in the asset management industry.

<sup>4</sup> See e.g. Gaspar, Massa, and Matos (2006), Bhattacharya, Lee, and Pool (2013), and Chaudhuri, Ivkovic, and Trzcinka (2014)

support for a key implication of our model, which relates the portfolio illiquidity of a family-affiliated fund to the style diversity within the family.

Most mutual fund families in the U.S. offer a myriad of investment styles, covering different asset classes (e.g., bonds vs. equity) as well as securities with different characteristics within the same asset class (e.g., large-cap value and small-cap growth within the equity class). It is well documented that investors in the different styles chase funds that outperform their style benchmarks.<sup>5</sup> The compensation of fund managers is typically tied to the amount of assets under management. This gives managers an incentive to alter the risk profile of their portfolios in order to beat their style benchmarks and maximize the likelihood of future inflows, as investors respond to the fund's recent benchmark-adjusted performance (Brown, Harlow, and Starks (1996)). Moreover, funds targeting different investment styles may shift risk by taking opposite idiosyncratic bets on similar assets (e.g. Basak, Pavlova, and Shapiro (2007), Chen and Pennacchi (2009)). When these funds belong to the same family organization, it is reasonable to expect that opposite idiosyncratic bets in illiquid stocks could avoid costly external markets by taking place internally.

Our model aims to capture this benchmark-induced motive for cross-trading in mutual fund families. We consider a family consisting of two funds that follow different benchmarks. Managers maximize expected utility of end-of-period compensation, which is a proportion of their assets under management. In turn, assets under management are affected by investors' money flows. Following Sirri and Tufano (1998) and Basak, Pavlova, and Shapiro (2007), we assume that investors' flows are a convex function of past performance of the funds relative to their style benchmarks.

To motivate trading in internal versus external markets, we assume that family-affiliated funds share some of their portfolio holdings (see Elton, Gruber, and Green (2007)) and that these common holdings are relatively more illiquid than the rest of their holdings. We approximate the price impact component of the trading costs that funds would incur if dealing with public markets by following the thin trading approach in Longstaff (2001). This modelling feature captures the idea that participants in a thin market are limited in their ability to buy or sell an illiquid security, so that building up or unwinding a large position in this security can take an extended period of time.

---

<sup>5</sup> See, e.g., DelGuercio and Tkac (2002).

We then allow the two family-affiliated fund managers to meet at the start of the investment period and decide whether it is in their mutually best interest to cross-trade some of their illiquid common holdings. After this initial meeting, each manager independently chooses her fund’s investment policy throughout the rest of the investment period.

Our first result is that style diversity alone can induce cross-trading within a fund family. This cross-trading is not coordinated at the family-level but results from the optimal decentralized investment decision of the individual fund managers. Despite sharing the same information and preferences in our model, family-affiliated fund managers find it optimal to place opposite buy and sell orders with each other in anticipation of a future need to deviate from their benchmarks. We show that cross-trading increases monotonically with style dispersion, and can be substantial in diverse enough families (reaching more than 40% of the family’s holdings of illiquid asset, in certain settings). Thus, style diversity is both a necessary condition for, and is positively related to, decentralized cross-trading in our model.

After cross-trading, family-affiliated funds in the model alter their portfolio liquidity relative to the benchmark in a way that differs significantly from their standalone counterparts. When following a low-liquidity style, a family-affiliated fund increases liquidity relative to the benchmark more than an equivalent standalone fund would do. Conversely, funds following a high-liquidity style decrease portfolio liquidity more when belonging to a family. Moreover, the deviation of fund liquidity from the liquidity in the benchmark increases with style diversity within the family. We find empirical support for this novel prediction of our model.

The decentralized cross-trading that maximizes each of the fund manager’s utility also increases the benefits accruing to the family as a whole, even though no family-level decision is made. This suggests that there is an “invisible hand” associated with internal markets in the mutual fund industry.<sup>6</sup> As a result of the optimal decentralized decisions by the individual fund managers, the family enjoys a better diversified portfolio, and generally a positive growth in future assets under management. Furthermore, the diversification benefits and the growth in future assets both

---

<sup>6</sup> The “invisible hand” is a term used by Adam Smith to describe his belief that individuals seeking their economic self-interest actually benefit society more than they would if they tried to benefit society directly.

increase with the dispersion in the liquidity of the styles in the family. In this sense, our results provide a novel rationale for the diversity in investment styles offered by mutual fund families in practice.

We next show that it is the fund investors who bear the burden of cross-trading, with net costs of delegation that can be as high as 6% of their initial wealth per year. Family-affiliated funds may appear to beat their standalone counterparts when evaluated by common performance measures such as the return-to-risk ratio. However, an evaluation that accounts for the dynamic deviations of funds' investment policies from their benchmarks reveals that the investors of at least one of the family-affiliated funds would be better off by delegating their portfolios to a standalone fund instead. The reason is that cross-trading allows for excessive risk-shifting by family-affiliated funds, relative to equivalent standalone funds. Hence, our results suggest that the decision of investing in family-affiliated versus standalone funds involves trading off the lower transaction costs versus the higher agency costs that result from the possibility of cross-trading.

Lastly, we use a sample of U.S. domestic equity mutual funds to examine empirically the testable implications of our model. We focus on the predictions that are novel to our model and central to all our results, namely that the deviation in funds' portfolio liquidity from their style benchmark liquidity is related to (i) the dispersion in the style menu offered by the family, and (ii) the average liquidity of the style benchmark. First, we find that style dispersion is positively associated with the absolute deviation of liquidity between the funds and their style benchmarks. Second, we find that funds further change the liquidity of their portfolios relative to the benchmarks depending on whether the benchmarks have low or high average liquidity. Finally, we provide evidence that the effect we find is different from what one would expect from alternative explanations advanced by prior research.

Our paper is related to the growing literature on cross-trading within asset management companies. In a seminal contribution, Gaspar, Massa, and Matos (2006) find that one way in which mutual fund families can transfer performance across member funds ('cross-fund subsidization') to favour those funds with a higher expected contribution to family profits is to have them cross-trade at below or above market prices. Chaudhuri, Ivkovic, and Trzcinka (2014) find evidence of a similar

strategic performance re-allocation across the different products offered by an institutional asset management company, with stronger effects occurring within illiquid investment styles. Casavecchia and Tiwari (2014) document a similar effect for brokers and other clients of the fund advisor, at the expense of the mutual fund clients. Eisele, Nefedova, and Parise (2014) provide intraday evidence of cross-subsidization via transfer prices, as hypothesized by Gaspar, Massa, and Matos (2006).<sup>7</sup>

We contribute to this empirical literature in at least two ways. First, prior studies emphasize centralized decisions that favour some funds at the expense of others, whereas we study cross-trading that can happen without a centralized decision maker. In prior studies, the compensation and reputation of the out-of-favour fund managers, and the performance of the fund, could be seriously hurt. What is the incentive of these fund managers to participate in such hurtful intra-family deals? Are there side-payments to these fund managers to induce participation, or does their compensation depend on the performance of the family as a whole? We present circumstances under which these intra-family deals can be rationalized. Most of the prior research also emphasizes agency costs stemming from ‘transfer prices’. In the U.S., these trades go clearly against SEC Rule 17(a)-7 of the Investment Act of 1940, which allows cross-trading as long as the transaction is effected at the “independent current market price of the securities” and is in the best interests of *both* the selling and the buying funds. However, the cross-trades that we focus on are executed at fair market prices, and can improve the performance of both funds. To the extent that investors in one or both funds are made worse off, these cross-fund trades still go against Rule 17(a)-7. The breach of fiduciary duty in our setting is, although equally significant, much more subtle than in the cross-subsidization case. As a result, it could be much harder to detect by regulatory authorities.

Our paper is also related to a more incipient theoretical research on the investment decisions of family-affiliated funds. Closest to ours is the paper by Binsbergen, Brandt, and Koijen (2008), who study the under-diversification and agency costs to an investor who delegates the administration of her savings to a fund family with multiple asset managers investing in different asset classes.

---

<sup>7</sup> Other empirical studies documenting family-coordinated strategies within mutual fund complexes include Nanda, Wang, and Zheng (2004) and Bhattacharya, Lee, and Pool (2013).

The problem these authors analyse is different from ours in important respects. Asset classes are mutually exclusive in their framework, ruling out cross-trading by design. We show that in the presence of illiquidity and risk-shifting incentives, families can benefit from allowing some overlap in asset holdings across their different investment styles, which is consistent with the evidence in Elton, Gruber, and Green (2007). Also related to ours is the work by Taylor (2003), who considers tournaments within fund families. Finally, although not in the context of fund families, the analysis in our paper draws on the characterization of mutual fund risk-shifting incentives induced by non-linear flow-performance relations studied by Basak, Pavlova, and Shapiro (2007). Our work can be seen as an application of their framework to funds affiliated with families in the presence of illiquid financial markets.

The rest of the paper proceeds as follows. We set up our model in Section 2. We solve the model using numerical methods and discuss the main results in Section 3. We provide empirical evidence consistent with the novel predictions of our model in Section 4. We conclude in Section 5.

## 2 Model Setup

Our goal is to study the asset allocation decision of mutual fund managers that seek to maximize the value of their compensation when (1) funds belongs to a family organization, and (2) the securities managers trade may be not perfectly liquid. The choice of our theoretical framework attempts to capture the distinct features introduced by (1) and (2) to our problem without departing significantly—thus avoiding results that arise mechanically from a completely different setup—from prior literature. First, we follow the partial-equilibrium, dynamic approach in the analysis of family-affiliated asset management companies of Binsbergen, Brandt, and Koijen (2008). Second, we incorporate the concerns about future assets under management of open-end mutual funds as examined by Basak, Pavlova, and Shapiro (2007). Third, we motivate the introduction of a cross-trading “platform” within the mutual fund family by modelling asset illiquidity in the spirit of Longstaff (2001). We expand on each of these modelling choices bellow.



## 2.1 The Economy

We consider an economy in which investors (households) delegate the administration of their savings to mutual funds over a certain investment horizon  $[0, T]$  (e.g., one calendar year). Mutual fund companies have access to financial markets consisting of three risky assets, with prices denoted by  $S_i(t)$ , for  $i \in \{1, 2, C\}$ :

$$dS_i(t) = \mu_i S_i(t) dt + \sigma_i S_i(t) dZ_i(t), \quad (1)$$

where  $\mu_i$  and  $\sigma_i$  are constant, as are the correlation coefficients among the assets  $0 \leq \rho_{kl} < 1$ , for  $k, l \in \{1, 2, C\}$ , and  $k \neq l$  (i.e.  $E[dZ_k(t)dZ_l(t)] = \rho_{kl} dt$ ). Uncertainty is governed by the standard Brownian motion processes  $Z_i(t)$ , for  $i \in \{1, 2, C\}$ . Asset prices are assumed to start the investment period at the value  $S_i(0) = s_i$ .

The first two assets trade in perfectly liquid markets, whereas asset  $C$  trades in a *thin* market. We follow Longstaff (2001) in considering a market thin when its participants can only adopt trading strategies that are of *bounded variation*, i.e., the number of shares  $\varphi_j(t)$  of asset  $C$  that can be bought or sold per unit of time is limited:

$$dN_C(t) = \varphi(t) dt, \quad (2)$$

where  $-\infty < -\alpha \leq \varphi(t) \leq \alpha < +\infty$ , for  $\alpha > 0$ , and  $N_C^j(t)$  is the number of shares of the illiquid asset  $C$  in a fund's portfolio as of time  $t$ .

Equation (2) reflects a situation where traders' ability to buy or sell asset  $C$  is limited or restricted, so that building up or unwinding a large position in this security may take an extended period of time. In this way, traders behave as if accounting for the transitory price impact of a their orders (see, e.g., Isaenko (2009)). This assumption represents a departure from the Black-Scholes-Merton complete financial market structure, and is key in our setup to motivate the convenience of cross-trading within a mutual fund family over trading in open, possibly thin, markets. Since the bulk of the transaction costs that mutual funds pay are in the form of price impact rather than in bid-ask spreads (see Edelen, Evans, and Kadlec (2013)), we choose this approach over the

traditional proportional costs approach to model this friction in our setup.<sup>8</sup> As highlighted by Longstaff (2001), modelling illiquidity according to (2) has the convenient additional feature of leading to endogenous borrowing and short-selling constraints, a pervasive restriction observed in the mutual fund industry in practice (see, e.g., Almazan, Brown, Carlson, and Chapman (2004)).

Agents in our economy have constant relative risk aversion (CRRA) preferences over consumption of wealth at the terminal date  $T$ :

$$U_k(w) = \frac{w^{1-\gamma_k}}{1-\gamma_k}, \quad (3)$$

where  $\gamma_k > 1$  and  $k \in \{m, c, h\}$  denotes, alternatively, a fund manager  $m$ , a Chief Investment Officer (CIO) of the family organization  $c$ ,<sup>9</sup> or a delegating household  $h$ . We further consider delegating investors' heterogeneous preferences for different investment styles by assuming that these investors are indexed by their relative risk aversion (RRA) coefficient  $\gamma_h$  in an interval  $[\underline{\gamma}, \bar{\gamma}]$ . We introduce fund managers and CIO and explain their objectives in subsection 2.4, after we introduce the mutual funds and the family organization in the next subsection.

## 2.2 Investment Styles and Mutual Funds

Investors' heterogeneity in appetite for risk translates into preference for funds with different risk-return profiles, or 'investment styles' (e.g., large-cap value), in our setup. Styles are identified by benchmark portfolios that invest in different, though not necessarily mutually exclusive, sets of assets. We consider two of such investment styles, L and S. Style L corresponds to a portfolio of assets 1 and  $C$ , with a higher weight on the liquid asset 1. Style S is characterized by a portfolio of assets 2 and  $C$ , with predominance of the illiquid asset  $C$ . Style L can be thought of as a conservative strategy focusing on liquid risky assets, such as well-known publicly traded large-cap ("L") stocks, while style S can be thought of as an aggressive strategy investing predominantly in illiquid risky assets, such as small-cap ("S") stocks.

---

<sup>8</sup> See, e.g., Constantinides (1986) and Dai and Yi (2009).

<sup>9</sup> In the asset management industry, a Chief Investment Officer is a board-level manager for an investment company. For most mutual fund families, CIOs have the responsibility for the investments and strategy of the overall group and oversee the team of investment professionals in charge of the individual funds' investments.

A style benchmark  $Y_j(t)$  for  $j \in \{1, 2\}$  is the value process of a (self-financing) traded portfolio holding  $B_C^i(t)$  shares of asset  $C$  and investing  $Y_j(t) - B_C^i(t)S_C(t)$  in asset  $j$ . We set  $B_C^j(t) = y_j \beta_C^j(0)/s_j$ , where  $\beta_C^j(0)$  is the initial weight of asset  $C$  on benchmark  $j$ , for  $j \in \{1, 2\}$ . Benchmark 1 represents the relatively liquid investment style L, while benchmark 2 represents the relatively illiquid style S:  $0 \leq \beta_C^1(0) < \beta_C^2(0) \leq 1$ . Benchmark  $j$ 's dynamics are given by:

$$dY_j(t) = \left[ Y_j(t)\mu_j + B_C^j(t)S_C(t)(\mu_C - \mu_j) \right] dt + Y_j(t)\sigma_j dZ_j(t) + B_C^j(t)S_C(t) [\sigma_C dZ_C(t) - \sigma_j dZ_j(t)], \quad (4)$$

along with the initial condition  $Y_j(0) = y_j$ ,  $j \in \{1, 2\}$ . Each benchmark portfolio represents a passive buy-and-hold strategy that keeps the *number of shares* of the illiquid asset as of time  $t = 0$  constant over the investment period.<sup>10</sup>

Given each benchmark's weight in the illiquid asset  $C$ , we consider that investment style L (respectively, S) is catered to an investor with RRA coefficient  $\gamma_{h1}$  ( $\gamma_{h2}$ ) if  $\beta_C^1(0)$  ( $\beta_C^2(0)$ ) equals the weight  $\beta_{unc,C}^1(\gamma_{h1})$  ( $\beta_{unc,C}^2(\gamma_{h2})$ ) of the illiquid asset  $C$  in the optimal unconditional portfolio that this investor would choose under self-management and perfect liquidity for all assets. For  $j \in \{1, 2\}$ , this weight can be easily solved (see, e.g., Chen and Pennacchi (2009)) as:

$$\beta_{unc,C}^j(\gamma_{hj}) = \frac{\mu_C - \mu_j + \sigma_j^2 - \rho_{jc}\sigma_j\sigma_C}{\gamma_{hj}(\sigma_j^2 + \sigma_C^2 - 2\rho_{jc}\sigma_j\sigma_C)}. \quad (5)$$

Our analysis of the utility implications of family-affiliated fund strategies in Section 3.5 focuses on the investor with RRA coefficient  $\gamma_{hj}$ ,  $j \in \{1, 2\}$ . Assuming that delegating investors self-select into the closest benchmark to their unconditionally efficient portfolio (5), we also examine the utility implications for investors in the corresponding *risk-appetite clientele*, i.e., on a risk-tolerance vicinity around the benchmark.

We consider two types  $j$  of mutual funds,  $j \in \{1, 2\}$ . Fund 1 follows investment style L and

---

<sup>10</sup> A fixed-weight benchmark would instead be continuously rebalanced to hold the *relative weights* of the assets in the portfolios constant over the investment period. Since asset  $C$  is illiquid in our framework, it can be infinitely costly for a manager to keep the relative weight of this asset in her portfolio constant. Keeping the *number of shares* constant (a buy-and-hold strategy on asset  $C$ ) instead is a more natural specification for a benchmark portfolio in our setup.

is administered by portfolio manager  $m1$ , whereas fund 2 follows style S and is administered by portfolio manager  $m2$ . The value of fund  $j$ 's self-financing portfolio,  $F_j(t)$ , is:

$$dF_j(t) = \left[ F_j(t)\mu_j + N_C^j(t)S_C(t)(\mu_C - \mu_j) \right] dt + F_j(t)\sigma_j dZ_j(t) + N_C^j(t)S_C(t) [\sigma_C dZ_C(t) - \sigma_j dZ_j(t)] \quad (6)$$

with initial value  $F_j(0) = f_j$ , for  $j \in \{1, 2\}$ . We constrain these portfolios to lie in the closed solvency region:

$$\mathcal{S} = \left\{ (S_j(t), S_C(t)) \in \mathbb{R}^2 : N_j(t)S_j(t) + N_C^j(t)S_C(t) \geq 0 \right\} \quad (7)$$

for all  $t \in [0, T]$ , where  $N_j(t)$  denotes the number of shares of the liquid asset  $j$  that fund  $j \in \{1, 2\}$  holds as of time  $t$ . Table 1 summarizes the main elements of the mutual fund structure introduced above.

[Table 1 about here]

Due to the assumption of illiquidity in our setup, a fund's initial number of shares is not a variable of choice but an additional parameter in the model. We fix the value of this parameter by assuming that fund  $j$ 's initial portfolio composition equals the composition of its benchmark:  $n_{C}^j s_C / f_j = \beta_C^j(0)$  (i.e.,  $N_C^j(0) = B_C^j(0)$  when  $f_j = y_j$ ), where  $n_C^j \geq 0$  is the initial number of shares of asset  $C$  in fund  $j$ 's portfolio, for  $j \in \{1, 2\}$ . This assumption means that each fund's initial excess performance with respect to the benchmark is exactly zero, and is necessary for our specification of managers' compensation in what follows.

**Manager's Compensation:** We set fund managers' compensation to be proportional to the value of their assets under management and due at the investment horizon  $t = T$ .<sup>11</sup> Apart from the initial delegation of wealth, we assume that external cash inflows to or outflows from the funds occur only at the investment horizon and are a pre-specified function of a fund's performance relative

---

<sup>11</sup> This compensation structure is in line with standard practice in the mutual fund industry and is justified on theoretical grounds by Berk and Green (2004).

to its style benchmark. More precisely, we borrow from Basak, Pavlova, and Shapiro (2007) the following flow-performance relationship:

$$\phi_j(T) = \begin{cases} \phi_j^L & \text{if } R_j^F(T) - R_j^Y(T) < \eta_j \\ \phi_j^L + \psi_j \left[ e^{R_j^F(T) - R_j^Y(T)} - e^{\eta_j} \right] & \text{if } R_j^F(T) - R_j^Y(T) \geq \eta_j, \end{cases} \quad (8)$$

where for  $j \in \{1, 2\}$ ,  $R_j^F(T) = \ln(F_j(T)/F_j(0))$  and  $R_j^Y(T) = \ln(Y_j(T)/Y_j(0))$  denote the continuously-compounded return of fund  $j$ 's portfolio and of benchmark  $j$ 's portfolio,  $\phi_j^L$  and  $\psi_j$  are positive constants, and  $\eta_j \in \mathbb{R}$ . We set  $Y_j(0) = F_j(0)$  without loss of generality.

According to relationship (8), flows depart (arrive) at the constant rate  $\phi_j(T) < 1$  ( $\phi_j(T) > 1$ ) when fund  $j$  underperforms its benchmark by a margin greater than  $\eta_j$ , and arrive at the increasing rate  $\psi_j(\exp(R_j^F(T) - R_j^Y(T)) - \exp(\eta_j))$  otherwise. This flow relationship is meant to capture Sirri and Tufano (1998)'s empirical finding that fund investors' flows penalize very poor and moderately poor performance at a similar rate but reward good performance at an increasingly higher rate.<sup>12</sup>

Specification (8) can alternatively be interpreted as an explicit compensation contract according to which the fund family remunerates a fund manager with a fixed fee over assets under management plus a bonus conditional on meeting a given performance target. Such compensation structure has been shown to be optimal in delegated asset management contexts by, e.g., Maug and Naik (2011), Basak, Pavlova, and Shapiro (2008), Li and Tiwari (2009) and Dybvig, Farnsworth, and Carpenter (2010). Moreover, they are standard in practice in the mutual fund industry as reported by Ma, Tang, and Gomez (2015).

---

<sup>12</sup> We choose this flow-performance relation over Chevalier and Ellison (1997)'s empirical specification in order to capture the greater cash inflows to star performers in a fund family as found empirically by Nanda, Wang, and Zheng (2004). However, we do not consider the latter authors' documented 'spillover' effect to other funds in the same family of a star performer, as this would introduce strategic interactions between the portfolio decision problems of the fund managers. Such a problem is significantly harder to solve, but would likely result in even stronger cross-trading than in our simplified setup.

### 2.3 Family-Affiliated vs. Standalone Funds

Mutual funds 1 and 2 can be affiliated to a family (FA funds) or standalone (SA funds).<sup>13</sup> What makes a family different from just a portfolio of two standalone funds in our setup is the possibility of avoiding public markets at  $t = 0$  by trading  $X$  shares of their common asset  $C$  across the two funds. In the U.S., SEC Rule 17(a)-7 of the Investment Act of 1940 allows cross-fund trading as long as the transaction is effected at the “independent current market price of the securities” and is in the best interests of *both* the selling and buying funds.<sup>14</sup> Although we recognize that cross-trading can happen at any point during the investment period  $[0, T]$ , we restrict it to occur only at the start of the period for tractability and interpret our results as a lower bound on the effects that can be expected in the real world.

After cross-trading each FA fund is managed independently according to its own style. The dynamics of the funds’ assets under management during  $t \in (0, T]$  are still given by equation (6) but the funds’ initial conditions change to:

$$N_C^1(0) = n_C^1 - X, \tag{9}$$

$$N_C^2(0) = n_C^2 + X. \tag{10}$$

Thus, a positive value of  $X$  is interpreted as a purchase of asset  $C$  by fund 2 from fund 1, whereas a negative value represents the opposite transaction. We impose the condition that  $-n_C^2 \leq X \leq n_C^1$  (no ‘internal’ short-selling). Equations (9) and (10) implicitly prevents cross-trading at ‘transfer prices’ in our setup, ruling out by assumption a situation in which one fund’s performance is boosted by having it sell (buy) shares from the other fund at above-market (below-market) prices.<sup>15</sup> Funds 1 and 2 stand alone when  $X = 0$ .

---

<sup>13</sup> A family is the prevalent organizational form in the U.S. mutual fund industry, see e.g., Gaspar, Massa, and Matos (2006). We abstract from looking into the reasons behind the emergence of families as an organizational form in the first place, or the factors determining families’ optimal size, and take them as given instead.

<sup>14</sup> The latter condition was stipulated in a no-action letter (Federated Municipal Funds (November 20, 2006)) in which the SEC staff provided clarification on Rule 17(a)-7 with respect to an investment adviser’s fiduciary duties in connection with Rule 17(a)-7 transactions. This condition prohibits transactions that are in the best interest of one fund but are otherwise neutral to the other fund.

<sup>15</sup> This type of strategies was first examined empirically by Gaspar, Massa, and Matos (2006).

## 2.4 Fund Managers' Problem

Family-affiliated mutual funds solve a two-stage recursive problem. Given the extent of cross-trading  $X$  at  $t = 0$ , in the second stage  $t \in (0, T]$  fund  $j$ 's manager solves the following problem:

$$V_j(F_j, Y_j, N_C^j, S_C, t) = \sup_{\varphi_j(t)} E_t \left[ \frac{[\phi_j(T)F_j(T)]^{1-\gamma_{mj}}}{1-\gamma_{mj}} \right], \quad (11)$$

subject to the price process (1) for asset  $C$ , the dynamics of the assets under management (6), the benchmark process (4), and the initial conditions (9) and (10).

In the first stage of the problem, at  $t = 0$ , managers decide whether they trade with each other, and the number of shares of asset  $C$  to exchange if they do. Rational managers solving problem (11) will be willing to engage in cross-trading only if by doing so they can increase their indirect utility  $V_j(f_j, y_j, n_C^j + x_j, s_C, 0)$ , with  $\{x_1(X), x_2(X)\} = \{-X, X\}$ , relative to no cross-trading,  $V_j(f_j, y_j, n_C^j, s_C, 0)$ ,  $j \in \{1, 2\}$ . Defining  $\Delta V_j(X) \equiv V_j(f_j, y_j, n_C^j + x_j(X), s_C, 0) - V_j(f_j, y_j, n_C^j, s_C, 0)$ , we thus assume that managers agree at the outset on the following constrained max-min rule to determine their time-0 cross-trading:

$$X = \arg \max_{X'} \left( \min\{\Delta V_1(X'), \Delta V_2(X')\} \right) \quad s.t. \quad \min\{\Delta V_1(X'), \Delta V_2(X')\} \geq 0. \quad (12)$$

According to this rule, FA fund managers will trade with each other only if (1) *both* managers' strictly benefit from so doing, and (2) the extent of cross-trading  $X$  is determined by the maximum improvement in utility achieved by the manager that benefits the least. Note that problem (12) does not necessarily have a solution, in which case managers will not trade with each other and will behave as SA funds throughout the entire investment period.

In case managers do decide to cross their trades, we assess the net impact on the overall family of the FA funds' decentralized cross-trading and investment policies by computing two performance measures. The first measure assumes that a Chief Investment Officer (CIO) runs the family company by deciding, for instance, the benchmarks that FA funds follow. We do not model this decision explicitly but, following Binsbergen, Brandt, and Koijen (2008), assume that

the CIO's compensation is proportional to the family's end-of-period assets under management  $W(T) \equiv \phi_1(T)F_1(T) + \phi_2(T)F_2(T)$ . The CIO's derived utility, for each level of cross-trading  $X$  is:

$$V_c(X) = E_0 \left[ \frac{W^*(T)^{1-\gamma_c}}{1-\gamma_c} \right], \quad (13)$$

where  $W^*(T)$  is the terminal value of the family's AUM when the FA funds agree on the extent of cross-trading  $X$  and implement the optimal policies  $\{\varphi_1^*(t), \varphi_2^*(t)\}$  during  $t \in (0, T]$ .<sup>16</sup>

Our second measure is the end-of-period (continuously-compounded) return of the family after flows or, equivalently, the expected growth of assets under management  $R^*(T) \equiv \ln(W^*(T)/W(0))$ . Both  $V_c(X)$  and  $R^*(T)$  assume that the family benefits from managing a larger pool of assets, the only difference being that  $V_c(X)$  accounts for higher-order moments in the distribution of total AUM growth rates. We examine both measures in Section 3 to determine whether managers' decentralized decisions are in the best interest of the family.

### 3 Model Solution

The assumption of imperfect liquidity in our model is a necessary condition to create cross-trading within fund families but prevents us from obtaining a closed-form solution to problem (11). Instead, we solve the model numerically using the Least-Squares Monte Carlo algorithm proposed in Longstaff and Schwartz (2001) and adopted in Longstaff (2001) to find the solution of a portfolio choice problem. Succinctly, this method involves replacing the conditional expectation function in (11) by its orthogonal projection on the space generated by a finite set of basis functions of the values of the state variables that are part of the managers' problem.<sup>17</sup> From this explicit functional approximation, we can solve for the optimal control variable  $\varphi_j^*(t)$  for  $t \in (0, T]$  for any starting

---

<sup>16</sup> The CIO in our setting cares about the overall value of the family but has no role in the investment decision. Our setting could easily be modified to allow for the CIO to decide on the extent of cross-trading that maximizes the value of the family, in the spirit of the empirical findings of Gaspar, Massa, and Matos (2006). We prefer to focus on a previously unexplored type of cross-fund transactions in the rest of the paper, namely those that are determined at the individual fund level with no intervention of a centralized decision maker. We nevertheless explore the CIO's problem in Subsection 3.4.

<sup>17</sup> We used up to the third order power polynomials of all the state variables (accounting for their interactions) and the first three powers of the utility function as basis functions.



value of  $N_C^j(0)$  as given by the solution to manager  $j$ 's Hamilton-Jacobi-Bellman equation. Because the control is constrained, this solution is as follows: if  $\partial V_j / \partial N_C^j > 0$  then  $\varphi_j^*(t) = \alpha$ , and if  $\partial V_j / \partial N_C^j < 0$  then  $\varphi_j^*(t) = -\alpha$ , whenever such amount of trading  $\alpha$  is admissible, otherwise  $\varphi_j^*(t) = 0$ , for  $j \in \{1, 2\}$ .

Given the optimal trading strategies  $\varphi_j^*(t)$  managers decide the extent of cross-trading at the start of the period according to the rule (12). Portfolio weights held in the risky illiquid asset  $C$  by fund  $j$  can then be easily retrieved, for each time  $t \in (0, T]$ , from the relation:

$$\omega_C^j(t) = \omega_C^j(0) + \int_0^t \frac{S_j(u)}{F_j(u)} \varphi_j(u) du \quad (14)$$

where  $\varphi_j(0) = 0$ ,  $\omega_C^j(0) = N_C^j(0) s_C / f_j$ , and the remainder  $(1 - \omega_C^j(t))$  is invested in the liquid asset  $j$  that is specific to the investment style of fund  $j \in \{1, 2\}$ .

The numerical results presented throughout the rest of this Section are based on 100 time steps—the discretization period is 0.01 years—and 100,000 simulated paths for the state variables. Without loss of generality, we normalize the values of the funds' assets under management and of their corresponding benchmarks to unity at  $t = 0$ , i.e.  $s_C = s_j = f_j = y_j = 1$ , for  $j \in \{1, 2\}$ . We set the initial value of the total holdings of illiquid asset  $C$  within the family  $(n_C^1 + n_C^2) s_C$  to one, so that  $\beta_C^2(0) = 1 - \beta_C^1(0)$ .

We consider a baseline and several alternative scenarios for the rest of the model parameters as described in Appendix A. In our baseline scenario we approximately match historical market data in the U.S. where available, and use calibrated values based on prior literature otherwise. The alternative scenarios allow us to check the robustness of our results to different parameterizations of the model and to derive testable predictions from comparative statics analyses.

### 3.1 Opposite Trades Induced by Relative Performance Concerns

As first pointed out theoretically and empirically by Basak, Pavlova, and Shapiro (2007) and Chen and Pennacchi (2009), under a flow-performance relationship like (8) a manager can deviate systematically from her benchmark in response to expected money flows. These deviations are unrelated

to asset fundamentals, are time-varying, and can either overweight or underweight an asset in the manager's portfolio relative to the benchmark depending on the riskiness of the benchmark.<sup>18</sup>

Following this argument, when a family offers multiple investment styles some of its member funds can deviate from different benchmarks by simultaneously trading in common assets but in opposite directions. These trades need not be the result of opposite signals about the same security, but purely the consequence of performance concerns relative to the benchmarks characterizing their different investment styles. Figure 1 shows that this is likely the case for the two standalone (SA) funds 1 and 2 ( $X = 0$ ) in our setup.

[Figure 1 about here]

Indeed, Figure 1 shows that throughout the investment period the two SA funds in our baseline scenario will adopt opposite excess positions in the illiquid asset  $C$  in many situations, with fund 1 overweighting and fund 2 underweighting this asset in their portfolios. This opposite behaviour is the result of managers' optimal response to their flow-performance relationship as characterized by Basak, Pavlova, and Shapiro (2007), with the additional constraint on trading imposed by asset illiquidity. Initially, the liquid benchmark 1 underexposes manager  $m1$  to the illiquid asset  $C$ , to which  $m1$  responds over the first fifth of the period by increasing asset  $C$  in the portfolio relative to the benchmark. The opposite is true for manager  $m2$ , who is initially overexposed to the illiquid asset  $C$  and attempts to reduce instead the weight in this asset relative to the benchmark.

As each fund's performance relative to its respective benchmark varies over time, managers hedge against changes in expected flows by increasing or decreasing their excess exposure in asset  $C$ . Notably, average *changes* in the excess holdings of these two funds exhibit opposite signs during most of the period. For instance, between  $t = 0.35$  and  $t = 0.45$  manager 1 increases her excess position in  $C$  relative to the benchmark by more than 2%, while manager 2 decreases it by about the same proportion. Moreover, the correlation coefficient between the two funds' changes for the entire period in Figure 1 is -0.09. Since asset  $C$  is illiquid and thus costly to trade in external

---

<sup>18</sup> In line with this argument, Huang, Sialm, and Zhang (2011) find empirically that active mutual fund managers change their portfolio risk dynamically over time, increasing or decreasing it depending on their interim position relative to peers. Dai, Goncalves-Pinto, and Xu (2015) show that these effects weaken in the presence of liquidity restrictions, although managers' deviations from the benchmark can still remain significant.

markets, Figure 1 suggests that there could be many instances in which trading with each other could be in the best interest of *both* SA funds.

### 3.2 Optimal Cross-Trading

If asset  $C$  were perfectly liquid managers would have no need to cross their opposite trades within the family because they could trade at zero cost in the open market. However, the restricted ability to buy or sell shares of asset  $C$  forces managers to pay a positive transaction cost, equal to the shadow price of illiquidity, when placing opposite and relatively large orders in the open market for this asset. Under SEC Rule 17(a)-7, both managers could then reduce these costs by ‘crossing’ their trades within the family.<sup>19</sup>

Figure 2 illustrates how benchmark concerns can give rise to substantial decentralized cross-trading between FA mutual funds in our model. When managing SA funds (NCT), both managers would decrease their indirect utility if fund 1 sought to enhance its profile as a style-L fund by *selling* shares of asset  $C$  to fund 2. However, both fund managers could increase their utility if fund 1 *bought* shares of asset  $C$  from fund 2 instead. Specifically, manager  $m1$  would agree to buy  $|X| = 0.07$  shares of  $C$  from fund 2 as the resulting portfolio would maximize her indirect utility over all possible levels of cross-trading. Manager  $m2$  would agree to the cross trade—and would be willing to sell even more shares of  $C$ —because the transaction allows her to improve her utility relative to the NCT case. That is, cross-trading in our setup is a mutually beneficial outcome arising from each manager’s pursuit of her self-interest. Internal markets within the family can therefore improve both managers’ utility.

**[Figure 2 about here]**

While improving managers’ utility, the cross-fund trading also improves the risk-return profiles of the funds involved. This result is illustrated in Figure 3, which plots the end-of-period return-to-risk ratio (RRR)  $E(R^j(T))/\sigma_j$  of funds  $j = 1, 2$  for all levels of cross-trading. Whereas the

---

<sup>19</sup> We assume that managers do not compete with each other within the fund family. See Taylor (2003) and Basak and Makarov (2014) for an analysis of the risk-shifting incentives induced by the strategic interaction among managers of the same family when their compensation is based on tournaments.

optimal level of cross-trading  $X = -0.07$  improves each fund's RRR over NCT (i.e.,  $X=0$ ), further cross-trading (e.g.,  $X = -0.20$ ) could improve the funds' measured performance even further. The improvement in measured performance seems to suggest that the cross trade is in the best interest of *both* the selling and the buying funds. Therefore, the cross-trading between the FA funds in our baseline scenario appears to represent no obvious violation of SEC Rule 17(a)-7 regulating fund advisors' fiduciary duty in relation to cross-fund transactions.<sup>20</sup> However, we show in subsection 3.5 that the cross-trading can still lead to large utility losses for at least one of the funds' investors, and thus still violates the spirit of SEC Rule 17(a)-7.

**[Figure 3 about here]**

The purchase of 0.07 shares of asset  $C$  by fund 1 from fund 2 deviates these funds' initial portfolio composition from the benchmark. In particular, it makes the liquid fund 1 more illiquid, and vice versa for fund 2. We show in Table 2 that managers' dynamic trading during the investment period does not fully revert, and may even increase, this initial deviation: relative to otherwise equivalent SA funds, fund 1's average holdings of asset  $C$  are 5.4% higher whereas fund 2's are 7.6% lower in our baseline scenario.

**[Table 2 about here]**

Table 2 also shows the extent of cross-trading, changes in the RRR and average illiquid asset holdings of FA funds relative to SA funds across our alternative parameterizations. In general, the qualitative results in our baseline case hold throughout the alternative scenarios, with important nuances in some cases. In particular, decentralized decisions by family-affiliated funds can lead to large cross-trading (e.g., 29% of each fund's initial AUM for  $\mu_1 = \mu_2 = .085$ ) only as a consequence of managers' concerns relative to different investment styles. As cross-trading intensifies, so does the average deviation of each fund's portfolio from its style benchmark. We remind the reader that the parameter values that lead to high cross-trading also imply larger values of the distance  $|\beta_C^2(0) - \beta_C^1(0)|$  between the style benchmarks' weights in the illiquid asset according to (5). Since

---

<sup>20</sup> See Section 2.3.

larger values of this distance can be associated with greater style diversity, the results above suggest a positive relation between style diversity within a family, the extent of cross-trading and the differences in funds' average illiquidity relative to their benchmarks. We explore this relation in the next subsection and derive a novel testable implication for which we find empirical validation in Section 4.

### 3.3 Style Diversity and Funds' Portfolio Liquidity

Our assumptions on mutual funds' risk-appetite clienteles in Section 2.2 and on the family's total holdings of asset  $C$  allow us to identify style diversity also with the distance  $|\gamma_{h1} - \gamma_{h2}|$  between the RRA coefficients of investors 1 and 2. Indeed, by (5) a higher RRA coefficient  $\gamma_{h1}$  implies a lower weight  $\beta_C^1(0)$  of benchmark 1 in asset  $C$ , which in turn implies a higher weight  $\beta_C^2(0)$  of benchmark 2 and greater style diversity  $|\beta_C^2(0) - \beta_C^1(0)|$ .<sup>21</sup> Figure 4 plots the optimal level of cross-trading (Panel A) and the associated average excess holdings of the illiquid asset (Panel B) as a function of style diversity:<sup>22</sup>

[Figure 4 about here]

Cross-trading increases monotonically with style diversity. As in Table 2, the possibility of cross-trading leads fund 1 to buy shares of  $C$  from fund 2. The optimal crossing goes from  $|X| = 0.03$  for the lowest style diversity to  $|X| = 0.42$  for the most dispersed styles within the family. The intuition is that high style diversity leads managers to follow more extreme benchmark compositions and exposes them to insufficient diversification, with manager  $m1$  having too little and manager  $m2$  too much exposure to asset  $C$ . Managers then require a larger rebalancing of their initial portfolios to achieve a more desired portfolio composition. Given that  $m1$  demands more shares of  $C$  while  $m2$  is willing to supply those shares, they can satisfy their need for rebalancing by trading with each other a large amount of the illiquid asset.

---

<sup>21</sup> Using (5) once again to back out the investor's 2 RRA coefficient  $\gamma_{h2}$ , we see that a higher risk aversion for  $h1$  maps one-to-one onto a lower risk aversion for  $h2$  in our model.

<sup>22</sup> We keep the baseline assumption  $\gamma_{h1} > \gamma_{m1} = \gamma_{m2} > \gamma_{h2}$  throughout.

Conversely, insufficient diversity leads to little or no cross-trading. This would be the case, for instance, if the liquid style  $L$  was catered to households with slightly larger RRA coefficient than the manager (e.g.,  $\gamma_{h1} = 3$ ). Thus, within-family diversity in investment styles not only strengthens the extent of cross-trading in our model but is a necessary condition for managers to consider crossing their trades in the first place.

As diversity in the investment styles offered by a family increases, Panel B shows that so does each fund’s average deviation from its respective benchmark in the direction of the other fund’s benchmark. That is, *the FA fund that follows the more liquid style  $L$  (fund 1) increases its portfolio illiquidity more than an equivalent SA fund, while the converse—excess portfolio illiquidity falls more than for an equivalent SA portfolio—is true for the fund that follows the illiquid style  $S$  (fund 2)*. This relation between style diversity within a family and the illiquidity of a fund relative to the illiquidity of its benchmark has important implications on the derived utility of delegating investors as we show in Section 3.5. This is a novel prediction of our model, which we test on a sample of U.S. actively managed mutual funds in Section 4.

### 3.4 Decentralized Cross-Trading and Family Performance

Our model assigns no asset allocation role to a centralized decision maker or, as introduced in Section 2.4, a Chief Investment Officer (CIO) of the overall family.<sup>23</sup> In practice, CIOs of mutual fund families may not have a direct say on the investment decisions of the affiliated funds but still have the responsibility for the strategy of the overall group. Under the family arrangement, it could be argued that the CIO would not allow—by, e.g., offering a narrow set of investment styles—the funds to cross their trades of asset  $C$  unless it satisfies not only the interests of the affiliated fund managers but also her own interest. Thus, our result so far can represent an equilibrium situation within the family only if the family CIO is also better off—or at least not worse off—after the internal trading.

---

<sup>23</sup> In this sense, our setup differs from Binsbergen, Brandt, and Koijen (2008), who assume the CIO’s role is to allocate capital to the different investment styles. Although realistic in the context of, e.g., pension funds, their assumption does not necessarily hold in the context of open-end mutual funds for which the assets under management are decided by households instead. To reduce the layers of agency involved in our problem, we thus assume that the CIO does not make any investment decision (see Section 2.4).

Table 2 shows that the CIO does benefit from the funds’ cross-trading in our baseline scenario, making the trade consistent with the goals not only of fund managers but also of the manager of the family. Even though the after-flow assets of the overall family do not grow, the CIO increases her utility (certainty equivalent) substantially from allowing the direct trading between the funds.<sup>24</sup> Given that the CIO derives utility from the family’s overall assets, this result implies that cross-trading allows for better diversification of the family’s portfolio of funds. Intuitively, the possibility of cross-trading induces managers to take an overall exposure on all three assets that is closer, compared with the no-cross-trading scenario, to the CIO’s desired exposure. Indeed, two equivalent SA funds would invest too little in one of the liquid assets (asset 2 in our baseline case), whereas the CIO prefers similar holdings in both liquid assets for diversification purposes.<sup>25</sup> Notably, the better diversification of the family’s portfolio implies almost no loss in future AUM, as each fund’s policy optimally exploits the positive and convex relation between performance and future fund flows.

The net benefits accruing to the CIO hold across all of our alternative scenarios, and increase significantly for large optimal cross-trading (e.g., for  $\rho_{iC} = 0.6$ ). In the latter cases, the expected end-of-period AUM (after-flow returns) of the family organization exceed the sum of AUM of two otherwise equivalent SA funds. As if led by an “invisible hand” of internal markets, the decentralized cross-trading that maximizes the utility of each manager also increases the benefits accruing to the family and its CIO, *even though no family-coordinated decision* is made. This is the case even when the performance of one of the FA funds, as measured by RRR, does not improve relative to its SA counterpart (e.g., for  $\mu_1, \mu_2 = 0.085$ ).

Since higher style diversity within the family leads to larger cross-trading in our model, the results above suggest that families can derive larger benefits from increasing the menu of investment styles (i.e., the style diversity) available to investors. We look into this prediction in Figure 5, which plots the CIO’s certainty equivalent (CE) and after-flow returns to the family as a function of style

---

<sup>24</sup> We describe the computation of agents’ certainty equivalent (CE) in subsection 3.5.

<sup>25</sup> Since the liquid assets offer similar—the same, in our setup—risk-return tradeoff but are not perfectly correlated, a CIO investing in the three assets will invest similar weights in each of these assets.

diversity.

[Figure 5 about here]

As expected, the CIO's CE from running funds 1 and 2 under a family organization increases monotonically with style diversity, deriving high CE returns from allowing for cross-fund transactions relative to the no-cross-trading (NCT) scenario. The expected growth in the AUM of the family also increases monotonically with style diversity, from marginally negative values for low diversity (e.g.,  $\gamma_{h1} = 4$ ) to a few percentage points for large enough style dispersion. Altogether, these results imply that a centralized decision maker for a family of mutual funds (i) will not discourage internal trading across FA funds, and (ii) will offer a menu of investment styles that is as diverse as possible. The latter result seems to indicate that an optimal strategy for the family is to maximize the style diversity and encourage FA funds to follow more extreme benchmarks. However, we argue in the next subsection that such strategy can inflict high agency costs on delegating households. These costs should impose tight limits on the implementation of such strategy in practice.

**Cross-Subsidization Hypothesis and Centralized vs. Decentralized Decisions.** Gaspar, Massa, and Matos (2006) conjecture, and empirically verify, that the convexity of funds' flow-performance relationship can encourage fund families to 'play favourites' among affiliated funds in order to maximize the family's total amount of assets under management. In particular, they find that fund families can cross-subsidize some member funds over others within the family through interfund transactions at *below or above market prices*. Such behaviour goes clearly against SEC Rule 17(a)-7 and should thus be illegal.<sup>26</sup>

As described above, a novel prediction of our model is that families can still increase the amount of assets under management without playing favourites by choosing to offer diverse enough styles. The affiliated funds following these styles will indirectly satisfy the family objective while engaging in *mutually beneficial* cross trades. This cross-trading is not decided by a centralized decision maker playing favorites among affiliated funds, it is instead decided by the decentralized optimal

---

<sup>26</sup> Note that the 'non-market' feature of prices is key for this hypothesis to remain valid in perfectly liquid markets, since at fair market prices each fund would be just indifferent between cross-trading with another fund in the same family and trading in the public markets.



trading of the individual fund managers. As a result, no fund is “sacrificed” for the greater good of the family as documented by Gaspar, Massa, and Matos (2006).<sup>27</sup> Still, another ‘invisible hand’, one working in internal markets, ensures that the family also derives significant benefits from the decentralized cross-trading.

Notwithstanding the difference between the approach of these authors and our approach, a slight modification of our setup shows that when illiquidity costs are non-negligible, families can play favourites among affiliated funds by having them cross trade even *at fair market prices*. In Figure 6, we show the level of cross-trading that a CIO would choose if deciding to play favourites between funds 1 and 2 in our model. Clearly, because the CIO’s derived utility is increasing in the number of shares of asset  $C$  sold from fund 2 to fund 1, the CIO benefits by maximizing manager  $m_2$ ’s utility ( $OM_2$ ) rather than  $m_1$ ’s utility ( $OM_1$ ). At this level of cross-trading, the CIO enables the manager of the favoured fund 1 to reduce the costs of illiquidity by making the other fund in the family adopt a suboptimal investment policy. The end-of-period RRR of fund 1 is higher than that of a comparable SA fund ( $NCT$ ), while the RRR of fund 2 is significantly lower. Relative to the no-cross-trading ( $NCT$ ) scenario, the resulting funds’ measured performance resembles the cross-subsidization of fund 1 by fund 2. Although such cross-trading would still violate the spirit of Rule 17(a)-7—both funds have to benefit from the trade according to this rule—this strategy would be arguably more difficult to detect in practice.

**[Figure 6 about here]**

Figure 6 also shows the performance costs associated with centralized decision making. Under the ‘playing favourites’ strategy, in which case the CIO decides the level of cross-trading ( $OM_2$ ), the end-of-period performance of both funds, as measured by RRR, is lower than the performance attained under the decentralized optimum  $OCT$ . This result is consistent with the empirical findings of Kacperczyk and Seru (2012), who show that funds from decentralized families have higher performance than their centralized counterparts.

---

<sup>27</sup> See also Bhattacharya, Lee, and Pool (2013) and Casavecchia and Tiwari (2014).

### 3.5 Agency Costs of Cross-trading

To the extent that lower transaction costs translate into higher after-fee risk-adjusted returns, investors should be better off by delegating their portfolios to family-affiliated (FA) funds compared to delegating to (otherwise identical) standalone (SA) funds. However, the availability of an internal market can also facilitate opportunistic trades that would otherwise be too costly to execute in the open market. When the incentives of the managers are not fully aligned with those of their delegating investors, cross-fund transactions can benefit both managers involved while at the same time impose agency costs on the funds' investors.

For instance, if at an interim point during the investment period manager  $m1$  in our setup underperforms her relatively liquid benchmark, she may attempt to 'gamble for resurrection' by placing large bets on small illiquid stocks (asset  $C$ ). Due to the assumed flow-performance relationship (8),  $m1$  will enjoy large investors' inflows in the future if these bets pay off, and disproportionately lower outflows if these bets go sour. High transaction costs may deter a SA fund from gambling on illiquid stocks, but need not discourage a FA fund who can circumvent these costs by trading with a sibling fund with a similar but opposite intended order.

We assess the net effect to investors from portfolio delegation by examining their certainty equivalent (CE) rate of return. For an investment policy  $\pi$ , this is the risk-free rate of return  $CE(\pi)$  that makes an agent indifferent between following the policy  $\pi$  over the investment horizon  $T$  or alternatively earning this risk-free rate on the same initial investment over the same period. For an agent with CRRA coefficient  $\gamma$  and initial wealth  $z$ ,  $CE(\pi)$  solves:

$$\frac{[CE(\pi)z]^{1-\gamma}}{1-\gamma} = E \left[ \frac{(Z^\pi(T))^{1-\gamma}}{1-\gamma} \right], \quad (15)$$

where the superscript ' $\pi$ ' denotes that the final wealth  $Z(T)$  is attained under the optimal investment policy.

We report the net effect of delegation to family-affiliated vs. standalone funds in Table 3.<sup>28</sup>

---

<sup>28</sup> In all cases, investor's derived utility under delegation is obtained by plugging her managed fund's terminal wealth in utility function (3).

These net effects arise only due to the potential cross-fund trades under the family arrangement, so they can equivalently be interpreted as net effects of cross-trading. As explained in Section 2.2, we consider two types of investors. First, the households  $h1$  and  $h2$  to which funds 1 and 2 are catered, in the sense that their benchmarks are the unconditional efficient portfolios that these investors would choose under self-management and perfect liquidity. Second, the alternative investors  $h11$  and  $h12$  in fund 1's clientele, and  $h21$  and  $h22$  in fund 2's clientele, with RRA coefficients as specified in Appendix A.

[Table 3 about here]

Agency costs of cross-trading can be substantial. Under our baseline case, investor  $h1$  in fund 1 suffers significant costs (0.44% per year) from delegating to a FA fund relative to a SA fund. Except for those investors with similar or higher risk tolerance than  $h2$  (e.g.,  $h21$ ), all other investors in a neighbourhood of  $h1$  and of  $h2$  also suffer the costs of family affiliation. In this case, the decentralized cross-trading effectively *cross subsidizes* some investors in fund 2 at the expense of other investors in fund 2 and of investors in fund 1. Thus, the investors in fund 2 enjoy the savings on illiquidity costs that the other family investors pay via the suboptimal investment policy that their funds adopt.

Our alternative scenarios show that the costs of delegation can be even larger, and up to 5.86% per year in some cases (i.e., for  $\rho_{iC} = 0.6$ ). For large levels of cross-trading, manager's optimal policies can be such that *both investors  $h1$  and  $h2$  are worse off after cross-trading*, even though *the measured performance for both of their funds improves*—the RRR is higher for both funds at *OCT* relative to *NCT*, see Figure 6. This situation occurs because managers dynamically change the risk of their portfolios in a way that exposes their investors to inadequate—either excessive or insufficient—risk, while this risk is not appropriately captured by conventional performance measures such as RRR (see Goetzmann, Ingersoll, Spiegel, and Welch (2007)). Investors' decision to delegate to a family-affiliated vs. a standalone fund then *trades off saving on transaction costs vs. paying additional agency costs*. In this sense, the extent of style diversity that a family could reach as described in subsection 3.4 should be limited by the agency costs that the resulting level

of cross-trading imposes on its delegating investors.

To the extent that investors in one or in both funds are worse off after cross-trading, the cross-fund trades that we examine in this Section go against Rule 17(a)-7. However, these internal trades are executed at fair market prices. Moreover, both fund managers benefit and the RRR of both funds improves after the trades. Arguably, regulatory authorities should find such breach of fiduciary duty harder to detect in spite of the substantial negative effects it accrues to delegating investors.

## 4 Empirical Tests

A central implication of our model is that higher style diversity within a family leads fund managers following high- and low-liquidity styles to deviate further away from their benchmarks in opposite directions. Specifically, the high-liquidity fund should increase portfolio illiquidity and conversely for the low-liquidity fund. Given the potentially negative consequences of this behaviour on delegating investors' utility, as we emphasized in subsection 3.5, we test the validity of this prediction in the following subsection.

### 4.1 Data and Methodology

We obtain fund returns, investment objectives, fees, total net assets (TNA), and other fund characteristics from the Center for Research in Security Prices (CRSP) Survivor-Bias-Free Mutual Fund Database. We use the Wharton Research Data Services MFLINKS file to merge this database with the Thomson Financial Mutual Fund Holdings dataset, which contains information on stock positions of funds and identifies fund families (see Wermers (2000)). We restrict our analysis to diversified domestic actively managed equity mutual funds.<sup>29</sup> We compute fund-level variables by aggregating across all the share classes. For instance, a fund's expense and turnover ratios are the TNA-weighted averages of the ratios of its different share classes. The age of the fund is equal to the age of its oldest share class, and the TNA of the fund is the sum of the TNAs of all its share

---

<sup>29</sup> More specifically, and following existing literature, we exclude international, balanced, sector, bond, money market, and index funds.

classes. We exclude funds that manage less than \$5 million.

We follow Reed and Wu (2005) and Basak, Pavlova, and Shapiro (2007) who use daily fund returns to study how mutual fund managers respond to beating their benchmarks. Our final sample covers the period 1999-2009. It is limited below by the CRSP daily mutual fund returns data, which starts in 1999, and it is limited above by the active share sample of Cremers and Petajisto (2009) and Petajisto (2013), which ends in 2009.<sup>30</sup>

We assign a DGTW benchmark to each fund-quarter according to the following procedure. First, we create the daily series of returns for each of the 125 DGTW benchmarks, following the methodology in Daniel, Grinblatt, Titman, and Wermers (1997).<sup>31</sup> Second, we compute the pairwise tracking errors (annualized) between the daily returns of the largest share class of each fund (following Petajisto (2013)) and all the 125 DGTW portfolios. Finally, we assign to a fund the DGTW benchmark that has a tracking error closest to the tracking error that Petajisto (2013) computed between the prospectus benchmark and the fund's returns.<sup>32</sup> Following Petajisto (2013), we use quarterly rolling windows of 6-month daily returns in the computation of such tracking errors.

We assign a DGTW portfolio to each mutual fund, instead of using directly their prospectus benchmarks and respective constituents. This allows us to more easily match the fund's portfolio holdings' characteristics with those of the assets in their respective DGTW benchmarks. Our model's main testable implication relates a FA fund's relative illiquidity to the diversity of styles

---

<sup>30</sup> Even if our model predictions are borne in the data, we expect our empirical findings to weaken after the introduction of SEC rules 38a-1 and 206(4)-7 (introduction of chief compliance officer independent from fund management) and the amendments to rule 204-2 (advisers must maintain copies of their compliance policies and procedures and copies of any records documenting the adviser's annual review of those policies) in 2004, see Eisele, Nefedova, and Parise (2014).

<sup>31</sup> Some of the key steps of the methodology in Daniel, Grinblatt, Titman, and Wermers (1997) involve a triple sort of common stocks into buckets of size, book-to-market (B/M), and momentum (quintiles for each of these three dimensions), in the month of July of each year. Moreover, it requires availability of COMPUSTAT data for at least 2 years, a minimum of 6 months of returns in CRSP, size weights are constructed using the market value in June (using the NYSE size breakpoints), the B/M ratio uses the market cap in prior fiscal-year end (and are adjusted with industry averages, using the 49 Fama-French industry classification available in Ken French's webpage), and the momentum factor is the 12 month return with one month reversal gap. More details on the methodology are available in Russ Wermers' website: <http://www.smith.umd.edu/faculty/rwermers/ftpsite/Dgtw/coverpage.htm>

<sup>32</sup> This list of official benchmarks that mutual funds disclose in their prospectuses is available in Antti Petajisto's website for the period between 1980 and 2009: <http://www.petajisto.net/data.html>. It includes 5 indices from S&P, 12 indices from Russell, and 2 indices from Dow Jones / Wilshire. All the variables in this dataset are defined and the sample of funds is selected as in Petajisto (2013). The sample used in Petajisto (2013) covers the period 1980-2009.

a family offers. Therefore, we would like to compare the liquidity of the fund’s portfolio and the liquidity of its benchmark. Following Lou (2012), we use the liquidity cost estimates of Hasbrouck (2009) as our illiquidity measure, which we value-weight across all the stocks in each of the 125 DGTW benchmarks. We then compare the liquidity cost of the DGTW benchmarks assigned to each mutual fund with the liquidity cost of the fund’s portfolio, which is also a value-weighted average across all the stocks held by the fund. Due to the quarterly frequency of mutual fund holdings data, we aggregate all variables at the quarterly level. The resulting total sample consists of 1,861 distinct mutual funds.

To approximate the style diversity offered by a family, two natural candidate variables are the number of funds *NFUNDS* and the total assets under the family’s management *MFFSIZE*. However, a small two-fund family can offer more extreme (e.g., one very conservative and one very risky) investment styles than a large multi-fund family offering similar styles. To the extent that more funds or a larger pool of assets in the family do not capture more dispersion in fund styles our model has no say on the funds’ predicted deviations from their benchmark. Therefore, we construct an alternative variable, *DIVERS*, to better reflect the style diversity in our model. More precisely, for each family we compute all the pairwise correlations across all the different benchmarks that funds affiliated with the same family follow. We define *DIVERS* as the minimum of those pairwise correlations.<sup>33</sup>

## 4.2 Main Results

The first hypothesis we examine is how much a FA fund deviates its portfolio illiquidity from the illiquidity of its benchmark for different levels of family diversity. According to our model, larger family diversity should lead to larger deviations from benchmark liquidity.

We run the following specification to test this hypothesis:

$$\text{ABSILLIQDIFF} = \beta_0 + \beta_1 \text{DMEASURE} + \beta_2 \text{Controls} + \epsilon \quad (16)$$

---

<sup>33</sup> We take the negative of the minimum pairwise correlation within the family so that a decrease in correlation can be interpreted as an increase in diversity.

where  $ABSILLIQDIFF$  is the absolute difference between the fund illiquidity ( $FILLIQ$ ) and its benchmark illiquidity ( $BILLIQ$ ),  $DMEASURE$  is our family diversity measure as captured, alternatively by  $NFUNDS$ ,  $MFFSIZE$ , or  $DIVERS$ , and  $Controls$  is a vector of other determinants of  $ABSILLIQDIFF$ . In particular, we control for the age (in years) of the fund since inception ( $FAGE$ ) and the TNA of the fund ( $SIZE$ ). These two variables are very skewed. We use a logarithmic transformation of those variables to deal with the skewness. We also include indicator variables  $DG$  and  $DGI$ , which equal one if the fund is in the Growth and in the Growth and Income categories, respectively, and equals zero otherwise, to control for the style of the funds. Our controls also include the past 12-month volatility of the fund’s monthly returns,  $QVOL$ , the fund’s expense ratio,  $QER$ , a dummy that equals one if the fund ranked in the bottom half across all the funds in the same investment category in the previous quarter,  $DLOSE$ , and the average illiquidity of the benchmarks within the fund’s family,  $FBILLIQ$ . We run Fama and MacBeth (1973) regressions with heteroskedasticity-consistent standard errors and Newey and West (1987) correction using four lags. We present the results of this estimation in Table 4.

As predicted, there is a positive relationship between the style diversity within fund families and the deviations of illiquidity between the funds and their respective benchmarks. Moreover, the diversity measure suggested by the model,  $DIVERS$ , has the highest statistical significance and remains strong even after controlling for  $NFUNDS$  and  $MFFSIZE$ .<sup>34</sup>

[Table 4 about here]

Although these results lend support to our theoretical results, we can take advantage of our model predictions to further refine our hypothesis by examining deviations from high- and low-liquidity benchmarks separately. We expect to find that funds deviate by either increasing or decreasing portfolio illiquidity depending, respectively, on whether they follow low- or high-illiquidity benchmarks. This is a more stringent test than the one in the above, but can lend stronger empirical support to our model if successful. We run the following specification to test our refined

---

<sup>34</sup> We note that, when included in the more general model (4)  $NFUNDS$  changes sign and its coefficient value offsets the value of the coefficient on  $MFFSIZE$ . This suggests a potential multicollinearity problem when both variables are included in the same regression.

hypothesis:

$$\begin{aligned}
\text{ILLIQDIFF} &= \alpha_0 + \alpha_1 \text{DMEASURE} + \alpha_2 \text{LOWBILLIQ} + \alpha_3 \text{HIGHBILLIQ} + & (17) \\
&+ \alpha_4 \text{DMEASURE} \times \text{LOWBILLIQ} + \alpha_5 \text{DMEASURE} \times \text{HIGHBILLIQ} + \\
&+ \alpha_6 \text{Controls} + \psi
\end{aligned}$$

where the dependent variable is now ILLIQDIFF, the difference of illiquidity between the funds and their respective benchmarks. This variable can take both positive and negative values, while the dependent variable in model (16) only takes positive values. The variable DMEASURE is as defined in model (16). The variable LOWBILLIQ (HIGHBILLIQ) is an indicator function that equals one if the fund benchmark ranks in the bottom (top) tercile of illiquidity across all the benchmarks in a given quarter.

Our model implications relate specifically to the interactions between DMEASURE and the indicators LOWBILLIQ and HIGHBILLIQ, predicting a positive sign for the first and a negative sign for the second interaction terms. We are particularly interested in the interaction terms involving DIVERS. We report the results of the estimation of this model in Table 5.

**[Table 5 about here]**

As predicted, the coefficients  $\alpha_4$  and  $\alpha_5$  on the interactions  $\text{DIVERS} \times \text{LOWBILLIQ}$  and  $\text{DIVERS} \times \text{HIGHBILLIQ}$  are positive and negative, respectively, and statistically significant at 1% level (column (3)). The coefficients on NFUNDS and MFFSIZE and their interactions with LOWBILLIQ are also positive and statistically significant at the 1% level (columns (1) and (2)), consistent with the results for DIVERS. Unlike the case of DIVERS, though, the interactions of NFUNDS and MFFSIZE with HIGHBILLIQ are not statistically different from zero. Moreover, when we include all the alternative measures of style diversity in one single specification, as well as their multiple interactions with LOWBILLIQ and HIGHBILLIQ in column (4), only  $\text{DIVERS} \times \text{LOWBILLIQ}$  and  $\text{DIVERS} \times \text{HIGHBILLIQ}$  remain economically and statistically significant. These findings confirm the importance of style diversity in the choice of portfolio liquidity



for family-affiliated funds, in agreement with our model.

### 4.3 Comparison to Cross-Subsidization Hypothesis

We argue that the effects documented above are the result of decentralized strategies at the level of the individual fund managers. However, Gaspar, Massa, and Matos (2006) document that families can also play strategies at a centralized level. In particular, they show that families can play favourites and maximize the amount of assets for the whole group, assuming the spillover effects documented in Nanda, Wang, and Zheng (2004), by transferring performance from low-value funds to high-value funds ('cross-fund subsidization') via cross-trading.

To prove the incremental relevance of the decentralized cross-trading in our model, we seek to rule out the possibility that our empirical findings are driven by family-coordinated trades between affiliated funds. Specifically, we need to reject the hypothesis that the deviations in funds' liquidity from their benchmarks' illiquidity is a consequence of the cross-subsidization of high-value funds at the expense of their low-value siblings within the family. Gaspar, Massa, and Matos (2006) consider high-value funds to be young funds, smaller funds, and funds that charge higher fees. According to the theoretical work in Berk and Green (2004) and Huang, Wei, and Yan (2007), the first two variables (age and size) are endogenous determinants of the flow-performance relationship, which is essential in our theoretical model. Therefore, it would not be adequate to use age and size as additional covariates in our model. We thus focus our analysis on total fees only.

In Table 6, we re-run the specification of column (3) in Table 5 with two additional indicator variables, LOWFEE and HIGHFEE, their interactions with LOWBILLIQ and HIGHBILLIQ, as well as triple interactions involving DIVERS. The variable LOWFEE (HIGHFEE) is an indicator that equals one if the fee charged by the fund is in the bottom (top) quartile of the fees charged across all the funds within a given family.

[Table 6 about here]

If the effect we documented in column (3) of Table 5 is indeed the result of a strategy played at the family-level, we would expect to observe (i) that deviations in funds' liquidity from their bench-

marks are stronger for high- and low-value funds, both before and after controlling for DIVERS, and (ii) that DIVERS cannot explain deviations in funds' liquidity after controlling for high- and low-value fund effects. The results in Table 6 show that we can reject this alternative explanation. Indeed, the coefficients on LOWFEE and HIGHFEE, and their interactions with LOWBILLIQ, HIGHBILLIQ and DIVERS are all statistically insignificant in columns (1) and (2). By contrast, the coefficients on the interactions DIVERS $\times$ LOWBILLIQ and DIVERS $\times$ HIGHBILLIQ retain the sign and statistical significance of column (3) in Table 5. We conclude that our empirical findings cannot be attributed to the cross-subsidization hypothesis, lending further support to our model.

## 5 Conclusion

We propose a model of internal markets in fund families and explore the possibility that benchmark concerns generate opportunistic internal trades between family-affiliated funds. When the family offers a diverse enough menu of investment styles, and such styles represent portfolios with common holdings of illiquid assets, cross-trading can result as the optimal outcome of a decentralized strategy implemented at the individual fund-level. Fund managers have an incentive to distort their portfolios away from their style benchmarks in an attempt to attract investors' flows. When the styles that these funds follow are sufficiently different from one another, managers will likely trade common holdings in opposite directions. If such funds belong to the same family, they have access to an internal market, which they can use to cross those opposite trades at low cost, instead of having to deal with illiquid open markets. The ability that family-affiliated funds have to circumvent the costs of illiquidity by trading in the internal market, allows them to more aggressively change the excess liquidity of their portfolios relative to their respective benchmarks throughout the investment period. The overall family stands to benefit from this strategy nearly as much as if the cross-trading were centrally decided. However, the increased risk-shifting that family-affiliated funds can perform due to the presence of internal markets, relative to equivalent standalone funds that have no access to such markets, creates additional agency costs to fund investors who choose to invest under such a family arrangement. Lastly, we provide empirical evidence consistent with

the novel implications of our model. In particular, we use a sample of U.S. actively managed equity mutual funds to test the hypothesis that style diversity within a fund family is associated with the degree to which funds deviate the liquidity of their portfolios relative to the liquidity of their style benchmarks.

Our results offer several implications. First, they draw attention to a novel agency cost of cross-trading and to a subtle, although economically significant, potential breach of fiduciary duty of family-affiliated fund managers with respect to their shareholders. Second, they provide an additional rationale for the diversity in investment styles within fund families that we observe in practice. Third, they relate differences in the performance of centralized versus decentralized mutual funds to the extent of cross-trading executed within a fund family. Finally, our results point to style diversity within fund families as a potential source of cross-sectional variation in the liquidity management by mutual funds.

## References

- Almazan, A., K. C. Brown, M. Carlson, and D. A. Chapman, 2004, “Why Constrain Your Mutual-Fund Manager?,” *Journal of Financial Economics*, 73, 289–321.
- Basak, S., and D. Makarov, 2014, “Strategic Asset Allocation in Money Management,” *Journal of Finance*, 69, 179–217.
- Basak, S., A. Pavlova, and A. Shapiro, 2007, “Optimal Asset Allocation and Risk Shifting in Money Management,” *Review of Financial Studies*, 20, 1583–1721.
- , 2008, “Offsetting the Implicit Incentives: Benefits of Benchmarking in Money Management,” *Journal of Banking and Finance*, 32, 1882–1993.
- Berk, J. B., and R. C. Green, 2004, “Mutual Fund Flows and Performance in Rational Markets,” *Journal of Political Economy*, 112, 1269–1295.
- Bhattacharya, U., J. H. Lee, and V. K. Pool, 2013, “Conflicting Family Values in Mutual Fund Families,” *Journal of Finance*, 68, 173–200.
- Binsbergen, J. H. V., M. W. Brandt, and R. S. J. Koijen, 2008, “Optimal Decentralized Investment Management,” *Journal of Finance*, 63, 1849–1895.
- BIS, 2003, “Incentive Structures in Institutional Asset Management and their Implications for Financial Markets,” Report submitted by a Working Group established by the Committee on the Global Financial System.
- Brown, K. C., W. V. Harlow, and L. T. Starks, 1996, “Of Tournaments and Temptations: An Analysis of Managerial Incentives in the Mutual Fund Industry,” *Journal of Finance*, 51, 85–110.
- Casavecchia, L., and A. Tiwari, 2014, “Cross Trading by Investment Advisers: Implications for Mutual Fund Performance,” forthcoming, *Journal of Financial Intermediation*.
- Chaudhuri, R., Z. Ivkovic, and C. Trzcinka, 2014, “Strategic Performance Allocation in Institutional Asset Management Firms: Behold the Power of Stars and Dominant Clients,” Working Paper, Oakland University.
- Chen, H.-I., and G. G. Pennacchi, 2009, “Does Prior Performance Affect a Mutual Fund’s Choice of Risk? Theory and Further Empirical Evidence,” *Journal of Financial and Quantitative Analysis*, 44, 745–775.
- Chevalier, J., and G. Ellison, 1997, “Risk Taking by Mutual Funds as a Response to Incentives,” *Journal of Political Economy*, 105, 1167–1200.
- Constantinides, G. M., 1986, “Capital Market Equilibrium with Transaction Costs,” *Journal of Political Economy*, 94, 842–862.
- Cremers, M., and A. Petajisto, 2009, “How active Is Your Fund Manager? A New Measure That Predicts Performance,” *Review of Financial Studies*, 22, 3329–3365.
- Dai, M., L. Goncalves-Pinto, and J. Xu, 2015, “Every Cloud Has a Silver Lining: The Deterrence Effect of Illiquidity on Mutual Fund Risk-Shifting,” Working Paper, National University of Singapore.

- Dai, M., and F. Yi, 2009, “Finite-Horizon Optimal Investment with Transaction Costs: A Parabolic Double Obstacle Problem,” *Journal of Differential Equations*, 246, 1445–1469.
- Daniel, K., M. Grinblatt, S. Titman, and R. Wermers, 1997, “Measuring Mutual Fund Performance with Characteristic-Based Benchmarks,” *Journal of Finance*, 52, 1035–1058.
- DelGuercio, D., and P. A. Tkac, 2002, “The Determinants of the Flow of Funds of Managed Portfolios: Mutual Funds versus Pension Funds,” *Journal of Financial and Quantitative Analysis*, 37, 523–557.
- Dybvig, P. H., H. K. Farnsworth, and J. N. Carpenter, 2010, “Portfolio Performance and Agency,” *Review of Financial Studies*, 23, 1–23.
- Edelen, R. M., R. Evans, and G. Kadlec, 2013, “Shedding Light on Invisible Costs: Trading Costs and Mutual Fund Performance,” *Financial Analysts Journal*, 69, 33–44.
- Eisele, A., T. Nefedova, and G. Parise, 2014, “Are Star Funds Really Shining? Cross-Trading and Performance Shifting in Mutual Fund Families,” Working Paper, Swiss Finance Institute.
- Elton, E. J., M. J. Gruber, and T. C. Green, 2007, “The Impact of Mutual Fund Family Membership on Investor Risk,” *Journal of Financial and Quantitative Analysis*, 42, 257–278.
- Fama, E. F., and J. D. MacBeth, 1973, “Risk, Return, and Equilibrium: Empirical Tests,” *Journal of Political Economy*, 81, 607–636.
- Gaspar, J.-M., M. Massa, and P. Matos, 2006, “Favoritism in Mutual Fund Families? Evidence on Strategic Cross-Fund Subsidization,” *Journal of Finance*, 61, 73–104.
- Goetzmann, W., J. Ingersoll, M. Spiegel, and I. Welch, 2007, “Portfolio Performance Manipulation and Manipulation-proof Performance Measures,” *Review of Financial Studies*, 20, 1503–1546.
- Hasbrouck, J., 2009, “Trading Costs and Returns for U.S. Equities: Estimating Effective Costs from Daily Data,” *Journal of Finance*, 64, 1445–1477.
- Huang, J., C. Sialm, and H. Zhang, 2011, “Risk Shifting and Mutual Fund Performance,” *Review of Financial Studies*, 24, 2575–2616.
- Huang, J., K. D. Wei, and H. Yan, 2007, “Participation Costs and the Sensitivity of Fund Flows to Past Performance,” *Journal of Finance*, 62, 1273–1311.
- Kacperczyk, M., and A. Seru, 2012, “Does Firm Organization Matter? Evidence from Centralized and Decentralized Mutual Funds,” Working Paper.
- Li, C. W., and A. Tiwari, 2009, “Incentive Contracts in Delegated Portfolio Management,” *Review of Financial Studies*, 22, 4681–4714.
- Longstaff, F. A., 2001, “Optimal Portfolio Choice and the Valuation of Illiquid Securities,” *Review of Financial Studies*, 14, 407–431.
- Longstaff, F. A., and E. S. Schwartz, 2001, “Valuing American Options by Simulation: a Simple Least-Squares Approach,” *Review of Financial Studies*, 14, 113–147.
- Lou, D., 2012, “A Flow-Based Explanation for Return Predictability,” *Review of Financial Studies*, 25, 3457–3489.
- Ma, L., Y. Tang, and J.-P. Gomez, 2015, “Portfolio Manager Compensation in the U.S. Mutual Fund Industry,” Working Paper, Northeastern University.

- Maug, E. G., and N. Y. Naik, 2011, "Herding and Delegated Portfolio Management: The Impact of Relative Performance Evaluation on Asset Allocation," *Quarterly Journal of Finance*, 1, 265–292.
- Nanda, V., Z. J. Wang, and L. Zheng, 2004, "Family Values and the Star Phenomenon: Strategies of Mutual Fund Families," *Review of Financial Studies*, 17, 667–698.
- Newey, W. K., and K. D. West, 1987, "A Simple, Positive Semi-Definite Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55, 703–708.
- Petajisto, A., 2013, "Active Share and Mutual Fund Performance," *Financial Analysts Journal*, 69, 73–93.
- Reed, A., and L. Wu, 2005, "Racing the Clock: Benchmarking or Tournaments in Mutual Fund Risk-Shifting?," Working Paper, University of North Carolina.
- Sirri, E. R., and P. Tufano, 1998, "Costly Search and Mutual Fund Flows," *Journal of Finance*, 53, 1589–1622.
- Taylor, J., 2003, "Risk-Taking Behavior in Mutual Fund Tournaments," *Journal of Economic Behavior and Organization*, 50, 373–383.
- Wermers, R., 2000, "Mutual Fund Performance: An Empirical Decomposition into Stock-Picking Talent, Style, Transaction Costs, and Expenses," *Journal of Finance*, 55, 1655–1703.

# Appendix

## A Model Parameterizations

Across all scenarios, we set  $\gamma_{mj} = 2.5 = \gamma_c$  to be in line with prior literature. We follow Basak, Pavlova, and Shapiro (2007) in calibrating funds' flow-performance relationship to match the estimates in Sirri and Tufano (1998) by setting  $\phi_j^L = 0.97$ ,  $\psi_j = 1.6$ , and  $\eta_j = -0.05$  for  $j \in \{1, 2\}$ .

Our baseline risk aversion coefficients for the investors  $h1$  and  $h2$  to which style L and S are catered according to (5) are  $\gamma_{h1} = 5$  and  $\gamma_{h2} = 1.5$ . We consider alternative investors  $h11$  and  $h12$  in fund 1's clientele, and  $h21$  and  $h22$  in fund 2's clientele, with values  $\gamma_{h11} = 1, \gamma_{h12} = 3, \gamma_{h21} = 3.5, \gamma_{h22} = 6.5$ . In our comparative statics analysis of funds' investment styles, we allow for further heterogeneity in risk-preferences and the resulting benchmarks' compositions by letting  $\gamma_{hj}$  adopt values in  $[\underline{\gamma}, \bar{\gamma}] = [1.01, 7.5]$ ,  $j \in \{1, 2\}$ .

We set market parameters to approximately match first- and second-order moments of the return distribution of the high (for the liquid assets 1 and 2) and low (for the illiquid asset  $C$ ) quintiles of the size-sorted Fama-French value-weighted portfolios over the period 1927-2009.<sup>35</sup> The baseline expected returns and return volatilities are  $\mu_1 = \mu_2 = 0.09$ ,  $\mu_C = 0.18$ ,  $\sigma_1 = \sigma_2 = 0.20$ , and  $\sigma_C = 0.37$ . These values imply a higher reward-to-risk ratio  $\mu/\sigma$  for the illiquid asset  $C$  relative to the liquid assets 1 and 2, which we assume as a non-negative premium for illiquidity.<sup>36</sup> Our alternative scenarios contemplate both an even higher illiquidity premium as implied by  $\mu_1 = \mu_2 = 0.085$  and a zero illiquidity premium as implied by  $\mu_1 = \mu_2 = 0.095$ . The baseline correlation coefficients are  $\rho_{12} = \rho_{1C} = \rho_{2C} = 0.77$ . Our alternative scenarios include the cases  $\rho_{iC} \in \{0.6, 0.85\}$  ( $i = 1, 2$ ) and  $\rho_{12} \in \{0.6, 0.9\}$ .

We set the baseline illiquidity parameter  $\alpha = 0.60$  to match the portfolio turnover in the mutual fund industry. Because  $\alpha$  can be seen as a fund portfolio's maximum turnover rate over the investment period, we chose this value so that the optimal portfolio turnover of the funds in our model approximately matches, on average, the turnover rate of equity mutual funds over the period 1974-2009.<sup>37</sup> Our alternative scenarios contemplate both higher and lower asset illiquidity:  $\alpha \in \{0.30, 0.90\}$ .

---

<sup>35</sup> We downloaded these data from Ken French's website: [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

<sup>36</sup> The historical average return for the large-cap Fama-French portfolio is closer to 11%. If we identify this portfolio with the purely liquid asset in our setup, the historical average implies a negative illiquidity premium that is difficult to reconcile with economic intuition. We therefore set  $\mu_1 = \mu_2 = 0.09$  in our baseline case.

<sup>37</sup> See the 2009 *Investment Company Fact Book* published by the Investment Company Institute (ICI).

## B Tables and Figures

**Table 1: Mutual Fund Structure in the Model**

This table summarizes the main ingredients of our theory model, which consists of two mutual funds that follow two different investment styles. These investment styles are associated with different performance benchmarks and different sets of assets. Each fund holds two risky assets, one perfectly liquid, and the other illiquid. They both hold the illiquid asset. Each fund is managed by a different manager and caters its product to a different investor/household.

Mutual Fund	Investment Style	Benchmark	Invests in Assets	Specialized in Asset	Manager	Catered to Household
$F_1$	L	$Y_1$	1, $C$	1	$m1$	$h1$ (and $h11$ , $h12$ )
$F_2$	S	$Y_2$	2, $C$	$C$	$m2$	$h2$ (and $h21$ , $h22$ )



**Table 2: Comparative Statics for Fund-Level and Family Outcomes**

This table reports comparative statics on optimal cross-trading (OCT), differences in portfolio weights for asset  $C$  between family-affiliated (FA) and standalone (SA) funds, differences in expected returns, the return-to-risk ratio (RRR), the change from NCT to OCT in the certainty-equivalent (CE) rate of return for the CIO, and the difference in expected returns for the family as a whole. The comparative statics are provided for different values of the return of the liquid assets ( $\mu_1$  and  $\mu_2$ ), the illiquidity of asset  $C$  ( $\alpha$ ), and the correlation between the illiquid asset  $C$  and the liquid assets ( $\rho_{iC}$ ). The first column provides the results for the baseline case.

	Baseline	$\mu_1 = \mu_2$		$\alpha$		$\rho_{iC}$	
		$\mu=8.5\%$	$\mu=9.5\%$	$\alpha=0.3$	$\alpha=0.9$	$\rho_{iC}=0.6$	$\rho_{iC}=0.85$
OCT	-0.07	-0.29	-0.05	-0.04	-0.29	-0.42	-0.01
$\Delta$ CE from NCT to OCT for m1 (%)	5.48	25.41	3.9	3.59	22.85	33.64	0.79
$\Delta$ CE from NCT to OCT for m1 (%)	-7.63	-29.9	-5.73	-4.25	-30.49	-44.35	-1.03
$\Delta$ Ret from NCT to OCT for fund 1 (bps)	50	243	33	33	205	308	7
$\Delta$ Ret from NCT to OCT for fund 2 (bps)	-0.67	-2.82	-0.48	-0.38	-2.71	-3.97	-0.09
$\Delta$ RRR from NCT to OCT for fund 1 (%)	0.95	3.61	0.47	0.69	2.05	2.93	0.12
$\Delta$ RRR from NCT to OCT for fund 2 (%)	0.52	-0.71	0.62	0.32	0.68	4.07	0.02
$\Delta$ CE from NCT to OCT for CIO (bps)	25	257	17	6	270	586	1
$\Delta$ Ret from NCT to OCT for CIO (bps)	-2	233	-7	-5	206	483	-2

**Table 3: Comparative Statics on Portfolio Delegation Costs**

This table reports comparative statics on the costs accruing to the households from delegating to a family-affiliated fund compared to delegating to a standalone one. This is captured by the differences in certainty-equivalent (CE) rate of return between NCT and OCT for households 1 ( $h1$ ) and 2 ( $h2$ ), as well as their alternatives ( $h11$ ,  $h12$ ,  $h21$ , and  $h22$ ). The comparative statics are provided for different values of the return of the liquid assets ( $\mu_1$  and  $\mu_2$ ), the illiquidity of asset  $C$  ( $\alpha$ ), and the correlation between the illiquid asset  $C$  and the liquid assets ( $\rho_{iC}$ ). The first column provides the results for the baseline case.

	Baseline	$\mu_1 = \mu_2$		$\alpha$		$\rho_{iC}$	
		$\mu=8.5\%$	$\mu=9.5\%$	$\alpha=0.3$	$\alpha=0.9$	$\rho_{iC}=0.6$	$\rho_{iC}=0.85$
$\Delta$ CE from NCT to OCT for h1 (bps)	-44	-297	-30	-25	-289	-362	-8
$\Delta$ CE from NCT to OCT for h11	-15	-127	-11	-9	-123	-153	-3
$\Delta$ CE from NCT to OCT for h12	-75	-484	-53	-42	-483	-590	-12
$\Delta$ CE from NCT to OCT for h2 (%)	2	-26	1	2	-32	-100	2
$\Delta$ CE from NCT to OCT for h21	25	30	23	16	49	101	3
$\Delta$ CE from NCT to OCT for h22	-27	-146	-18	-17	-135	-197	-4

**Table 4: Style Diversity and Deviations from Benchmark Illiquidity**

This table provides coefficient estimates for four different specifications of Fama and MacBeth (1973) cross-sectional regressions. The dependent variable is ABSILLIQDIFF, the absolute difference between the illiquidity of the fund (FILLIQ) and the illiquidity of the benchmark (BILLIQ). In column (1), the main independent variable of interest is NFUNDS (the log of 1 plus the number of funds affiliated with a given family). In column (2), the main independent variable is MFFSIZE (the log of the fund family's AUM), and in column (3) the main independent variable is DIVERS, which we compute as the minimum of the pairwise correlations of returns across all the benchmarks in a given family. The minimum correlation is multiplied by -1 so that a decrease in the minimum correlation corresponds to an increase in the style diversity within a fund family. The specification in column (4) includes all the three proxies for the style diversity (NFUNDS, MFFSIZE, and DIVERS). The remaining covariates in all the four specifications are FAGE (the log of 1 plus the number of years since the inception of the fund), SIZE (the log transformation of the fund TNA), DG (indicator for Growth investment style), DGI (indicator for Growth and Income investment style), QVOL (the monthly return volatility of the fund for the past 12 months), QER (the fund's expense ratio), DLOSE (an indicator that equals one if the fund ranked in the bottom half of performance across all the funds in the same investment style), and FBILLIQ (the average illiquidity of all the benchmarks across the funds affiliated with the same family). The t-statistics (in parenthesis) are computed using Newey and West (1987) standard errors. We denote by \*\*\*, \*\*, \* the significance at the 1%, 5%, and 10% levels, respectively.

Dependent Variable:	ABSILLIQDIFF [= ABS(FILLIQ - BILLIQ)]			
	(1)	(2)	(3)	(4)
INTERCEPT	-0.012 (-1.41)	-0.029* (-1.95)	0.049*** (2.94)	0.026 (1.62)
NFUNDS	0.007* (1.77)			-0.005** (-2.51)
MFFSIZE		0.005** (2.11)		0.005** (2.51)
DIVERS			0.070*** (3.09)	0.065*** (3.27)
FAGE	0.004 (1.30)	0.005 (1.41)	0.004 (1.29)	0.004 (1.36)
SIZE	-0.003 (-1.65)	-0.006* (-1.94)	-0.003* (-1.77)	-0.006** (-2.08)
DG	-0.017** (-2.07)	-0.016** (-2.10)	-0.015** (-2.08)	-0.015** (-2.04)
DGI	-0.028*** (-2.93)	-0.027*** (-2.97)	-0.026*** (-3.00)	-0.025*** (-2.98)
QVOL	0.010** (2.25)	0.010** (2.29)	0.009* (2.01)	0.009* (1.99)
QER	0.012*** (3.45)	0.010*** (3.62)	0.012*** (3.37)	0.009*** (3.55)
DLOSE	-0.001 (-0.13)	0.000 (-0.02)	0.000 (0.06)	0.001 (0.18)
FBILLIQ	0.275*** (6.83)	0.279*** (6.86)	0.263*** (7.55)	0.269*** (6.92)
Observations	42	42	42	42
Adjusted $R^2$	0.170	0.171	0.175	0.177

**Table 5: Extreme Benchmark Illiquidity and the Effects of Style Diversity**

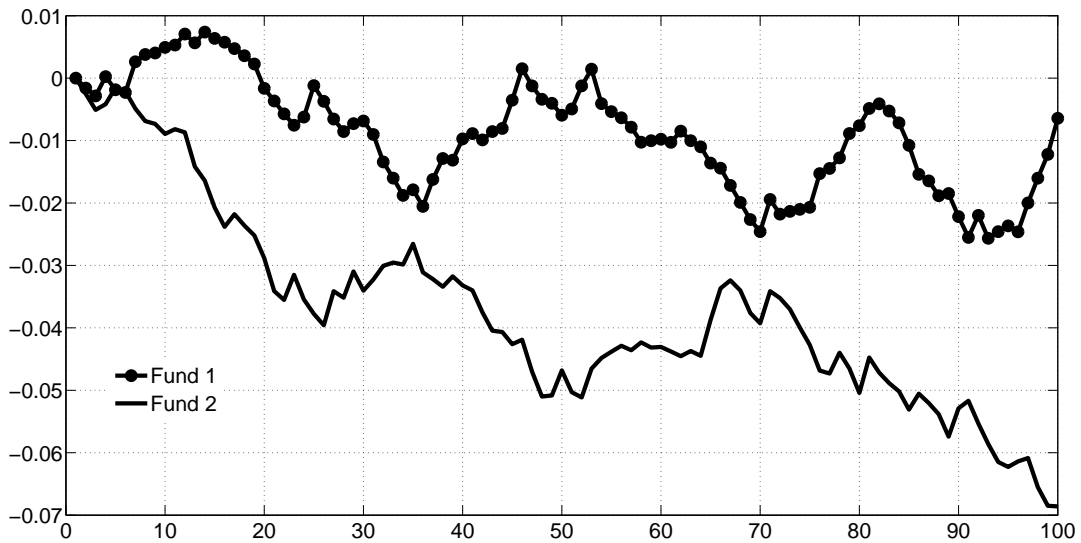
This table provides coefficient estimates for four different cross-sectional regressions similar to those in Table 4. In addition to the variables in Table 4, this table includes two extra indicators. The indicator LOWBILLIQ (HIGHBILLIQ) equals one if the fund benchmark ranks in the bottom (top) tercile of illiquidity across all benchmarks in a given quarter. The specifications in this table also include the interaction terms between these two extra indicators and each of the proxies for style diversity, i.e. NFUNDS, MFFSIZE, and DIVERS. The dependent variable is ILLIQDIFF, the simple difference between the illiquidity of the fund (FILLIQ) and the illiquidity of the benchmark (BILLIQ), which can take positive and negative values. In columns (1) to (3) the main independent variables of interest are NFUNDS, MFFSIZE, and DIVERS, respectively. The specification of column (4) includes all the three diversity proxies. The definition of these proxies is provided in Table 4. The remaining covariates (which coefficients are omitted in order to save space) are the same as in Table 4 (FAGE, SIZE, DG, DGI, QVOL, QER, DLOSE, and FBILLIQ). The t-statistics (in parenthesis) are computed using Newey and West (1987) standard errors. We denote by \*\*\*, \*\*, \* the significance at the 1%, 5%, and 10% levels, respectively.

Dependent Variable:	ILLIQDIFF [= FILLIQ - BILLIQ]			
	(1)	(2)	(3)	(4)
INTERCEPT	-0.068*** (-3.32)	-0.056** (-2.35)	-0.118*** (-4.58)	-0.076*** (-2.71)
LOWBILLIQ	0.084*** (6.01)	0.074*** (3.89)	0.152*** (7.16)	0.126*** (5.15)
HIGHBILLIQ	-0.279*** (-5.75)	-0.294*** (-4.90)	-0.438*** (-4.17)	-0.484*** (-3.29)
NFUNDS	-0.011*** (-3.70)			-0.008 (-1.58)
NFUNDS × LOWBILLIQ	0.007*** (3.17)			0.003 (0.76)
NFUNDS × HIGHBILLIQ	-0.004 (-0.68)			0.012 (1.09)
MFFSIZE		-0.005*** (-4.56)		-0.001 (-0.45)
MFFSIZE × LOWBILLIQ		0.003*** (3.02)		0.001 (0.34)
MFFSIZE × HIGHBILLIQ		0.001 (0.25)		0.001 (0.28)
DIVERS			-0.051** (-2.47)	-0.02 (-1.22)
DIVERS × LOWBILLIQ			0.081*** (3.80)	0.061*** (3.46)
DIVERS × HIGHBILLIQ			-0.293** (-2.32)	-0.317** (-2.09)
Controls	Yes	Yes	Yes	Yes
Observations	42	42	42	42
Adjusted R2	0.686	0.685	0.694	0.704

**Table 6: Fund-Level versus Family-Level Strategies**

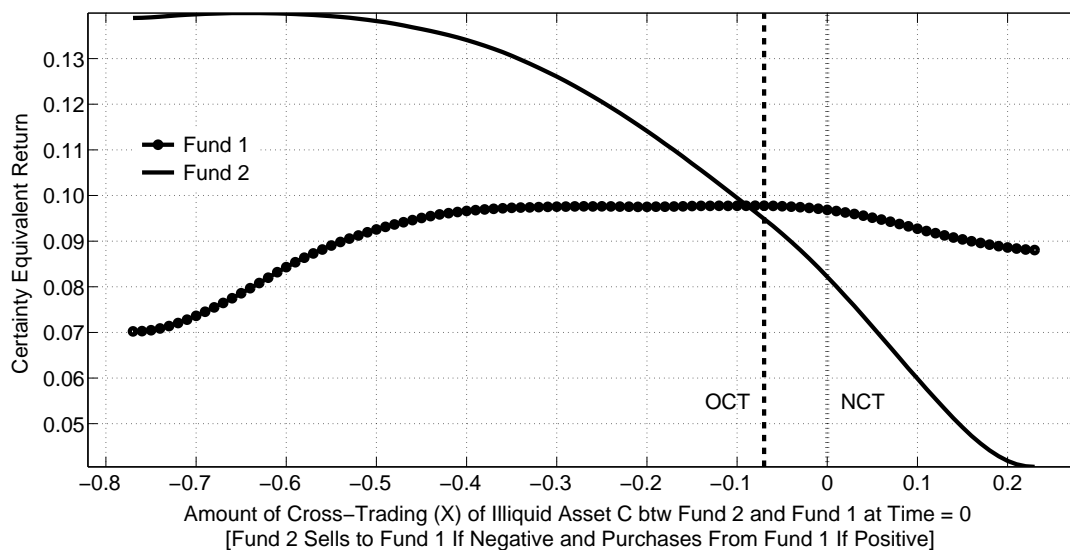
This table provides coefficient estimates for two different cross-sectional regressions similar to those in column (3) of Table 5. In addition to the variables included in column (3) of Table 5, this table includes two additional indicators (LOWFEE and HIGHFEE), as well as their multiple interactions with the indicators LOWBILLIQ and HIGHBILLIQ as well as the DIVERS proxy for style diversity within a fund family. The indicator LOWFEE (HIGHFEE) equals one if the fees charged by the funds rank in the bottom (top) quartile of fees charged across all the funds affiliated with the same family. The dependent variable is ILLIQDIFF. The remaining covariates (which coefficients are omitted in order to save space) are the same as in Table 4 (FAGE, SIZE, DG, DGI, QVOL, QER, DLOSE, and FBILLIQ). The definition of those variables is provided in Table 4. The t-statistics (in parenthesis) are computed using Newey and West (1987) standard errors. We denote by \*\*\*, \*\*, \* the significance at the 1%, 5%, and 10% levels, respectively.

Dependent Variable:	ILLIQDIFF	
	(1)	(2)
INTERCEPT	-0.080*** (-3.84)	-0.105*** (-3.94)
LOWBILLIQ	0.100*** (6.50)	0.158*** (6.89)
HIGHBILLIQ	-0.281*** (-6.36)	-0.416*** (-3.85)
LOWFEE	-0.014 (-1.68)	-0.066 (-0.80)
HIGHFEE	0.004 (1.22)	-0.088 (-0.78)
LOWFEE $\times$ LOWBILLIQ	0.013 (1.52)	0.058 (0.70)
LOWFEE $\times$ HIGHBILLIQ	-0.046* (-1.95)	0.121 (0.45)
HIGHFEE $\times$ LOWBILLIQ	-0.006 (-1.55)	0.091 (0.79)
HIGHFEE $\times$ HIGHBILLIQ	-0.011 (-0.68)	0.119 (0.73)
DIVERS		-0.053** (-2.43)
DIVERS $\times$ LOWFEE		-0.06 (-0.47)
DIVERS $\times$ HIGHFEE		-0.134 (-0.85)
DIVERS $\times$ LOWBILLIQ		0.092*** (3.99)
DIVERS $\times$ HIGHBILLIQ		-0.273** (-2.07)
DIVERS $\times$ LOWFEE $\times$ LOWBILLIQ		0.051 (0.39)
DIVERS $\times$ LOWFEE $\times$ HIGHBILLIQ		1.302 (0.93)
DIVERS $\times$ HIGHFEE $\times$ LOWBILLIQ		0.139 (0.87)
DIVERS $\times$ HIGHFEE $\times$ HIGHBILLIQ		0.108 (0.38)
Controls	Yes	Yes
Observations	42	42
Adjusted $R^2$	0.689	0.708



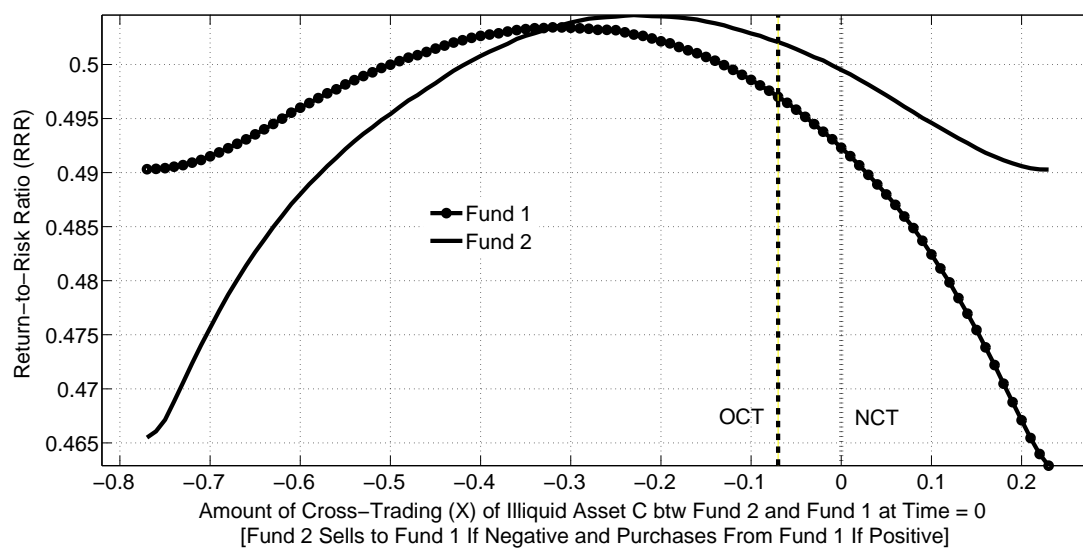
**Figure 1: Average Deviations from the Benchmark**

This plots the time-series of the cross-sectional average of the spread between the funds' portfolio weights on asset  $C$  and the weights of asset  $C$  in their respective benchmarks. The cross-section has 100,000 observations (the number of simulations we performed in our numerical analysis), while the time-series is divided in 100 time-steps, as represented by the length of the x-axis. The funds represented in this figure are standalone funds, which start off at time  $t=0$  with portfolio compositions that exactly match those of their respective benchmarks (i.e.  $X=0$ ), and are managed independently from that point onwards. The correlation between these two series is negative -0.09.



**Figure 2: Optimal Cross-Trading**

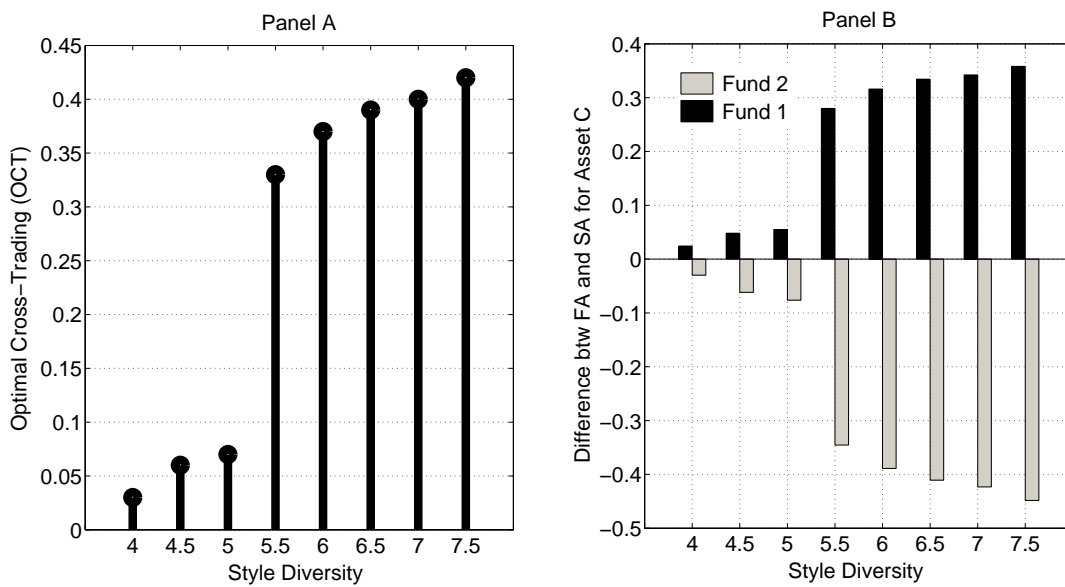
This plots the certainty equivalent (CE) rate of return each fund obtains for different levels of cross-trading. The vertical dotted line labelled as NCT represents the case in which both funds start off the investment period with portfolio compositions that exactly match those of their respective benchmarks. The vertical dashed line labelled as OCT identifies the level of cross-trading ( $X=-0.07$ ) that improves the CE for both funds relative to NCT (i.e.,  $X=0$ ). The funds follow the constrained max-min rule proposed in (12) to agree on the optimal level of cross-trading (OCT).



**Figure 3: Funds' Return-to-Risk Ratio**

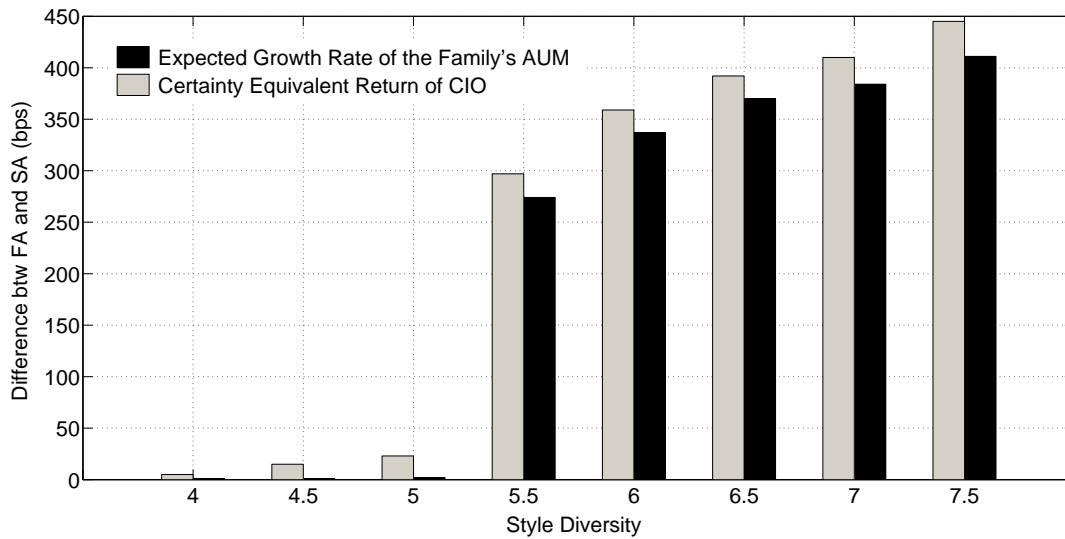
This plots the return-to-risk ratio (RRR) that each fund obtains for different levels of cross-trading. The vertical dotted line labelled as NCT represents the case in which both funds start off the investment period with portfolio compositions that exactly match those of their respective benchmarks. The vertical dashed line labelled as OCT identifies the level of cross-trading ( $X=-0.07$ ) that improves the CE for both funds relative to NCT (i.e.,  $X=0$ ). The funds follow the constrained max-min rule proposed in (12) to agree on the optimal level of cross-trading (OCT).





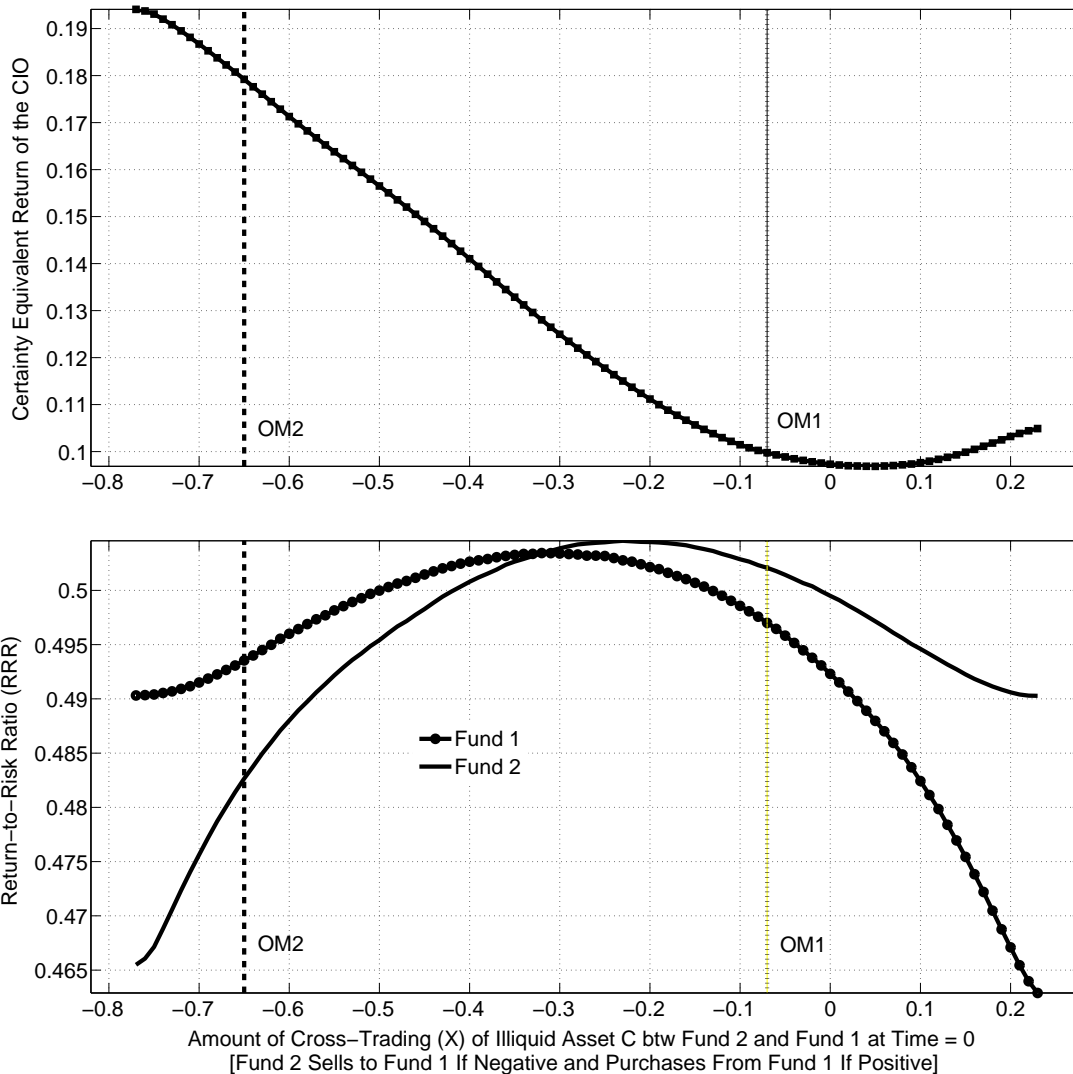
**Figure 4: Style Diversity, Optimal Crossing, and Benchmark Deviations**

Panel A reports the optimal level of cross-trading for different degrees of diversity within the fund family. Panel B plots the average excess holdings of illiquid asset  $C$  (comparing family-affiliated funds with their respective standalone versions) as a function of style diversity within the fund family. The style diversity in the x-axis is determined by the risk aversion parameter of investor  $h1$  ( $\gamma_{h1}$ ).



**Figure 5: Certainty Equivalent Return to the Fund Family**

This plots the certainty-equivalent (CE) rate of return for the CIO and the after-flow returns to the family (i.e. the expected growth rate of the assets under management of the fund family) as a function of style diversity. The style diversity in the x-axis is determined by the risk aversion parameter of investor  $h1$  ( $\gamma_{h1}$ ).



**Figure 6: CER of the CIO and RRR of the Individual Funds**

The top graph plots the certainty-equivalent (CE) rate of return that accrues to the CIO, and the bottom graph plots the return-to-risk ratio (RRR) of the individual funds, for different levels of cross-trading (X) between the two funds. The vertical dotted line labelled as OM1 identifies the amount of cross-trading between the two funds that would maximize the utility of fund manager 1, while the vertical dashed line labelled as OM2 indicates the amount of cross-trading that would maximize the utility of fund manager 2.