Market-Based Executive Compensation under Asymmetric Information

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Abstract

The paper investigates how a publicly traded firm's liquidation value and stock price are used in the executive compensation contract when information acquisition in the stock market is endogenized. We find that the inside owners can increase their payoff by incorporating the stock price into the contract even when the stock price does not contain any information about the managerial effort. It is because the increase in the firm's liquidation value always dominates that in expense of the managerial compensation. We analyze comparative statistics of the optimal contract. If information cost in the stock market displays an intermediate value, external factors generate direct effects and indirect effects via information acquisition. Otherwise, the indirect effects disappear and only the direct effects have influence on the optimal contract. Finally, we find that the market-based contract leads to a higher social welfare compared to the contract excluding the stock price.

KEYWORDS: principal-agent problem; executive compensation; information acquisition; price informativeness; price volatility

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1 Introduction

During the last decades, managerial compensations for top managements drastically increase in U.S. compared to economic growth.[1] Extremely high level of the CEOs payments cause debates about optimal managerial incentive scheme. Principals want CEOs to take actions which maximize firm values. However, since principals cannot perfectly observe managerial activities, CEOs seek for their private interests. Traditional studies such as Mirrlees (1976) and Holmström (1979) discuss managerial compensation models based on firms' liquidation values for risk sharing purposes. On the other hand, Hayek (1945) emphasises that price system plays a role of transmitting information about economic states. From this point of view, Holmström and Tirole (1993) and Baiman and Verrecchia (1995) show that incorporating stock prices into managerial compensation is helpful to monitoring managerial performance.[2] In the empirical study of Jensen and Murphy (1990), it is shown that managerial compensations tend to increase in stock prices.

The purpose of this paper is to investigate how a publicly traded firm's liquidation value and stock price are used in the executive compensation contract when information acquisition of ex ante identical traders is endogenized. To do this, we incorporate a standard principal-agent problem (e.g., Holmström, 1979) into Grossman and Stiglitz's (1980) asset pricing model with asymmetric information. There exist risk-neutral inside owners who own large shares of the firm's equity and a risk-averse manager who operates the firm. The inside owners offer a linear compensation contract which is based on the firm's liquidation value and the stock price. If the managers accepts the contract, he makes costly unobservable effort and the effort affects the firm's liquidation value and the stock price. Information about the fundamental value is endogenously acquired by outside rational traders. We derive and characterize the optimal compensation contract between the inside owners and the manager.

We find that the inside owners can achieve a higher profit by offering the market-based compensation contract to the manager. Compared to managerial contracts excluding the stock price, the manager takes more effort and this increases both the firm's expected liquidation value and the expense of the managerial compensation. However, since the increase in the liquidation value dominates that in managerial compensation, the inside owners can expect a higher income. This result seems to be similar with that of Holmström and Tirole (1993). In their study, the inside owners can obtain a higher income from the market-based compensation since they extract additional information about managerial effort from the stock price. Similarly, the contract is related to the price informativeness in our model, but the inside owners still have incentive to incorporating the stock price into the contract even when any information about managerial effort level is not impounded in the stock price.

[1] See Bebchuk and Grinstein (2005).

[2] See also Kim and Suh (1993), Kang and Liu (2010), and Calcagno and Heider (2014) among others.

We examine the effects induced by external factors such as information cost, market liquidity, and the volatility of the firm's fundamental value on the optimal contract. If information cost has an intermediate value, changes in the external factors affect the proportion of informed traders and thus they can generate direct effects and indirect effect via information acquisition on the optimal contact. On the other hand, if information cost is sufficiently high or sufficiently low, all rational traders choose to be informed or uninformed and small changes do not affect the proportion of informed traders. Then the indirect effects disappear and only the direct effects have influence on the optimal contract.

This study interprets the overall effects of the external factors on the contract in terms of informativeness and variability of the stock price. One can expect that the inside owners prefer a higher price informativeness since they want to extract more information about the managerial performance and the risk-averse manager wishes to reduce the uncertainty of his income. The optimal contract may reflect the interests of both sides. We investigate how such features in the stock market are incorporated in the contract.

We show that incorporating the stock price into the contract contributes to the increase of social welfare. Similar to Kang and Liu (2010), we measure social welfare with the sum of ex ante expected utilities of the inside owners, the manager, and the rational traders in the stock market. Since we assume rational expectations, the equilibrium stock price perfectly reflects changes in managerial performance and the ex ante utilities of rational traders are not affected by the contract schemes. The ex ante expected utility of the manger also independent from the contract schemes and is also fixed with his residual value due to individual rationality constraints. Meanwhile, the inside owners can expect a higher payoff by offering the market-based compensation contract. As a result, the increase in the expected utility of the inside owners raises up the social welfare.

This paper is closely related to Holmström and Tirole (1993), who also propose a linear compensation model based on the liquidation value and the stock price. They adopt the asset pricing model of Kyle (1985), in which competitive market makers set stock prices given aggregate order flows. Moreover, they assume that the inside owners choose the precision of his private information which increases in his expenditure. This study complements Holmström and Tirole (1993) along two dimensions. First, by adopting the asset pricing model of Grossman and Stiglitz (1980), we examine how information market equilibrium affects the optimal contract when all rational traders are ex ante identical and the proportion of informed traders is endogenously determined. Facing changes in the external factors, the inside owners take into account the indirect effects induced by information acquisition when they offer the contract. Second, we simplify their compensation scheme. In addition to the fixed wage, Holmström and Tirole (1993) consider performance incentives based on the stock price, the short-term performance, and the liquidation value. We exclude incentives for the short-term performance from our managerial contract for more tractable analysis.[3]

This paper is also related to Kang and Liu (2010) in that the population of informed traders in the stock market is endogenously determined. Similar to this paper, they attempt to divide overall effects of external factors on the optimal contract into direct effects and indirect effect via information acquisition. However, their information acquisition process in the stock market is different from ours. In Kang and Liu (2010), a finite number of rational traders who purchase information and noise traders participate in stock trading and thus there is no asymmetric information between rational traders. Thus, as long as information cost is finite, it is impossible to incorporate the case where there is no informed trader and the stock price does not contain any information of informed traders. In our model, on the contrary, information market equilibrium is determined when the ex ante expected utilities of informed and uninformed traders are equal, and all rational traders trade the stock based on their information. Consequently, our model encompasses the case where all rational traders refuse to purchase information.

The rest of the paper is organized as follows. In Section 2, we introduce the model of principal-agent problem while asymmetric information is present in the stock market. The stock market equilibrium is derived in Section 3. In Section 4, we analyze managerial compensation contract between the owner and the manager. The effect of incorporation the stock price into the contract on social welfare is examines in Section 5. Concluding remarks are given in Section 6. All the proofs are relegated to Appendix.

2 The Model

We consider an economy where there are three dates, indexed by $t = 0, 1, 2$. There is an inside owners who hold a constant large fraction of the firm's equity until the final date ($t = 2$). At the initial date ($t = 0$), a publicly traded firm is established and the firm's inside owners offer a manager a compensation contract, which is based on the firm's stock price and liquidation value. Accepting the contract, the manager makes costly effort. At date 1, the stock is issued and traded. Traders decide whether to purchase information or not before trading and make portfolio choices conditional on their information. At date 2, the firm's liquidation value is determined and the manager and traders are paid.

2.1 Executive Compensation

We consider two performance measures for the manager: the firm's liquidation value v and stock price p. The risk-neutral inside owners offer a linear compensation contract to the manager,

[3] See also Baiman and Verrecchia (1995), Kang and Liu (2010), and Calcagno and Heider (2014) among others.

which is given by

$$I = a_0 + a_1 v + a_2 p,$$

where a_0 represents a fixed wage, and a_1 and a_2 denote the weights on the firm's liquidation value v and stock price p, respectively. If the manager accepts the contract, he chooses effort level e with monetary cost $ke^2/2$ at date 0. We assume that the fundamental value θ of the firm consists of the effort level e chosen by the manager and external source η beyond the manager's control: $\theta = e + \eta$. Random variable η is normally distributed with mean zero and variance σ_η^2. Note that information about managerial effort e is impounded in the fundamental value θ. The resulting liquidation value v of the firm is given by

$$v = \theta + \varepsilon$$

where noise ε has a normal distribution with mean zero and variance σ_ε^2. It is assumed that the volatility of ε is less than that of η: $\sigma_\eta^2 > \sigma_\varepsilon^2$. The inside owners hold a constant fraction δ of shares during all the periods. We assume that the manager is prohibited from trading and has a CARA utility function with risk aversion coefficient $\gamma_m > 0 : u_m(w) = -\exp(-\gamma_m w)$.

2.2 Asset Markets

There are two assets: a risky stock issued by the firm and a risk-free bond. At $t = 1$, the prices of the stock and the bond are given by p and 1, respectively. The bond is assumed to be a numeraire. A rational trader τ invests his initial wealth w_0 in b_τ shares of the bond and x_τ shares of the stock with the budget constraint $b_\tau + px_\tau = w_0$. At $t = 2$, the stock gives payoff $\tilde{v}$ to traders. For simplicity, we assume that traders in the stock market value the firm with gross payoff instead of net payoff deducting managerial compensation.[4]

There is a continuum of rational traders indexed in interval $[0, 1]$. All rational traders are ex ante identically uninformed and have CARA utility function with risk aversion coefficient $\gamma > 0$: $u(w) = -\exp(-\gamma w)$. After the manager makes effort with the level of e, rational traders may purchase information about fundamental value θ which contains information about e. Informed traders observe realization (p, θ) of $(\tilde{p}, \tilde{\theta})$ at cost c, while uninformed traders only observe p when $t = 1$. Thus, at $t = 2$, portfolio (b_τ, x_τ) yields wealth $w_\tau = w_0 - c + (\tilde{v} - p)x_\tau$ if trader τ is informed and yields wealth $w_\tau = w_0 + (\tilde{v} - p)x_\tau$ otherwise. Let $\lambda \in [0, 1]$ denote the fraction of informed traders among rational traders.

There also exist liquidity traders who participate in trading for exogenous reasons. Their demand is denoted by z which is normally distributed with mean zero and variance σ_z^2. We measure market liquidity by σ_z^2 as in Holmström and Tirole (1993). It is assumed that both informed and uninformed traders have rational expectations in that they understand the functional relationship

[4] See also Baiman and Verrecchia (1995), Milbourn (2003), and Kang and Liu (2005).

$\tilde{p}$ between p and (θ, z). Note that informed traders can find the realization of $\tilde{z}$ by observing asset price p and fundamental value θ while uninformed traders cannot extract information about z from p.

The sequence of events is illustrated in Figure 1.

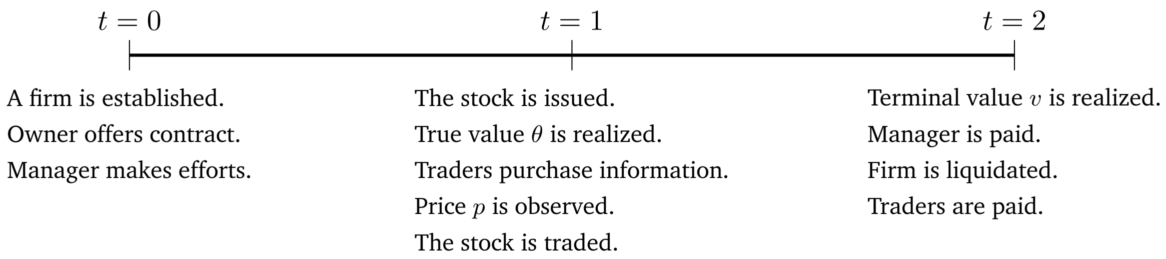

Figure 1: Sequence of Events

3 Asset Market Equilibrium

In the stock market, we adopt the rational expectations equilibrium of Grossman and Stigltiz (1980). First, we derive the equilibrium stock price for a given proportion λ of informed traders and then incorporate information acquisition of rational traders. For the optimal portfolio choice, informed trader i with initial wealth w_0 solves

$$\max_{x_i} \mathbb{E}[-\exp\left(-\gamma[w_0 - c + (\tilde{v} - p)x_i]\right) | (\tilde{p}, \tilde{\theta}) = (p, \theta)]$$

and his demand for the stock is given by

$$x_i(p, \theta) = \frac{\theta - p}{\gamma \sigma_\varepsilon^2}.$$

Similarly, uninformed trader u with initial wealth w_0 solves

$$\max_{x_u} \mathbb{E}[-\exp\left(-\gamma[w_0 + (\tilde{v} - p)x_u]\right) | \tilde{p} = p]$$

and his demand for the stock is given by

$$x_u(p, \tilde{p}) = \frac{\mathbb{E}[\tilde{v}|\tilde{p} = p] - p}{\gamma \text{Var}[\tilde{v}|\tilde{p} = p]}.$$

Equilibrium stock price function $\tilde{p}$ satisfies the market clearing condition: for every $p = \tilde{p}(\theta, z)$,

$$\lambda x_i(p, \theta) + (1 - \lambda) x_u(p, \tilde{p}) + z = 1 - \delta.$$

Following Grossman and Stigltiz (1980), we define a compound signal function $\tilde{s} : (\theta, z) \mapsto s$, which encapsulates θ and z:

$$\tilde{s}(\theta, z) = \begin{cases} \theta - \dfrac{\gamma\sigma_\varepsilon^2}{\lambda}(1-\delta-z) & \text{if } \lambda \in (0,1], \\ -(1-\delta-z) & \text{if } \lambda = 0. \end{cases}$$

Clearly, $\tilde{s}$ is normally distributed with mean $e - \gamma\sigma_\varepsilon^2(1-\delta)/\lambda$ and variance $\sigma_s^2 \equiv \sigma_\eta^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2/\lambda^2$ if $\lambda \in (0,1]$ and is normally distributed with mean $-(1-\delta)$ and variance σ_z^2 if $\lambda = 0$. We define equilibrium price function $P : \mathbb{R} \to \mathbb{R}$ by $P(\tilde{s}(\theta, z)) := \tilde{p}(\theta, z)$ and conjecture that P strictly increases in signal s, which is verified by Proposition 3.1 below.

Proposition 3.1. *Let $\bar{e}$ be the equilibrium effort level of the manager. For a given $\lambda \in [0,1]$, equilibrium price function P is given by*

$$P(s) = \begin{cases} (1-\alpha)\bar{e} + \alpha s & \text{if } \lambda \in (0,1], \\ \bar{e} + \gamma\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)s & \text{if } \lambda = 0, \end{cases}$$

where

$$\alpha = \frac{\lambda^2\sigma_\eta^2 + \lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \lambda\gamma^2\sigma_\varepsilon^4\sigma_z^2}{\lambda^2\sigma_\eta^2 + \lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2}. \tag{3.1}$$

Now we analyze endogenous information acquisition. To do this, we need to compute ex ante expected utilities of would-be informed traders and would be uninformed traders. Let w_i and w_u denote the wealths of informed traders and uninformed traders at $t = 2$. The ex ante expected utility of would-be informed traders and uninformed traders are given by

$$\mathbb{E}[u(w_i)] = e^{\gamma c}u(w_0)\sqrt{\frac{\kappa}{1+\nu}} \quad \text{and} \quad \mathbb{E}[u(w_u)] = u(w_0)\sqrt{\frac{1}{1+\nu}}, \tag{3.2}$$

respectively, where

$$\begin{aligned} \kappa &= \frac{\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2}{\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2}, \\ \nu &= \frac{\gamma^4\sigma_\varepsilon^6\sigma_z^4\left(\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)}{\left(\lambda^2\sigma_\eta^2 + \lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)^2}. \end{aligned}$$

Note that the ex ante expected utilities are independent from the managerial contract and the effort level chosen by the manager. From (3.2), the ratio of expected utility between would-be informed and uninformed traders is given by

$$\varphi(\lambda) := \frac{\mathbb{E}[u(w_i)]}{\mathbb{E}[u(w_u)]} = e^{\gamma c}\sqrt{\kappa}. \tag{3.3}$$

Since

$$\frac{\partial\kappa}{\partial\lambda} = \frac{2\lambda\gamma^2\sigma_\eta^4\sigma_\varepsilon^2\sigma_z^2}{\left(\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)^2} > 0, \tag{3.4}$$

ratio function $\varphi : [0,1] \to \mathbb{R}$ is a strictly increasing function of λ. Thus as λ increases, relative expected utility of informed traders with respect to uninformed ones decreases.[5] This implies that as more traders acquire information about θ, traders have less incentives to purchase it and information acquisition exhibits strategic substitutability. If $\varphi(0) \geq 1$, no traders become informed, i.e., $\lambda = 0$ and if $\varphi(1) \leq 1$, all traders become uninformed, i.e., $\lambda = 1$. Otherwise, for $\lambda \in (0,1)$ to be an information market equilibrium, it must be the case $\varphi(\lambda) = 1$. Then it is straightforward to obtain Proposition 3.2.

Proposition 3.2. *The following hold.*

1. *If information cost c is sufficiently low such that*

$$c \leq \frac{1}{2\gamma} \ln\left(1 + \frac{\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2}{\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2}\right) \equiv \underline{c}, \tag{3.5}$$

all traders become informed, i.e., $\lambda = 1$.

2. *If information cost c is sufficiently high such that*

$$c \geq \frac{1}{2\gamma} \ln\left(1 + \frac{\sigma_\eta^2}{\sigma_\varepsilon^2}\right) \equiv \bar{c}, \tag{3.6}$$

all traders remain uninformed, i.e., $\lambda = 0$.

3. *If information cost c takes intermediate value such that*

$$\underline{c} < c < \bar{c}, \tag{3.7}$$

a fraction of traders become informed, i.e., $\lambda \in (0,1)$.

Suppose that information market has an interior equilibrium, i.e., $\lambda \in (0,1)$. In the equilibrium, φ should be fixed with 1. Thus an increase of c should decrease κ. This implies that if information cost c increases, proportion λ of informed traders decreases by (3.4). Moreover, if $e^{\gamma c}$ is fixed, so is κ. Thus, for a fixed c, changes in σ_η^2 and σ_z^2 do not move the value of κ. Note that κ can be rewritten as

$$\kappa = \left(1 + \frac{\gamma^2\sigma_\varepsilon^2\sigma_z^2}{\zeta_n}\right)^{-1} = \left(1 + \frac{\gamma^2\sigma_\eta^2\sigma_\varepsilon^2}{\zeta_z\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^4}\right)^{-1}$$

where

$$\zeta_\eta \equiv \lambda^2 + \frac{\gamma^2\sigma_\varepsilon^4\sigma_z^2}{\sigma_\eta^2} \tag{3.8}$$

and

$$\zeta_z \equiv \frac{\lambda^2}{\sigma_z^2}. \tag{3.9}$$

[5]Recall that we assume a negative utility function of rational traders.

In equilibrium $\lambda \in (0,1)$, ζ_η is fixed when σ_η^2 moves, and ζ_z is fixed when σ_z^2 moves. Thus an increase in volatility σ_η^2 of the firm's fundamental value or market liquidity σ_z^2 induces more rational traders to be informed. We summarize the features of information market equilibrium with $\lambda \in (0,1)$ in Proposition 3.3.

Proposition 3.3. *Suppose that information cost c satisfies (3.7), then the following hold.*

1. *If information cost c increases such that (3.7) holds, proportion λ of informed traders decreases.*
2. *If volatility σ_η^2 of the firm's true value or market liquidity σ_z^2 increases, λ increases.*

Before we discuss the optimal contract in Section 4, we need to characterize the price informativeness and the price volatility. Following Grossman and Stiglitz (1980), we define price informativeness by squared correlation coefficient ρ^2 between fundamental value θ and the stock price P:

$$\rho^2 = \frac{\lambda^2\sigma_\eta^2}{\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2}, \quad \lambda \in [0,1]. \tag{3.10}$$

Clearly, ρ^2 is the increasing function of λ. Since fundamental value θ impounds information about managerial effort e, the inside owners extract information about effort level e from observed prices. Note that if all the rational traders are uninformed, stock prices do not contain any information about the fundamental value, and even when all rational traders are informed, stock prices can transmit only partial information about θ due to noise traders.

From the first claim of Proposition 3.3 and (3.10), we know that as information cost increases, the stock price becomes less informative. If the volatility σ_η^2 of the firm's fundamental value increases, both the direct effect and indirect effect via information acquisition increase ρ^2. Therefore, as the firm's fundamental value becomes more unpredictable, stock prices convey more information about θ. On the other hand, a change in market liquidity σ_z^2 does not have influence on ρ^2.[6] An increase of market liquidity σ_z^2 directly decreases ρ^2, but indirectly increases it via changes in λ, and both effects are exactly offset. We summarize theses features of the price informativeness at equilibrium $\lambda \in (0,1)$ in Proposition 3.4.

Proposition 3.4. *Suppose that information cost c satisfies (3.7). The following hold.*

1. *As c increases such that (3.7) holds, price informativeness ρ^2 decreases.*
2. *If σ_η^2 increases, ρ^2 increases.*
3. *A change in market liquidity σ_z^2 does not affect ρ^2.*

[6]See also Grossman and Stiglitz (1980).

From (3.1), we obtain the volatility of the stock price given by

$$\sigma_P^2 = \frac{\left(\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)\left(\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)^2}{\left(\lambda^2\sigma_\eta^2 + \lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)^2}, \quad \lambda \in [0,1]. \tag{3.11}$$

Then we have the following proposition.

Proposition 3.5. *Suppose that information cost c satisfies (3.7). The following hold.*

1. *If information cost c increases where (3.7) holds, price volatility σ_P^2 increases if rational traders are sufficiently averse to risk such that*

$$\gamma > \frac{1}{\sqrt{\sigma_\eta\sigma_z}}. \tag{3.12}$$

2. *If market liquidity σ_z^2 increases, price volatility σ_P^2 increases.*

Price volatility σ_P^2 does not always decrease in proportion λ of informed traders.[7] This means that an increase of price informativeness ρ^2 does not always ensure the decrease the precision of the stock price (the reciprocal of price volatility). However, if rational traders are sufficiently averse to risk, σ_P^2 decreases in λ. Thus the stock price becomes more volatile as information cost increases. On the other hand, an increase of market liquidity σ_z^2 always increases price volatility σ_P^2 due to endogenous information acquisition.[8]

4 Equilibrium Contract

This section is devoted to analyzing the optimal compensation contract. To maximize income, the inside owners collectively design the contract, which will be accepted by the manager. First, we examine effects induced by incorporating the stock price into the contract. To do this, we derive the equilibrium contracts with and without the stock price and observe differences in effort levels taken by the manager and the liquidation values. Second, we investigate how external factors such as information cost, market liquidity, and volatility of the firm's fundamental affect the market-based contract. Since endogenous information acquisition is assumed in the stock market, changes in external parameters can generate indirect effects via a change in the proportion of informed traders as well as direct effects. We attempt to explain the overall effects in terms of price informativeness and price volatility.

[7]See also Black and Tonks (1992). They analyze effects of λ on price volatility in the framework of Grossman and Stiglitz (1980) excluding information acquisition.

[8]Note that in the model without endogenous information acquisition, σ_z^2 does not always increase σ_P^2.

4.1 Market-Based Compensation Contract

Now we assume that the reservation value of the manager equals one. The inside owners maximize his expected utility under the conditions of individual rationality and incentive compatibility by solving the following problem:

$$\begin{aligned}\max_{a_0,a_1,a_2,e}\ & \mathbb{E}[\tilde{v}-I] \\ s.t.\ & \mathbb{E}[I]-\frac{\gamma_m}{2}\text{Var}[I]-\frac{1}{2}ke^2\geq 0, \\ & e=\operatorname*{argmax}_{\bar{e}}\mathbb{E}[I]-\frac{\gamma_m}{2}\text{Var}[I]-\frac{1}{2}k\bar{e}^2.\end{aligned}$$

For every $c>0$, we obtain the equilibrium contract as follows.

Theorem 4.1. *In equilibrium, for every $c>0$ (and thus for every $\lambda\in[0,1]$), the optimal compensation contract is given by*

$$\begin{aligned} a_0 &= -\frac{y+2(1-\delta)\left((1-\alpha)\sigma_\eta^2+\sigma_\varepsilon^2\right)kx\lambda\gamma\sigma_\varepsilon^2}{2kx^2}, \\ a_1 &= \frac{(\lambda\sigma_\eta^2+\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2+\gamma^2\sigma_\varepsilon^4\sigma_z^2)\gamma^2\sigma_\varepsilon^2\sigma_z^2}{x}>0, \\ a_2 &= \frac{\lambda^2\sigma_\eta^2+\lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2+\gamma^2\sigma_\varepsilon^4\sigma_z^2}{x}>0. \end{aligned}$$

and the equilibrium effort level is

$$\bar{e}=\frac{a_1+a_2}{k}=\frac{\left[(1+\gamma^2\sigma_z^2(\sigma_\eta^2+\sigma_\varepsilon^2))\sigma_\varepsilon^2+2\lambda\sigma_\eta^2\right]\gamma^2\sigma_\varepsilon^2\sigma_z^2+\lambda^2\sigma_\eta^2}{kx} \tag{4.1}$$

where

$$\begin{aligned} x &= (1+k\gamma_m\sigma_\eta^2)\lambda^2\sigma_\eta^2 \\ &\quad +\left[2(1+k\gamma_m\sigma_\eta^2)\lambda\sigma_\eta^2+(1+k\gamma_m(\sigma_\eta^2+\sigma_\varepsilon^2))(\sigma_\eta^2+\sigma_\varepsilon^2)\gamma^2\sigma_\varepsilon^2\sigma_z^2+(1+2k\lambda\gamma_m\sigma_\eta^2)\sigma_\varepsilon^2\right]\gamma^2\sigma_\varepsilon^2\sigma_z^2 \\ y &= \left[\left((1-\alpha)^2\sigma_\eta^2+(1-k\alpha^2\gamma_m\sigma_\eta^2)\sigma_\varepsilon^2\right)\lambda^2+\alpha^2\gamma^2\sigma_\varepsilon^4\sigma_z^2\left(1-k\gamma_m(\sigma_\eta^2+\sigma_\varepsilon^2)\right)\right] \\ &\quad \times\left[\left((1-\alpha)^2\sigma_\eta^2+\sigma_\varepsilon^2\right)\lambda^2+\alpha^2\gamma^2\sigma_\varepsilon^4\sigma_z^2\right] \end{aligned}$$

Note that both weigh a_1 on the liquidation value and weigh a_2 on the stock price have positive values as in Holmström and Tirole (1993).[9] This implies that the inside owners can expect to be better off by incorporating the stock price into the contract. Otherwise, they would offer a contract with $a_2=0$. In our model, the manager has an incentive to raise both the liquidation value and the stock price to increase his payoff. On the other hand, Baiman and Verrecchia (1995) and Kang and Liu (2010) show that the managerial compensation decreases in the liquidation value.

[9] See also Kim and Seo (1993) and Calcagno and Heider (2014) among others.

They support this idea by arguing that since stock prices impound extra information in addition to managerial efforts, managerial compensation should be adjusted by the negative weight on the liquidation value.

4.2 Comparison with Non Market-Based Compensation

As a benchmark case, we suppose that the owners offer a compensation contract I' which adopts only the liquidation value v as a performance measure:

$$I' = a_0' + a_1' v.$$

Similar to our model, the inside owners solve the following problem considering individual rationality and incentive compatibility:

$$\begin{aligned} &\max_{a_0', a_1', e} \mathbb{E}[\hat{v} - I'] \\ &s.t.\ \ \mathbb{E}[I'] - \frac{\gamma_m}{2}\text{Var}[I'] - \frac{1}{2}ke^2 \geq 0, \\ &\qquad e = \operatorname*{argmax}_{\hat{e}} \mathbb{E}[I'] - \frac{\gamma_m}{2}\text{Var}[I'] - \frac{1}{2}k\hat{e}^2. \end{aligned}$$

Then we obtain the following equilibrium contract.

Proposition 4.1. *In equilibrium, the optimal compensation contract is given by*

$$\begin{aligned} a_0' &= \frac{-1 + k\gamma_m\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)}{2k\left(1 + k\tau\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)\right)^2} \\ a_1' &= \frac{1}{1 + k\gamma_m\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)}, \end{aligned}$$

and the equilibrium effort level is

$$\hat{e} = \frac{a_1'}{k} = \frac{1}{k\left(1 + k\gamma_m\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)\right)}. \tag{4.2}$$

According to Theorem 4.1 and Proposition 4.1, weights a_1, a_2, and a_1' decrease in the manager's degree γ_m of risk aversion and his cost per effort level. Irrespective of contract schemes, as the manager cares more about the uncertainty of his payoff or as his effort becomes more inefficient, the inside owners have an incentive to reduce all the weights on performance measures.

From

$$\mathbb{E}\left[\tilde{v} - I\right] - \mathbb{E}\left[\hat{v} - I'\right] = \frac{\left(\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)\gamma_m\sigma_\varepsilon^2}{2\left(1 + k\gamma_m\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)\right)x} > 0, \tag{4.3}$$

we confirm that the inside owners can obtain a higher profit by adopting the market-based compensation as long as the manager is risk-averse. Note that (4.3) holds even when $\lambda = 0$. This implies that, although the inside owners cannot extract any additional information about managerial effort from the stock price, they still can be better off by offering the market-based compensation contract to the risk-averse manager. The following proposition specifically shows how the higher profit of the inside owners is generated.

Proposition 4.2. *The following hold.*

1. *Compared to the benchmark, the manager chooses a higher effort level (i.e., $\bar{e} > \hat{e}$) and thus the expected fundamental value of the firm increases.*
2. *Compared to the benchmark, the expected managerial compensation increases (i.e., $I > I'$).*

The first claim of Proposition 4.2 shows that moral hazard problem between the inside owners and the manager is relived by incorporating the stock price into the contract. As long as the manager is risk-averse, he chooses a higher effort level under the market-based compensation than otherwise.[10] It is easy to believe that the increase of the effort level taken by the manager is due to the role of the stock price as a monitor of managerial performance.[11] However, even though there is no informed trader and the stock price transmits no information about the effort (i.e., $\rho^2 = 0$), the manager still chooses a higher effort level compared to the benchmark. Note that

$$a_1 - a_1' = -\frac{\left(1 + k\gamma_m\sigma_\eta^2\right)\lambda^2\sigma_\eta^2 + \left[\left(1 + k\gamma_m\sigma_\eta^2\right)\lambda\sigma_\eta^2 + \left(1 + k\lambda\gamma_m\sigma_\eta^2\right)\sigma_\varepsilon^2\right]\gamma^2\sigma_\varepsilon^2\sigma_z^2}{\left(1 + k\gamma_m\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)\right)x} < 0.$$

Thus, in the optimal contract, the weight on the liquidation value is reduced while that on the stock price increases compared to the benchmark. The first claim of Proposition 4.2 implies that the former effect is dominated by the latter one and thus the manager takes the higher effort.[12] Clearly, the higher effort level taken by the manager raises up the firm's expected fundamental value.

From the second claim, we know that the manager obtains a higher payoff under the market-based compensation contract than in the benchmark. This is related to the combined effect of the payoff volatility and effort cost. Irrespective of contract schemes, the expected utility of the manager is fixed at his residual value due to the individual rationality constraint in the inside owners' problems. Thus his expected payoff increases in the payoff volatility and in his effort cost. As the stock price is incorporated into the contract, the payoff volatility may increase or decrease while the effort cost always increases. However, even when the payoff volatility decreases, the

[10]Note that if the manager is risk-neutral, he chooses the same effort level irrespective of the contract scheme. See the proof for the first claim of Proposition 4.2.

[11]See Holmström and Tirole (1993) among others.

[12]Recall equilibrium effort levels in (4.1) and (4.2).

increase of the effort cost dominates the decrease, and thus expected payoff the manager always increases.

By offering the market-based compensation contract, the inside owners can expect a higher liquidation value of the firm by making the manager choose a higher effort level. Meanwhile, they anticipate an increased expense of the managerial compensation. However, Theorem 4.1 implies that the market-base contract induces a more increase of the liquidation value than that of the payoff expense and this leads to the increase of the expected inside owners' payoff.

4.3 Comparative Statics

In this subsection, we examine the effects of external factors such as information cost c, market liquidity σ_z^2, and the volatility σ_η^2 of the firm's fundamental value on the optimal contract. We interpret the overall effects of the external factors on the optimal contract in terms of price informativeness and price volatility. The inside owners may prefer to extract more information about the managerial performance from the stock price while the risk-averse manager may want to reduce uncertainty of his income. The optimal contract reflects such interests of both sides. We investigate how such features in the stock market are incorporated in the contract.

Let us define the relative weight on the liquidation (with respect to the stock price) as

$$\xi \equiv \frac{a_1}{a_2} = \frac{\left(\lambda\sigma_\eta^2 + \gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)\gamma^2\sigma_\varepsilon^2\sigma_z^2}{\lambda^2\sigma_\eta^2 + \lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2}. \tag{4.4}$$

Since $a_1 > 0$ and $a_2 > 0$, as ξ increases, the optimal contract gives more relative weight to liquidation value v, and as ξ decreases, the contract gives more relative weight to stock price p.

First, we consider when information cost c has intermediate value such that (3.7) holds. In the stock market, informed and uninformed traders coexist and changes in the external factors move the proportion λ of informed traders. Thus, the changes can generate indirect effects via information acquisition as well as direct effects on the optimal contract.

Proposition 4.3. *Suppose that (3.7) and (3.12) hold. As information cost c increases where (3.7) holds, ξ increases.*

Changes in information cost c indirectly affect ξ without direct effects. As we have shown in the first claims of Proposition 3.4 and Proposition 3.5, an increase of information cost in the stock market induces a decrease of price informativeness and an increase of price volatility when traders are sufficiently risk-averse. Then the inside owners can extract only a small amount of information about the managerial performance from the price while the risk-averse manager wants to reduce the uncertainty of his payoff. Consequently, when traders in the stock market have a sufficiently high risk aversion, as information cost increases, the liquidation value becomes relatively more important than stock prices in the optimal contract.

Proposition 4.4. *Suppose that (3.7) holds. As market liquidity σ_z^2 increases, ξ increases.*

The effect of market liquidity σ_z^2 on the relative weight ξ is divided into two effects:

$$\frac{d\xi}{d\sigma_z^2} = \underbrace{\frac{\partial \xi}{\partial \sigma_z^2}}_{\text{direct effect of } \sigma_z^2} + \underbrace{\frac{\partial \xi}{\partial \lambda}\frac{\partial \lambda}{\partial \sigma_z^2}}_{\text{indirect effect of } \sigma_z^2}.$$

The direct effect always increases in σ_z^2, but the indirect effect induced by information acquisition on ξ is unclear. However, the direct effect dominates the indirect one when both effects oppositely respond to σ_z^2. Therefore the increase of the market liquidity always increases the relative weight ξ.

The intuition is as follow. According to the third claim of Proposition 3.4, a change in market liquidity σ_z^2 does not affect price informativeness ρ^2. Thus, although market liquidity increases, the inside owners can extract only the same level of information about managerial effort by observing the stock price. Meanwhile, an increase in σ_z^2 always increases price volatility σ_P^2 by the second claim of Proposition 3.5. Since the stock price becomes more unpredictable while it gives the same level of information, the optimal contract gives an increased weight on the firm's fundamental value relative to the stock price.

Proposition 4.5. *Suppose that (3.7) and (3.12) hold. As the volatility σ_η^2 of the firm's fundamental value increases, ξ decreases.*

Similar to the case where σ_z^2 changes, the effect of market liquidity σ_η^2 on the relative weight ξ is divided by two effects:

$$\frac{d\xi}{d\sigma_\eta^2} = \underbrace{\frac{\partial \xi}{\partial \sigma_\eta^2}}_{\text{direct effect of } \sigma_\eta^2} + \underbrace{\frac{\partial \xi}{\partial \lambda}\frac{\partial \lambda}{\partial \sigma_\eta^2}}_{\text{indirect effect of } \sigma_\eta^2}.$$

If the traders are sufficiently averse to risk, the direct effect increases in σ_η^2, but the indirect effect via information acquisition decreases in σ_η^2. However, the former effect is dominated by the latter one and thus the overall effect decreases ξ.

An increase in the volatility of the firm's fundamental value leads to a higher price informativeness as we have seen in the second claim of Proposition 3.4 and it makes the liquidation value more volatile. Then both inside owners and the manager may prefer to give a higher weight on the stock price. On the other hand, the effect of σ_η^2 on price volatility σ_P^2 is unclear. From Proposition 4.5, however, we find that the inside owners and the manager care more about the combined effect of the price informativeness and the volatility of the liquidation value than the price volatility when they offer the contract.

Now we consider extreme cases where information cost is sufficiently low and is sufficiently high. Suppose that (3.5) or (3.6) holds. According to Proposition 3.2, all rational traders purchase information about the firm's fundamental value in the former case and no one becomes informed in the latter one. Then the proportion λ of informed traders is fixed with 1 or 0 although the external factors slightly change. Thus the optimal contract is directly affected by external factors without indirect effects. Clearly, if information cost changes such that (3.5) or (3.6) holds, the optimal contract is independent from the movements of information cost.

Proposition 4.6. *Suppose that information cost c is sufficiently low or sufficiently high such that (3.5) or (3.6) holds. As market liquidity σ_z^2 increases, ξ increases.*

As the market liquidity increases, the direct effect increases relative weight ξ as in the case where $\lambda \in (0,1)$, while there is no indirect effect. This leads to the increase of ξ. The intuition is as follows. According to (3.10), when $\lambda = 0$, price informativeness ρ^2 is also zero, and when $\lambda = 1$, an increase of market liquidity σ_z^2 decreases ρ^2. On the other hand, price volatility increases in σ_z^2 both when $\lambda = 1$ and when $\lambda = 0$ since

$$\left.\frac{\partial \sigma_P^2}{\partial \sigma_z^2}\right|_{\lambda=1} = \gamma^2 \sigma_\varepsilon^4 > 0, \quad \text{and} \quad \left.\frac{\partial \sigma_P^2}{\partial \sigma_z^2}\right|_{\lambda=0} = \gamma^2 \left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)^2 > 0.$$

In the extreme cases, an increase in the market liquidity has no effect on the price informativeness or decreases it while always makes the stock price more uncertain. Consequently, the inside owners and the manager agree with the contract which gives a more relative weight on the liquidation value.

Proposition 4.7. *The following hold.*

1. *If (3.5) holds, volatility σ_η^2 of the firm's fundamental value has no effect on ξ.*
2. *If (3.6) holds, as σ_η^2 increases, ξ increases.*

If all rational traders are informed, the direct effect induced by changes in σ_η^2 moves weights a_1 and a_2 exactly the same amount since

$$\left.\frac{\partial a_1}{\partial \sigma_\eta^2}\right|_{\lambda=1} = \left.\frac{\partial a_2}{\partial \sigma_\eta^2}\right|_{\lambda=1} = -\frac{\left(1 + \gamma^2 \sigma_\varepsilon^2 \sigma_z^2\right) k\gamma_m}{\left[1 + \left(1 + k\gamma_m \left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)\right) \gamma^2 \sigma_\varepsilon^2 \sigma_z^2 + k\gamma_m \sigma_\eta^2\right]^2}.$$

Therefore, relative weight ξ remains unchanged. For $\lambda = 1$, as σ_η^2 increases, the stock price becomes more volatile since

$$\left.\frac{\partial \sigma_P^2}{\partial \sigma_\eta^2}\right|_{\lambda=1} = 1 > 0,$$

which can give incentive to increase the weight on the liquidation value. Meanwhile, the price informativeness increases by (3.10), and clearly, volatility $\left(\sigma_\eta^2 + \sigma_z^2\right)$ of the liquidation value also increases. Such changes in price informativeness and the volatility of the liquidation value can increase the weight on the stock price. The first claim of Proposition 4.7 implies that the first effect is offset by the last two effects.

Suppose that λ is fixed with 0. An increase in σ_η^2 directly increases relative weight ξ as in the case where $\lambda \in (0,1)$. However, there is no indirect effect to reduce the direct effect. Thus ξ increases in σ_η^2 on the contrary to the case where information cost has intermediate values. Note that as σ_η^2 increases, the volatility of the stock price increases since

$$\left.\frac{\partial \sigma_P^2}{\partial \sigma_\eta^2}\right|_{\lambda=0} = 2\gamma^2\sigma_z^2\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right) > 0,$$

and so does volatility of the liquidation value, while the price informativeness is fixed with zero. From the second claim of Proposition 4.7, we see that the risk-averse manager cares more about the uncertainty of the stock price than that of the liquidation value.

5 Welfare Analysis

In this section, we examine the effect of market-based compensation on social welfare. Similar to Kang and Liu (2010), we measure social welfare with the sum of ex ante expected utilities of inside owners, the manager and the rational traders.[13] In the benchmark, the social welfare is given by

$$\mathsf{SW}' = \hat{e} - \mathbb{E}[I'] + \mathbb{E}\left[u_m\left(I' - \frac{1}{2}k\hat{e}^2\right)\right] + \lambda\mathbb{E}[u(w_i)] + (1-\lambda)\mathbb{E}[u(w_u)]$$

and in our model, it is given by

$$\mathsf{SW} = \bar{e} - \mathbb{E}[I] + \mathbb{E}\left[u_m\left(I - \frac{1}{2}k\bar{e}^2\right)\right] + \lambda\mathbb{E}[u(w_i)] + (1-\lambda)\mathbb{E}[u(w_u)].$$

From (3.2), we know that ex ante expected utilities of rational traders are independent from the optimal contract. Furthermore, their expected utilities are not affected by the firm's fundamental value which contains information about managerial effort. It is because the stock price exactly reflects changes in the fundamental value. The ex ante expected utility of the manager is also independent from the compensation scheme since the individual rationality constraints always fix the manager's expected utility with 1. As we have seen in Subsection 4.1, the inside owners can expect a higher profit with market-based compensation than without it since the increase in expected liquidation value dominates that in managerial compensation. Consequently, incorporating the stock price into the managerial compensation contract raises up social welfare.

[13]We ignore the expected utility of noise traders since they are not utility maximizers.

6 Concluding Remarks

The paper investigates how a publicly traded firm's liquidation value and stock price are used in the executive compensation contract when information acquisition in the stock market is endogenized. To do this, we incorporate a standard principal-agent problem into Grossman and Stiglitz's (1980) asset market model. We show that the inside owners can increase their payoff by offering a market-based compensation contract. Compared to the contract excluding the stock price, the manager chooses a higher effort levels under the market-based compensation contract, and this leads to the increases of both the expected liquidation value and the expected expense of managerial compensation. However, since the former increase dominates the latter one, the inside owners can forecast the additional profit. It is easy to believe that the monitoring role of the stock price makes the inside owners offer the market-based compensation contract as in Holmström and Tirole (1993). In our model, however, even when the stock price does not transmit any information about the managerial effort level, the inside owners still have incentive to choose the market-based compensation contract.

We also examine how external factors such as information cost, market liquidity, and volatility of the firm's fundamental affect the optimal contract. If information cost in the stock market exhibits an intermediate value, the optimal contract is affected by both indirect effects via information acquisition and direct effects. On the other hand, if information cost is sufficiently high or low, they only directly affect on the contract. Since the inside owners prefer to extract more information about the managerial performance from the stock price while the manager wants to reduce the variability of his income, we interpret the effects of the external factors on the contract by using price informativeness and price volatility.

Finally, this study shows that incorporating the stock price into the contract contributes to the increase of social welfare. Since the stock price perfectly reflects the managerial performance, the ex ante utilities of rational traders are not affected by the contract schemes, and moreover, the individual rationality constraint fixes the manager's expected wealth with his residual value. Meanwhile, the inside owners can increase their expected utility by the market-based compensation contract.

In the future research, we can consider ambiguous information about the firm's future value. Practically, there are plenty of information in financial markets, and the inside owners and external traders may have difficulty in estimating the quality of observed information, while the manager has relatively precise information. Thus, we can construct a model in which the inside owners and the manager have multiple beliefs about the firm's fundamental value.

Appendix

Proof of Proposition 3.1. We conjecture that P is strictly increasing in s. Suppose $\lambda = 0$. Then the market clearing condition becomes $x_u = z$. Since v and s are independent, we obtain

$$P(s) = e + (\sigma_\eta^2 + \sigma_\varepsilon^2)\gamma s.$$

Now suppose $\lambda \in (0, 1]$. Then information generated by equilibrium price $P(s)$ is equivalent to that by $\tilde{s}$. Since $\tilde{s}$ and $\tilde{v}$ are normally distributed, we have

$$\mathbb{E}[\tilde{v}|P(s) = p] = \mathbb{E}[\tilde{v}|\tilde{s} = s] = \frac{\gamma^2\sigma_\varepsilon^4\sigma_z^2 e + \lambda^2\sigma_\eta^2 s}{\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2},$$
$$\text{Var}[\tilde{v}|P(s) = p] = \text{Var}[\tilde{v}|\tilde{s} = s] = \frac{\sigma_\varepsilon^2(\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2)}{\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2}.$$

From the market clearing condition, we obtain

$$P(s) = (1 - \alpha)e + \alpha s$$

where

$$\alpha = \frac{\lambda^2\sigma_\eta^2 + \lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \lambda\gamma^2\sigma_\varepsilon^4\sigma_z^2}{\lambda^2\sigma_\eta^2 + \lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2}.$$ ■

Proof of Proposition 3.4 Suppose that (3.7) holds.

1. From (3.10), it is clear that ρ^2 increases in λ and is not directly affected by c. Furthermore, λ decreases in c according to the first claim of Proposition 3.3. Therefore, ρ^2 decreases in c.

2. Note that ζ_η in (3.8) remains constant. We have

$$\frac{d\rho^2}{d\sigma_\eta^2} = \frac{\partial\rho^2}{\partial\sigma_\eta^2} + \frac{\partial\rho^2}{\partial\lambda}\frac{\partial\lambda}{\partial\sigma_\eta^2}$$

where

$$\frac{\partial\rho^2}{\partial\sigma_\eta^2} = \frac{\lambda^2\gamma^2\sigma_\varepsilon^4\sigma_z^2}{\left(\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)^2} > 0,$$
$$\frac{\partial\rho^2}{\partial\lambda}\frac{\partial\lambda}{\partial\sigma_\eta^2} = \frac{2\lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^4\sigma_z^2}{\left(\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)^2}\frac{\gamma^2\sigma_\varepsilon^4\sigma_z^2}{2\sigma_\eta^2\sqrt{\zeta_\eta\sigma_\eta^4 - \gamma^2\sigma_\eta^3\sigma_\varepsilon^4\sigma_z^2}} > 0.$$

   Thus ρ^2 increases in σ_η^2.

3. Note that ζ_z in (3.9) remains constant. We have

$$\frac{d\rho^2}{d\sigma_z^2} = \frac{\partial\rho^2}{\partial\sigma_z^2} + \frac{\partial\rho^2}{\partial\lambda}\frac{\partial\lambda}{\partial\sigma_z^2}$$

where

$$\begin{aligned}\frac{\partial\rho^2}{\partial\sigma_z^2} &= -\frac{\lambda^2\gamma^2\sigma_\eta^2\sigma_\varepsilon^4}{\left(\lambda^2\sigma_\eta^2+\gamma^2\sigma_\varepsilon^4\sigma_z^2\right)^2}\\ \frac{\partial\rho^2}{\partial\lambda}\frac{\partial\lambda}{\partial\sigma_z^2} &= \frac{2\lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^4\sigma_z^2}{\left(\lambda^2\sigma_\eta^2+\gamma^2\sigma_\varepsilon^4\sigma_z^2\right)^2}\frac{\sqrt{\zeta_z}}{2\sqrt{\sigma_z^2}}.\end{aligned}$$

Plugging (3.9) into $\partial\rho^2/\partial\sigma_z^2$ and $(\partial\rho^2/\partial\lambda)(\partial\lambda/\partial\sigma_z^2)$, we obtain $d\rho^2/d\sigma_z^2 = 0$. ■

PROOF OF PROPOSITION 3.5 Suppose that (3.7) holds.

1. We have

$$\begin{aligned}\frac{\partial\sigma_P^2}{\partial\lambda} &= -\frac{2\gamma^2\sigma_\eta^2\sigma_\varepsilon^4\sigma_z^2\left(\lambda\sigma_\eta^2+\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2+\gamma^2\sigma_\varepsilon^4\sigma_z^2\right)}{\left(\lambda^2\sigma_\eta^2+\lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2+\gamma^2\sigma_\varepsilon^4\sigma_z^2\right)^3}\\ &\quad\times\left(\lambda^3\sigma_\eta^2+(\lambda+\gamma^2\sigma_\eta^2\sigma_z^2)\gamma^2\sigma_\varepsilon^4\sigma_z^2+\lambda\sigma_\eta^2(\gamma^2\sigma_\varepsilon^2\sigma_z^2-\lambda)+\gamma^2\sigma_\varepsilon^4\sigma_z^2(\gamma^2\sigma_\varepsilon^2\sigma_z^2-1)\right),\end{aligned}$$

   which decreases in λ when (3.12) holds. Then σ_P^2 increases in c since λ decreases in c by the first claim of Proposition 3.3.

2. Note that ζ_z remains constant regardless of changes of σ_z^2. Substituting λ into $\sqrt{\zeta_z\sigma_z^2}$ in (3.11), we have

$$\frac{\partial\sigma_P^2}{\partial\sigma_z^2} = \frac{\gamma^2\sigma_\varepsilon^4\left(\gamma^2\sigma_\varepsilon^4+\zeta_z\sigma_\eta^2\right)\left(\zeta_z\sigma_\eta^2+\gamma^2\sigma_\varepsilon^2(\sigma_\eta^2+\sigma_\varepsilon^2)\right)\left(\gamma^2\sigma_\varepsilon^2\sqrt{\zeta_z\sigma_z^2}(\sigma_\eta^2+\sigma_\varepsilon^2)+\zeta_z\sigma_\eta^2\right)}{\sqrt{\zeta_z\sigma_z^2}\left(\zeta_z\sigma_\eta^2+\gamma^2\sigma_\varepsilon^2\left(\sqrt{\zeta_z\sigma_z^2}\sigma_\eta^2+\sigma_\varepsilon^2\right)\right)^3} > 0. \quad ■$$

PROOF OF THEOREM 4.1 We consider two cases: (i) $c \geq \bar{c}$ and (ii) $c < \bar{c}$.

(i) From the second claim of Proposition 3.2, we know $\lambda = 0$. By Proposition 3.1, we have

$$\begin{aligned}\mathbb{E}[I] &= a_0+a_1e+\left(e-(1-\delta)(\sigma_\eta^2+\sigma_\varepsilon^2)\gamma\right)\\ \text{Var}[I] &= a_1^2(\sigma_\eta^2+\sigma_\varepsilon^2)+a_2^2\gamma^2\sigma_z^2\left(\sigma_\eta^2+\sigma_\varepsilon^2\right)^2.\end{aligned}$$

Note that there is no correlation between v and p when $\lambda = 0$. The first-order condition for the manager's incentive compatibility implies

$$\bar{e} = \frac{a_1+a_2}{k}.$$

Since the individual rationality condition for the manger holds as equality in equilibrium, we have

$$\mathbb{E}[v-I] = \frac{a_1+a_2}{k} - \frac{\gamma_m}{2}\left[a_1^2(\sigma_\eta^2+\sigma_\varepsilon^2)+a_2^2\gamma^2\sigma_z^2\left(\sigma_\eta^2+\sigma_\varepsilon^2\right)^2\right] - \frac{(a_1+a_2)^2}{2k}.$$

The first-order conditions for the owner are given by

$$\begin{aligned}
\frac{1-\left(1+k\gamma_m\left(\sigma_\eta^2+\sigma_\varepsilon^2\right)\right)a_1-a_2}{k} &= 0, \\
\frac{1-a_1-\left(1+k\gamma_m\gamma^2\sigma_z^2(\sigma_\eta^2+\sigma_\varepsilon^2)^2\right)a_2}{k} &= 0,
\end{aligned}$$

which implies

$$\begin{aligned}
a_1 &= \frac{\gamma^2\sigma_z^2\left(\sigma_\eta^2+\sigma_\varepsilon^2\right)}{1+\left(1+k\gamma_m(\sigma_\eta^2+\sigma_\varepsilon^2)\right)(\sigma_\eta^2+\sigma_\varepsilon^2)\gamma^2\sigma_z^2} \equiv A_1, \\
a_2 &= \frac{1}{1+\left(1+k\gamma_m\left(\sigma_\eta^2+\sigma_\varepsilon^2\right)\right)\left(\sigma_\eta^2+\sigma_\varepsilon^2\right)\gamma^2\sigma_z^2} \equiv A_2.
\end{aligned}$$

From the individual rationality condition, we obtain

$$a_0 = -\frac{y_0-2(1-\delta)\left(\sigma_\eta^2+\sigma_\varepsilon^2\right)^3k^2\gamma_m\gamma^3\sigma_z^2}{2\left[1+\left(1+k\gamma_m\left(\sigma_\eta^2+\sigma_\varepsilon^2\right)\right)\left(\sigma_\eta^2+\sigma_\varepsilon^2\right)\gamma^2\sigma_z^2\right]^2k} \equiv A_0.$$

(ii) From the first and third claims of Proposition 3.2, we know $\lambda \in (0,1]$. From Proposition 3.1, we have

$$\begin{aligned}
\mathbb{E}[I] &= a_0+a_1e+\left(e-\frac{\gamma\sigma_\varepsilon^2}{\lambda}(1-\delta)\right)a_2, \\
\text{Var}[I] &= a_1^2(\sigma_\eta^2+\sigma_\varepsilon^2)+a_2^2\alpha^2\left(\sigma_\eta^2+\frac{\gamma^2\sigma_\varepsilon^4\sigma_z^2}{\lambda^2}\right)+2a_1a_2\alpha\sigma_\eta^2.
\end{aligned}$$

The first-order condition for the manager's incentive compatibility implies

$$\hat{e} = \frac{a_1+a_2}{k}.$$

Since the individual rationality condition for the manger holds as equality in equilibrium, we have

$$\mathbb{E}[v-I] = \frac{a_1+a_2}{k}-\frac{\gamma_m}{2}\left[a_1^2(\sigma_\eta^2+\sigma_\varepsilon^2)+a_2^2\alpha^2\left(\sigma_\eta^2+\frac{\gamma^2\sigma_\varepsilon^4\sigma_z^2}{\lambda^2}\right)+2a_1a_2\alpha\sigma_\eta^2\right]-\frac{(a_1+a_2)^2}{2k}.$$

The first-order conditions for the owner are given by

$$\begin{aligned}
\frac{1-(1+k\gamma_m(\sigma_\eta^2+\sigma_\varepsilon^2))a_1-(1+k\alpha\gamma_m\sigma_\eta^2)a_2}{k} &= 0, \\
\frac{1-a_1-a_2}{k}-\alpha\gamma_m\left(a_1\sigma_\eta^2+a_2\alpha\left(\sigma_\eta^2+\frac{\gamma^2\sigma_\varepsilon^4\sigma_z^2}{\lambda^2}\right)\right) &= 0,
\end{aligned}$$

which implies

$$\begin{aligned}
a_1 &= \frac{(\lambda\sigma_\eta^2+\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2+\gamma^2\sigma_\varepsilon^4\sigma_z^2)\gamma^2\sigma_\varepsilon^2\sigma_z^2}{x} > 0, \\
a_2 &= \frac{\lambda^2\sigma_\eta^2+\lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2+\gamma^2\sigma_\varepsilon^4\sigma_z^2}{x} > 0.
\end{aligned}$$

From the individual rationality condition, we obtain

$$a_0 = -\frac{y + 2(1-\delta)\left((1-\alpha)\sigma_\eta^2 + \sigma_\varepsilon^2\right)kx\lambda\gamma\sigma_\varepsilon^2}{2kx^2}.$$

We also have $\lim_{\lambda\to 0} a_0 = A_0$, $\lim_{\lambda\to 0} a_1 = A_1$, and $\lim_{\lambda\to 0} a_2 = A_2$. ■

PROOF OF PROPOSITION 4.2

1. Note that

$$\begin{aligned} \bar{e} - \hat{e} &= \frac{1}{k}\left(a_1 + a_2 - a_1'\right) \\ &= \frac{\left(\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)\gamma_m\sigma_\varepsilon^2}{\left(1 + k\gamma_m\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)\right)x} > 0, \end{aligned}$$

Thus the manger do more effort when the stock price is incorporated into the managerial contract than otherwise. Since the fundamental θ of the firm increases in the managerial effort, incorporating the stock price increases θ.

2. In the benchmark case, expected managerial compensation is given by

$$\mathbb{E}\left[I'\right] = \frac{1}{2k\left(1 + k\gamma_m\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)\right)}.$$

If we incorporate the stock price into the contract, we obtain

$$\mathbb{E}[I] = \frac{\lambda^2\sigma_\eta^2 + \left(2\lambda\sigma_\eta^2 + \sigma_\varepsilon^2 + \gamma^2\sigma_\varepsilon^2\sigma_z^2\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)\right)\gamma^2\sigma_\varepsilon^2\sigma_z^2}{2k\left[\left(1 + k\gamma_m\sigma_\eta^2\right)\lambda^2\sigma_\eta^2 + \left(2(1 + k\gamma_m\sigma_\eta^2)\lambda\sigma_\eta^2 + (1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))(\sigma_\eta^2 + \sigma_\varepsilon^2)\gamma^2\sigma_\varepsilon^2\sigma_z^2 + (1 + 2k\lambda\gamma_m\sigma_\eta^2)\sigma_\varepsilon^2\right)\gamma^2\sigma_\varepsilon^2\sigma_z^2\right]}.$$

Since

$$\frac{\partial\mathbb{E}[I]}{\partial\lambda} = -\frac{(1-\lambda)\gamma_m\gamma^2\sigma_\eta^2\sigma_\varepsilon^4\sigma_z^2\left(\lambda\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^2\sigma_z^2(\sigma_\eta^2 + \sigma_\varepsilon^2)\right)}{\left[\left(1 + k\gamma_m\sigma_\eta^2\right)\lambda^2\sigma_\eta^2 + \left(2(1 + k\gamma_m\sigma_\eta^2)\lambda\sigma_\eta^2 + (1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))(\sigma_\eta^2 + \sigma_\varepsilon^2)\gamma^2\sigma_\varepsilon^2\sigma_z^2 + (1 + 2k\lambda\gamma_m\sigma_\eta^2)\sigma_\varepsilon^2\right)\gamma^2\sigma_\varepsilon^2\sigma_z^2\right]^2} < 0,$$

expected managerial compensation $\mathbb{E}[I]$ has the minimum value

$$\frac{1 + \gamma^2\sigma_\varepsilon^2\sigma_z^2}{2k\left[1 + (1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))\gamma^2\sigma_\varepsilon^2\sigma_z^2 + k\gamma_m\sigma_\eta^2\right]}$$

at $\lambda = 1$. Since

$$\begin{aligned} &\frac{1 + \gamma^2\sigma_\varepsilon^2\sigma_z^2}{2k\left[1 + (1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))\gamma^2\sigma_\varepsilon^2\sigma_z^2 + k\gamma_m\sigma_\eta^2\right]} - \frac{1}{2k\left(1 + k\gamma_m\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)\right)} \\ = \; &\frac{\gamma_m\sigma_\varepsilon^2}{2\left(1 + k\gamma_m\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)\right)\left[1 + (1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))\gamma^2\sigma_\varepsilon^2\sigma_z^2 + k\gamma_m\sigma_\eta^2\right]} > 0, \end{aligned}$$

we have $\mathbb{E}[I] > \mathbb{E}[I']$.

3. In the benchmark case, the expected utility of the inside owners is given by

$$W_I' \equiv \mathbb{E}\left[v - I'\right] = \frac{1}{2k\left(1 + k\gamma_m\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)\right)}.$$

In our model, their expected utility is given by

$$W_I \equiv \mathbb{E}[v - I] = \frac{\lambda^2\sigma_\eta^2 + \left(2\lambda\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^2\sigma_z^2\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right) + \sigma_\varepsilon^2\right)\gamma^2\sigma_\varepsilon^2\sigma_z^2}{2k\left[\left(1 + k\gamma_m\sigma_\eta^2\right)\lambda^2\sigma_\eta^2 + \left(2(1 + k\gamma_m\sigma_\eta^2)\lambda\sigma_\eta^2 + (1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))(\sigma_\eta^2 + \sigma_\varepsilon^2)\gamma^2\sigma_\varepsilon^2\sigma_z^2 + (1 + 2k\lambda\gamma_m\sigma_\eta^2)\sigma_\varepsilon^2\right)\gamma^2\sigma_\varepsilon^2\sigma_z^2\right]}.$$

Thus we have

$$\begin{aligned} & W_I' - W_I \\ = \;& \frac{k\gamma_m\sigma_\varepsilon^2\left(\lambda^2\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^2\sigma_z^2\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)\right)}{\left(1 + k\gamma_m\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)\right)\left[\left(1 + k\gamma_m\sigma_\eta^2\right)\lambda^2\sigma_\eta^2 + \left(2(1 + k\gamma_m\sigma_\eta^2)\lambda\sigma_\eta^2 + (1 + k\gamma_m(\sigma_\eta^2 + \sigma_\varepsilon^2))(\sigma_\eta^2 + \sigma_\varepsilon^2)\gamma^2\sigma_\varepsilon^2\sigma_z^2 + (1 + 2k\lambda\gamma_m\sigma_\eta^2)\sigma_\varepsilon^2\right)\gamma^2\sigma \cdots} \\ > \;& 0. \end{aligned}$$

Proof of Proposition 4.3 Suppose that (3.7) and (3.12) hold. If (3.12) holds, ξ decreases in λ because

$$\frac{\partial \xi}{\partial \lambda} = \frac{\left[\left((1 - 2\lambda)\sigma_\varepsilon^2 - 2\lambda\sigma_\eta^2 - (\sigma_\eta^2 + \sigma_\varepsilon^2)\gamma^2\sigma_\varepsilon^2\sigma_z^2\right)\gamma^2\sigma_\varepsilon^2\sigma_z^2 - \lambda^2\sigma_\eta^2\right]\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2}{\left(\lambda^2\sigma_\eta^2 + \lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)^2} < 0.$$

Since λ decreases in c from the first claim of Proposition 3.3, ξ increases in c. ■

Proof of Proposition 4.4 Suppose that (3.7) holds. As shown in Section 3, ζ_z in (3.9) is fixed in equilibria while σ_z^2 changes. Then we have

$$\frac{d\xi}{d\sigma_z^2} = \frac{\partial \xi}{\partial \sigma_z^2} + \frac{\partial \xi}{\partial \lambda}\frac{\partial \lambda}{\partial \sigma_z^2} \qquad \text{(A.1)}$$

where

$$\begin{aligned} \frac{\partial \xi}{\partial \sigma_z^2} &= \frac{\left[\lambda^3\sigma_\eta^4 + \left\{2\lambda^2\sigma_\eta^2 + (\lambda\sigma_\eta^2 + \sigma_\varepsilon^2)\gamma^2\sigma_\varepsilon^2\sigma_z^2\right\}(\sigma_\eta^2 + \sigma_\varepsilon^2)\gamma^2\sigma_\varepsilon^2\sigma_z^2\right]\gamma^2\sigma_\varepsilon^2}{\left(\lambda^2\sigma_\eta^2 + \lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)^2} \\ \frac{\partial \xi}{\partial \lambda}\frac{\partial \lambda}{\partial \sigma_z^2} &= \frac{\left[\left((1 - 2\lambda)\sigma_\varepsilon^2 - 2\lambda\sigma_\eta^2 - (\sigma_\eta^2 + \sigma_\varepsilon^2)\gamma^2\sigma_\varepsilon^2\sigma_z^2\right)\gamma^2\sigma_\varepsilon^2\sigma_z^2 - \lambda^2\sigma_\eta^2\right]\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2}{\left(\lambda^2\sigma_\eta^2 + \lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)^2}\frac{1}{2}\sqrt{\frac{\zeta_z}{\sigma_z^2}}. \end{aligned}$$

Plugging (3.9) into (A.1), we have

$$\frac{\partial \xi}{\partial \sigma_z^2} = \frac{\gamma^2\sigma_\varepsilon^2\left[\zeta_z^2\sigma_\eta^4 + \gamma^2\sigma_\varepsilon^2\left\{\zeta_z\sigma_\eta^2\left(2\sqrt{\zeta_z\sigma_z^2}(\sigma_\eta^2 + \sigma_\varepsilon^2) + \sigma_\varepsilon^2\right) + \gamma^2\sigma_\varepsilon^2(\sigma_\eta^2 + \sigma_\varepsilon^2)\left(\zeta_z\sigma_\eta^2\sigma_z^2 + 2\sigma_\varepsilon^2\sqrt{\zeta_z\sigma_z^2}\right)\right\}\right]}{2\sqrt{\zeta_z\sigma_z^2}\left(\zeta_z\sigma_\eta^2 + \gamma^2\sigma_\varepsilon^2\left(\sqrt{\zeta_z\sigma_z^2}\sigma_\eta^2 + \sigma_\varepsilon^2\right)\right)^2} > 0.$$

Therefore, ξ increases in σ_z^2. ■

PROOF OF PROPOSITION 4.5 Suppose that (3.7) and (3.12) hold. As shown in Section 3, ζ_η in (3.8) is fixed in equilibria while σ_η^2 changes. We can rewrite (3.8)

$$\lambda = \sqrt{\frac{\zeta_z \sigma_\eta^2 - \gamma^2 \sigma_\varepsilon^4 \sigma_z^2}{\sigma_\eta^2}}. \tag{A.2}$$

Note that $\sigma_\eta^2 - n\sigma_\varepsilon^2 > 0$. Then we have

$$\frac{d\xi}{d\sigma_\eta^2} = \frac{\partial \xi}{\partial \sigma_\eta^2} + \frac{\partial \xi}{\partial \lambda}\frac{\partial \lambda}{\partial \sigma_\eta^2} \tag{A.3}$$

where

$$\begin{aligned}
\frac{\partial \xi}{\partial \sigma_\eta^2} &= \frac{(1-\lambda)(\lambda + \gamma^2\sigma_\varepsilon^2\sigma_z^2)\gamma^4\sigma_\varepsilon^6\sigma_z^4}{\left(\lambda^2\sigma_\eta^2 + \lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)^2}, \\
\frac{\partial \xi}{\partial \lambda}\frac{\partial \lambda}{\partial \sigma_\eta^2} &= -\frac{\left[\lambda^2\sigma_\eta^2 + \left\{2\lambda\left(\sigma_\eta^2 + \sigma_\varepsilon^2\right) + \gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \left(\gamma^2\sigma_\varepsilon^2\sigma_z^2 - 1\right)\sigma_\varepsilon^2\right\}\gamma^2\sigma_\varepsilon^2\sigma_z^2\right]\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2}{\left(\lambda^2\sigma_\eta^2 + \lambda\gamma^2\sigma_\eta^2\sigma_\varepsilon^2\sigma_z^2 + \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)^2} \\
&\quad \times \frac{\gamma^2\sigma_\varepsilon^4\sigma_z^2}{2\sigma_\eta^2\sqrt{\sigma_\eta^2\left(\zeta_z\sigma_\eta^2 - \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)}} < 0.
\end{aligned}$$

Plugging (A.2) into (A.3), we have

$$\frac{\partial \xi}{\partial \sigma_\eta^2} = -\frac{\gamma^4\sigma_\varepsilon^6\sigma_z^4\left[\left(2\gamma^2\sigma_\varepsilon^2\sigma_z^2 - 1\right)\zeta_z\sigma_\eta^2 + \gamma^4\sigma_\varepsilon^4\sigma_z^4\left(\sigma_\eta^2 - \sigma_\varepsilon^2\right) + 2\zeta_z\sqrt{\sigma_\eta^2\left(\zeta_z\sigma_\eta^2 - \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)}\right]}{2\sigma_\eta^2\sqrt{\sigma_\eta^2\left(\zeta_z\sigma_\eta^2 - \gamma^2\sigma_\varepsilon^4\sigma_z^2\right)}\left(\zeta_z\sqrt{\sigma_\eta^2} + \gamma^2\sigma_\varepsilon^2\sigma_z^2\sqrt{\zeta_z\sigma_\eta^2 - \gamma^2\sigma_\varepsilon^4\sigma_z^2}\right)^2}.$$

Since $\sigma_\eta^2 > \sigma_\varepsilon^2$, we obtain $\partial\xi/\partial\sigma_\eta^2 < 0$. ■

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